Developing Problem-Solving Skills in Mathematics Teacher Education: Detecting Missing Ideas

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Abstract

The study sought to identify gaps in teachers' readiness to implement a problem-solving approach in mathematics. To achieve that, we analyzed the journals of in-service mathematics teachers enrolled in a graduate course at a Canadian university in 2022. The course focused on developing mathematical thinking through reading and practice. Participants wrote weekly journals describing their previous problem-solving experiences – both as students and teachers – and any ongoing changes resulting from the coursework. Nine teachers who perceived their past encounters with solving mathematical problems as positive were selected for the study. However, this favorable perception of problem-solving alone was insufficient for the confident and effective implementation of the approach in the classroom. We found that, prior to taking the course, the teachers were unaware of many ideas identified in research literature as fundamental and vital to the problem-solving process, let alone had the opportunity to test them in action. In this paper we discuss the six most common ideas cited by teachers in their journals and labeled as both new and useful. Our findings highlight a significant disconnect between research-based recommendations for mathematical problem-solving and the teachers' prior professional experiences. This deficiency in theoretical grounding may hinder pedagogical success.

Keywords: problem-solving, critical thinking, teacher education, mathematics

Introduction

For the purpose of our discussion, we define problem-solving as an activity in which the learner is required to find a solution to a question for which they know neither the answer nor an approach ahead of time, and thus the activity calls for some creative insight (Schoenfeld, 1992). Mathematical problem-solving involves logical reasoning, analytical skills, structured thinking and pattern recognition. These skills are not only crucial in mathematics but are also transferable to other areas of life and work (Wang et al., 2025). While mathematical problem-solving can be challenging for some students, it presents opportunities for developing perseverance, building confidence through overcoming difficulties, and fostering a growth mindset (Dinapoli & Miller, 2022). Therefore, numerous national curricula advocate for problem-solving as a teaching methodology (Stacey, 2005). For instance, the National Council of Teachers of Mathematics (NCTM, 1980, 2000) identifies problem-solving as an integral part of mathematics learning and suggests that problem-solving must be the focus of school mathematics. Similarly, the Programme for International Student Assessment (PISA) aims at identifying and promoting the ability of an individual to understand and solve problem situations, including “the willingness to engage in such situations to achieve one's potential as a constructive and reflective citizen” (OECD, 2012, p. 7).

Young students may enjoy solving problems and puzzles related to mathematics due to children’s natural curiosity and sense of accomplishment when solving simple problems. However, oftentimes this attitude changes as students grow and the volume, depth and abstractness of material significantly increase (Dowker et al., 2019). Thus, it is important to explore what can be done to address this phenomenon, in particular, from the perspective of teacher education and professional development. Specifically, the following question is of interest (Lester, 2013): What knowledge and experiences could be useful for teachers to have in order to help their students in learning mathematics through problem-solving activities?

To effectively guide students, teachers need mathematical contest knowledge that include a strong understanding of mathematical concepts and procedures. Teachers should be familiar with various problem-solving strategies and heuristics, such as making diagrams, looking for patterns, and working backwards (Piñeiro et al., 2022). As well, teachers should demonstrate problem-solving processes such as representing, analyzing, interpreting and communicating (Foster et al., 2014). Chapman (2015) observed that mathematics problem-solving knowledge for teaching is a complex network of interdependent knowledge. In addition to the problem-solving content knowledge, teachers benefit from knowledge of students as problem solvers, instructional practices for problem-solving and the role of affective factors and beliefs on students’ success with solving problems.

Despite many resent advances in research about problem-solving, there still a need for research about teaching through problem-solving (Weber & Leikin, 2016). In this paper we aim to detect some missing elements in problem-solving knowledge of practicing teachers. We considered a group of teachers who perceived themselves as capable problem solvers, had positive experiences with this way of learning mathematics and desired to implement problem-solving activities in their classes. At the same time, by enrolling in the course, these teachers sought to build confidence and enhance their effectiveness when implementing classroom problem-solving strategies. We were interested to find out to what extent these teachers were familiar with the most fundamental practices and ideas about problem-solving identified in the mathematics education research literature.

This paper is organised as follows: in section 2 we provide a brief summary of contemporary research on problem-solving. In section 3 we describe the context of data collection, methodology and the research question. In section 4 we present our findings. The last section contains our conclusions.

Literature Review on Problem-Solving

Research literature on problem-solving in mathematics reflects the epistemological, cognitive, metacognitive, affective, and pedagogical aspects of the problem-solving activity (Felmer et al., 2016; Liljedahl et al., 2016; Mason et al., 1982/2010; Schoenfeld, 1985, 1992; Silver 1985). In this section we briefly outline these aspects and main characteristics of this process.

A task that could be done by a familiar procedure (e.g. solving a quadratic equation using the formula for its roots) is an exercise (Schoenfeld, 1992). Solving many exercises in order to memorise certain procedures may be referred to as training. Indeed, extensive practice with given methods is useful for the learning about mathematical tools, but it is radically different from solving non-routine problems, because in the latter activity a method of solving needs to be found by the learner. More precisely, according to the heuristics proposed by Polya (1945/2004), the learner needs to understand the problem and develop a plan of their approach. Next, they implement the plan finding an answer, and finally, they check and elaborate on their solution. The experience of planning and consequently, of making one's own choices and inventions during the act of problem-solving helps students construct their own understandings out of many facts, methods, ideas, and other information available to them. This is consistent with the view on mathematics learning as a constructive rather than absorptive activity.

It has been noted that problem-solving at various levels relies upon mathematical creativity, the process which consists of four stages: initiation, incubation, illumination and verification (Hadamard, 1945). The initiation phase consists of deliberate and conscious work that includes understanding of and engagement with the problem along with recollection of past experience and study of related mathematical facts and tools. It creates the tension of unresolved effort when the solver is stuck and seemingly has no progress. The incubation stage signifies the continuation of work on the problem at the subconscious level. A rapid coming to mind of an idea how to proceed in order to attack the problem is referred to as the illumination stage or the AHA moment. If it happens, it usually results from the bridging of previous subconscious and conscious work on the problem. "These sudden inspirations never happen except after some days of voluntary effort which has appeared absolutely fruitless and whence nothing good seems to have come" (Poincaré, 1952, p. 56).

The correctness of the emerged idea is examined by the solver at the verification stage. This could include technical implementation of the idea as well as making judgement about the efficiency and possible modifications of the found approach. Problem solvers benefit from writing careful protocols of the work progress, especially at the initiation and verification stages.

The nature of mathematical knowledge is seen through scholars' participation in mathematical practices. This intertwines with growing understanding of mathematics as an empirical science that involves observation, experimentation and conjecturing besides calculations and deductions. In addition, mathematics is broadly viewed as a tool for searching for hidden patterns that helps to understand the world around us.

Cognitive approaches are concerned with mathematical content, specifically, with the roles of structural organization, mental representation, access to and processing of information in problem-solving. Metacognitive research highlights self-regulatory procedures, such as monitoring and decision making, and individual's abilities to control their cognitive processes. Students' beliefs about mathematics and self are also found to have an influence on the problem-solving outcomes. Problem-solving could be an emotional enterprise where students take risks and experience both disappointments and rewards. Developing affective and intellectual commitments to the solution helps students to stay on task and to display long term persistence in tackling problems.

It is proposed that students' mathematical dispositions, habits, viewpoints, and interpretation or meaning construction skills develop largely through the processes of socialization and enculturation rather than direct instruction (Resnick, 1988). Doing mathematics is increasingly seen as a social and collaborative act. This has pedagogical implications because the teaching of mathematics is viewed as giving students an opportunity to participate in a community of practice where they are not only familiarized with mathematical tools and trained with appropriate skills, but also form their mathematical dispositions and views. In any case, besides teachers' beliefs about mathematics and about capabilities of their students, the degree of their understanding of what constitutes the problem-solving process is an important factor in implementing this approach in schools. Below we will present some evidence of the formation of such understanding through teachers' participation in a graduate course on problem-solving.

Methodology

The data was collected in a graduate on-line course offered at Memorial University (Canada) for in-service K-12 teachers in spring of 2022. This 12-weeks course focused on mathematical thinking and problem-solving. During the course teachers participated in several types of activities consisting of individual problem-solving, discussion of problems in small groups, reading research-informed and practice-oriented articles about problem-solving, and writing a journal about their experiences and possible changes. More information about this course as well as examples of mathematical problems with teachers' collaborative inquiries could be found in Kondratieva (2020). All participants had at least 3 years of teaching experience. We selected only those teachers who reported having positive encounters with problem-solving in the past and later desired to implement this approach in their classes. Nine journals were analysed and reoccurring themes were identified. The research question was: What attitudes and ideas acquired by the teachers during the course were perceived by them as new and useful for conducting problem-solving activities in their classes?

Specifically, it was examined in which ways the teachers felt the influence of the ideas learned from the suggested reading on their ability to implement the problem-solving approach in their teaching practice.

Thematic analysis of students' journals employed in this research offers a method to identify common characteristics and attributes in the way people talk or write about a topic and thus allows the researcher to notice a shared experience of a group of people (Braun & Clarke, 2012). This approach allowed to obtain patterns in a set of data and to single out the topics and themes emerging in teachers' descriptions of new and useful ideas related to the problem-solving process as discussed in the literature that they read. The analysis and coding of the data was done by identifying words and phrases aligned with the ideas mentioned in the Literature Review section. The nine journals gave a good representation of teachers' answers as we found no additional common themes when adding more journals into the consideration. Each of the six identified ideas was discusses in at least five of the nine teachers' journals. The results are summarized and discussed in the next section.

Results

We followed the journals of teachers (hereafter referred to as T1, T2, etc.) who claimed their strong preference for the problem-solving approach from the beginning of the course. We identified them using their personal statements, for example, "As a student I really enjoyed problem-solving in mathematics" (T1) or "As a teacher of mathematics, I try to incorporate problem-solving into my practice" (T2) or "As the course progressed my love for challenging problems grew stronger" (T3). The main reading references for the course were Mason et al (1982/2010) and Polya (1945/2004). In this section we abbreviate them as "MBS" and "Polya" respectively and present quotations from these sources used by the teachers in their journals.

The following ideas and quotations were the most frequently noted and elaborated.

Idea 1: It Is Useful to Record Experiences While Solving Problems.

Teachers noticed the following advice, which several of them quoted in their journals:

> You should start recording your experiences so that they are not lost, but can be analysed and studied later. Recording your experiences will also help you to notice them and this contributes to developing your mathematical thinking. Aim to record three things: all the significant ideas that occur to you as you search for a resolution to a question; what you are trying to do; your feelings about it. (MBS, pp. 9–10)

Since one of the components of the course was actual problem-solving practice, the teachers had an opportunity to implement this idea right away. When recording their solution in progress, teachers started to use RUBRIC to pay attention to the following four moments: "The four key words that I suggest you begin using in your notes and in your thinking are STUCK!, AHA!, CHECK and REFLECT" (MBS, p. 16).

Most teachers explicitly stated that recording experiences "is an important step of the problem-solving process that I've never used before" (T4). In the following episode a teacher (T5) explains a typical situation when students record only computations:

> [..] they did not record their thinking/reasoning and how they got where they were. Thus, at the next class when they revisited their work, many could not remember how they arrived at the place they were in the problem. At the time, it did not occur to me to have the students record their reasoning and thinking in writing. I will be implementing the RUBRIC writing process with my new class in September and encouraging my colleagues to do the same.

From this and similar episodes found in teachers' journals it is evident that the strategy of recording own experience while solving a problem was unknown for the teachers and at the same time, they see a potential in the employment of this strategy in their classes.

Idea 2: It Is Useful to Check, Reflect On and Justify One's Own Answer.

For most teachers the idea of checking a solution was not new, but it was reduced to a numerical verification. Teachers also stated that working out examples and finding patterns was their main focus while the following practice was not common in their classrooms:

> Examples are, of course, not enough by themselves. They may convince your friend that your statements are plausible, but you must justify every step of your argument… You have to state the structural links which indicate why your conjecture is valid. (MBS, p. 87)

Admitting that "I've never really considered the why or how something works" (T4), several teachers quoted the following passage:

> The important thing to remember is that a conjecture is an informed guess about a possible pattern or regularity which might explain WHAT is puzzling in a particular question. Once formulated, the conjecture is investigated to see whether it must be modified or whether it can be convincingly justified. This is done by seeking WHY. The nature of I WANT changes, from trying to articulate what is true, to trying to see why the conjecture might be justified, that is, from seeking WHAT to seeking WHY. The answer to WHY is a structure which links what you KNOW to what you have conjectured. Your argument will be an exposition of that link. (MBS, p. 82).

Teachers related to the idea that "reflection is probably the most important activity to carry out" (MBS, p. 105). T5 continued in their journal, "We do not do this enough with students. Providing students with time to reflection on their own thinking helps students deepen their understanding."

We observe that refocusing on the WHY aspects of the solutions during the verification process, was identified by teachers as a novel and useful suggestion.

Idea 3: Getting Stuck Is Normal in Problem-Solving.

Many teachers' journals revealed that they usually would be very frustrated if they could not solve a problem because they associated "stuck" with a failure. In words of T6,

> this idea of failure can lead to a negative relationship with mathematics, where perhaps you don't want to attempt 'hard' problems because you fear you will fail. In this respect, 'intending or demanding to get the answer, and, worse, more elegantly and quicker than anyone else, is asking for failure eventually.' (MBS, p. 131)

However, teachers noted the following advice regarding the attitude, emotional and cognitive behavior:

> Being STUCK! is a healthy state, because you can learn from it [...] If you are STUCK!, […] Specializing is your best friend, (Mason, p. 56) […] then write down briefly what your idea is. If what you are going to try feels good, why not write AHA!? It makes you feel even better! (MBS, p. 57)

Teachers notice the importance of meta-cognitive skills allowing the solver to navigate their emotions during problem-solving. The recognition of being stuck and move forward trying new things signifies a shift from the attitude “I do not remember a formula” towards more adaptive reasoning and productive disposition. Teachers also noted the proposition “If still stuck, the best advice is to distil the question down to a form which can be held in the mind and mulled over” (MBS, p. 103), which is naturally connected to the next idea commonly identified in the journals.

Idea 4: Take Your Time to Think About a Challenging Problem.

Teachers started to appreciate the idea that solving a problem may take some longer time than usually is given in class for that activity. In words of T2, “I learned that it is okay to walk away from a problem and take a day (or many days!) to reflect on the problem.” Similarly, T1 wrote “The thing that resonated with me was that, ‘despite appearances, your thinking may still be going on, but below the level of awareness’” (MBS, p. 96).

Teacher T5 also suggested “we need to discuss, embrace, and teach that solving a challenging problem is usually ‘full of false steps, partial or mistaken insights, and groping toward articulating your sense of what is true’” (MBS, p. 87).

Of course, the literature on problem-solving does not give a universal rule of how to complete a solution of any problem, but it discusses how to deal with a stuck situation and make at least some progress. Taking more time to think about a problem and related questions was recognized as a new and beneficial approach in teachers’ journals.

Idea 5: Teacher Should Ask Guiding Questions in a Problem-Solving Class.

Several teachers reported that they learned the importance of not helping students too much with reference to the following rationale:

> If the teacher helps too much, nothing is left to the student. The teacher should help, but not too much and not too little, so that the student shall have a reasonable share of the work […] One way to do this is to ask the student WHAT they are trying to look for/solve, instead of telling them […] (Polya, p. 1)

Teachers noted the importance of selecting and introducing a problem for their students:

> It is sad to work for an end that you do not desire. Such foolish and sad things often happen, in and out of school, but the teacher should try to prevent them from happening in his class. […] If the student is lacking in understanding or in interest, it is not always his fault; the problem should be well chosen, not too difficult and not too easy, natural and interesting, and some time should be allowed for natural and interesting presentation. (Polya, p. 6)

Teachers realised the power of guiding questions instead of providing direct hints. As T9 summarized,

> MBS explain that being given hints in mathematics lend to blocking the opportunity for you to think. Rather than being given a hint, when stuck on a problem, or when you feel

> you're stuck but are not, it would help if you had someone ask you a question; this gets you going again.

From teachers' journals we see that they began to reconceptualize their role in the classroom when implementing problem-solving approaches.

Idea 6: There Are Benefits for Learners When Problem-Solving Is Done in Groups.

Teachers knew about this pedagogical approach but they seem not knowing how the group problem-solving works best. They noted several suggestions in their reading, such as "Writing out what is known and explaining it to a friend often releases a blockage [when you are stuck]" (MBS, p. 103).

Also, working in a group is helpful when one is trying to test their ideas and convince others about a solution's correctness. As T1 wrote, "I want to encourage [my students] to challenge their own conjecture and those of other classmates in a respectful way."

Teachers also learned from their own experience of solving problems in groups. As T2 stated, "there were times when I did overlook options [in my solution], but there were times when I didn't know they existed until they were presented to me by someone else."

Building on the newly learned concepts – such as being stuck, conjecturing, verifying and reflecting – teachers saw the benefits of group work in a new light. They began viewing working in groups as an opportunity to articulate doubts, test ideas, and compare various solutions.

Conclusion

Research-based and practice-oriented literature on problem-solving offers many important findings for teachers and students of mathematics. In this paper we consider a group of teachers who initially identified themselves as persons who had positive encounters with solving mathematical problems and who attempted to implement problem-solving approaches in their classes. During a graduate course on problem-solving, these teachers were exposed to the ideas fundamental for the success of the problem-solving process and the development of mathematical thinking with this approach. Our study demonstrates that teachers discovered many new facts about problem-solving from reading research-based literature along with personal problem-solving practice. In particular, their journals clearly show that many of them were not aware of such ideas as (1) the usefulness of proper documentation of the problem-solving process; (2) the roles of review, explanation and reflection on one's own solution; (3) strategies for navigating "stuck" situation; (4) the importance of continuous thinking about a challenging question; (5) the effects of the amount and type of help provided to students; and (6) effective methods for discussing problems in groups.

Our results confirm that teachers oftentimes remain unaware of the practice-oriented literature, let alone research findings related to problem-solving in mathematics. Given that teachers play the pivotal role in educating their students, the identified unawareness may not only slow down the improvement of students' problem-solving skills but also undermine development of their mathematical proficiency, i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al., 2001). Therefore, dissemination of research recommendations remains an important missing part in teacher education. The dissemination can occur by means of graduate programmes offered for in-service teachers. In addition, professional development should provide ongoing support and resources for teachers to improve their problem-solving instruction. Addressing the teachers' unawareness is necessary because even a positive personal history with solving problems, good intentions, and general teaching qualifications may not be sufficient for a teacher to confidently implement a problem-solving approach in the classrooms. By making the process of solving mathematical problems adequately present in the classroom instruction, teachers can help their students develop robust problem-solving skills and mathematical proficiency needed for success in school and beyond.

Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The author declares that no AI or AI-assisted technologies have been used to generate, refine, or correct the content in the manuscript. The ideas, design, procedures, findings, analyses, and discussion are originally written and derived from careful and systematic conduct of the research.

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