CRA-Based Learning in the 21st Century: Empowering Learners Through Inquiry and Conceptual Reasoning in *Mathematics in the Modern World*

Aileen A. Diansuy, Antipolo Institute of Technology, Philippines

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Abstract

In traditional mathematics education, many students are trained to manipulate symbols without being able to relate these manipulations to actual mathematical concepts in life; this is often a result of the disconnection between theorem application and the real world. The study explored how implementing the CRA (Concrete–Representational–Abstract) instructional framework applied to the *Mathematics in the Modern World* (MMW) course at the Antipolo Institute of Technology (AiTECH) in the Philippines assisted students in bridging the gap between active learning and using mathematics in a real-world context. Through a qualitative inquiry method, undergraduate students completed four experiential tasks: measuring the architecture of campus buildings for the Golden Ratio; documenting examples of natural Fibonacci sequences; modelling the Four-Color Theorem onto a map of Rizal Province, Philippines; and modelling weighted graphs for the purpose of optimizing transportation within their communities. Information from instructor narratives and digital portfolios of the students showed evidence of successful transitions from conceptual reasoning to abstract. Grounded in physical/visual representation of mathematical objects, students found that they could transition more easily to symbolic representations of mathematics, particularly in the case of complex modelling tasks. Additionally, the inclusion of the digital documentation promoted ethical authorship and community engagement by supporting the students' use of mathematical inquiry within their local communities. This study concludes that using the CRA model for contextualized mathematics instruction fosters cognitive trust, digital fluency, and a framework for scalable mathematics education in diverse institutional settings.

Keywords: CRA framework, Mathematics in the Modern World, inquiry-based learning, ethical authorship, graph theory, contextual mathematics

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Introduction

Mathematics in the Modern World (MMW) is the only mathematics general education subject intended within the Philippine K–12 curriculum. As stipulated by the Commission on Higher Education (CHED), MMW is the major vehicle for the development of mathematical literacy, with focus on applications to everyday life, critical thinking, and interdisciplinary connections. Unlike traditional mathematics themes which emphasize fluency and symbol manipulation, MMW will cause students to notice patterns, structures, and relationships within and across nature, society, and technology.

Modern pedagogy prescribes deeper conceptual engagement, situational context and a degree of student agency—especially in a subject area aimed at providing multiple pathways. The Concrete–Representational–Abstract (CRA) framework provides a scaffold for students' transition from concrete experience to symbolic abstraction, in cognitive as well as affective learning. The following paper shares four inquiry-based tasks undertaken in a college classroom in the Philippines. Each task was designed with the mathematical goal of linking abstract concepts to real-world applications while supporting ethical, creative and independent learning. In each activity, the aim of the math involved was to connect abstract ideas with practical applications. The activities were not just teaching tools; rather, they helped the students develop their abilities to question themselves and understand how they can represent the world using mathematics.

Methodology

The study took place in one class of the MMW subject offered by the Antipolo Institute of Technology for the Second Semester of the 2024–2025 academic year. In this class, the mode of instruction deviated from the conventional didactic method in favor of the inquiry method. The process occurred through an interactive learning experience where students formed small groups to work together using concrete objects from their surroundings to create meaning out of mathematical ideas.

The study came about through a pedagogy that acknowledges and appreciates students as contributors to their education. Students in undergraduate classes at the Antipolo Institute of Technology who used mathematical concepts with the help of the Concrete–Representational–Abstract (CRA) model while transitioning between the interior and exterior areas of the institution.

The task required documenting discoveries through digital media; they were required to reflect on their learnings through written documents and present them in class through presentations. The activities were consciously designed to not just deliver academic results, but also facilitate the promotion of ethical authoring, interdisciplinarity, and civic awareness. Incorporation of the concept of “Ethical Authorship” focuses on the mathematician's role as an honest reporter. Based on the International Center for Academic Integrity’s (ICAI, 2021) principles of honesty and responsibility, it instills a lesson that data integrity is an essential component of the discipline. Through a focused emphasis on establishing mathematics in real-life settings and exploration, the approach empowered students and made them realize the relevance of mathematics in their personal and civic lives.

The initial activity, Golden Ratio Exploration, initiated students to proportional understanding using empirical measurement. In the concrete stage, students toured the

campus setting to measure leaves, flowers, tiles, hollow blocks, and architectural details using measuring tapes and rulers. These hands-on experiences anchored mathematical exploration in environmental observation. During the representational phase, learners recorded their observations via photographs and sketches, marking measurements and comparing width to length ratios. Some examples were a tile with a ratio of 1.6, hollow blocks with 1.68, and motorcycle plate boards with 1.64—enabling learners to compare noted proportions with the Golden Ratio (around 1.618). The abstract stage further developed this question by inviting students to translate the mathematical and aesthetic meaning of their results, relating proportional relationships to more general ideas of harmony, balance, and repeating patterns.

Based on this, the second activity, Fibonacci Sequence Investigation, further developed students' interaction with mathematical patterns in nature and design. During the concrete phase, students noticed natural spirals and recurring patterns in flowers, leaf patterns, building features, and logos on campus. These visual experiences increased awareness of mathematical patterns present in their lives. The representational phase featured documenting all observations in sketches and photos, tallying the number of unexpected elements observed, and checking the total against the Fibonacci sequence familiar to groups of students (1, 2, 3, 5, 8, 13). In the abstract phase, students constructed Fibonacci sequences, working recursively to construct their results and comparing ratios of consecutive term, appreciating how ratios converge toward golden ratios, thereby solidifying their thinking of growth, proportion, and beauty.

To provide another context for mathematics reasoned, using risk as a framing, students were nudge toward productive abstract thinking through the third exercise, Map Coloring of Rizal Province, which pushes students from geographic annotation to graph-theoretic abstraction. Starting from the concrete phase, the students work with blank outline maps of Rizal Province, labeling all fourteen local government units, comprising Antipolo City and the thirteen municipalities of Rizal, such as Taytay, Binangonan, Cainta, Tanay, and others. The representational phase included the application of the Four Color Theorem such that neighboring regions do not share the same color, creating space for combinatorial thought and distributing color choices. In the abstract stage, learners converted their color maps into graph models, indicating each municipality as a vertex and depicting edges between neighboring regions. This enabled learners to investigate chromatic numbers and graph coloring concepts, linking mathematical modeling with spatial planning and civic engagement.

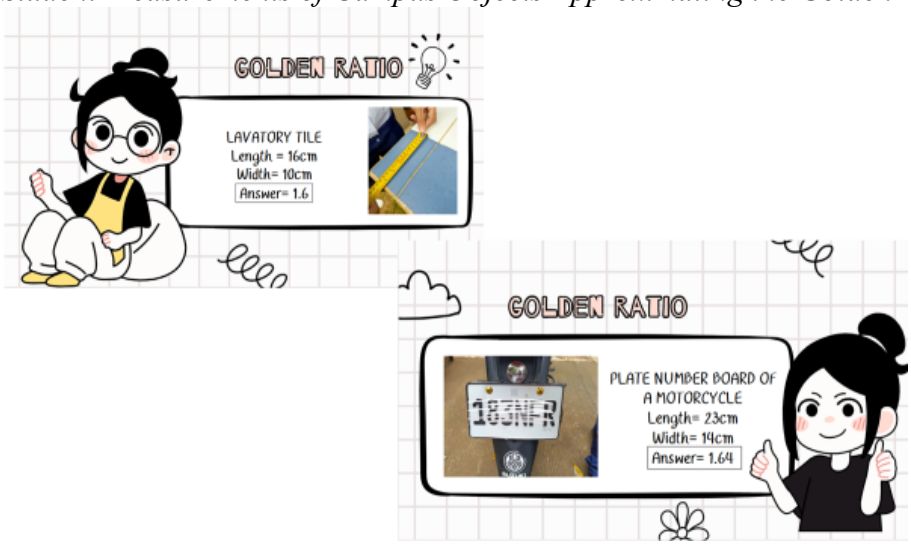
The last activity, Weighted Graph of Five Municipalities, integrated geographic observation with optimization and decision-making. During the concrete stage, students chose five municipalities in Rizal Province and determined direct road links between them. This set the task in everyday geography and prompted thinking about routes of travel and ease of access. The representational stage consisted of sketching graphs with municipalities as vertices and roads as edges with approximate distance labels. Students tinkered with circular arrangements and map-like settings, exercising imagination and spatial sense. During the abstract stage, students interpreted their graphs with formal graph theory concepts, investigating shortest paths, building spanning trees, and critiquing optimization models for travel efficiency.

In essence, the four practices have provided a consistent and prolonged methodological approach integrating observation from the real-world environment, documentation in images,

and symbolic representation. By consistently applying the CRA methodology in different situations, including environmental measurements, natural occurrences, civic geography, and transportation networks, the students developed proportional reasoning and recursive reasoning, spatial representations, and symbolic representations. The methodology further helped develop ethics in authorship, learner independence, and contextual relevance, thereby affirming that mathematics is indeed a living methodology of inquiry, interpretation, and engagement.

Figure 1

Student Measurements of Campus Objects Approximating the Golden Ratio



Note. Students documented lavatory tiles and motorcycle plates to verify ratios close to 1.618.

Results

Across all four CRA-based activities, students exhibited consistent patterns of engagement, mathematical understanding, and ethical authoring. The Golden Ratio activity encouraged students to observe nature and the built environment more deeply. Students were able to recognize, measure, and calculate ratios for objects such as leaves, doors, or design techniques we encountered in the built world that were “close” to the Golden Ratio ($\varphi \approx 1.618$). The activity also extended students' understanding of irrational numbers and concepts of beauty and introduced students to new ways of thinking across disciplines through connections made between mathematics and art or design.

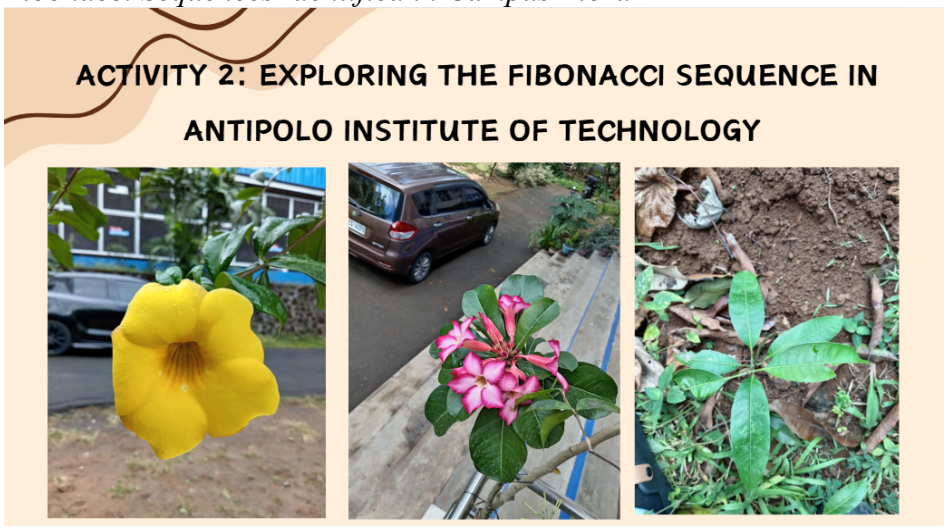
In the Fibonacci sequence activity, the students noted some recursions in their surroundings in the form of flower petal counts as well as spiral formations in the buildings. This was recorded using photos and drawings, and most of the students could show how these numbers exist in nature. It created an interest among students on the mathematical principles behind growth in nature, while reinforcing their understanding of the Fibonacci sequence and Golden Ratio.

The map coloring activity engaged students with graph theory from a common geographic discipline, Rizal Province. By labeling municipalities and applying the Four-Color Theorem, students engaged in spatial reasoning and combinatorial logic. They translated their colored maps into graph representations, correctly identifying adjacent regions and constructing vertex-edge models. This task enhanced their understanding of chromatic numbers and abstraction, while also promoting civic awareness of local geography.

Lastly, the weighted graph activity allowed students to model real-world connectivity by selecting five municipalities in Rizal and assigning edge weights based on approximate road distances. Students chose between circular and map-like layouts to draw their graphs, demonstrating both creativity and precision. They interpreted the graphs to identify shortest paths and potential optimization strategies, applying graph theory to simulate logistical decision-making. This exploration illuminated their strengths in identifying parameters, analyzing networks, and reporting findings clearly and meaningfully. Altogether, these student outputs and reflections suggest that CRA-based instruction - using contextual inquiry, integrated with digital tools - encourages mathematical understanding, learner independence, and ethical participation. Students were not only able to engage with abstract concepts, but they also developed the confidence to investigate, document, and communicate mathematics as an important part of a context to which they belong.

Figure 2

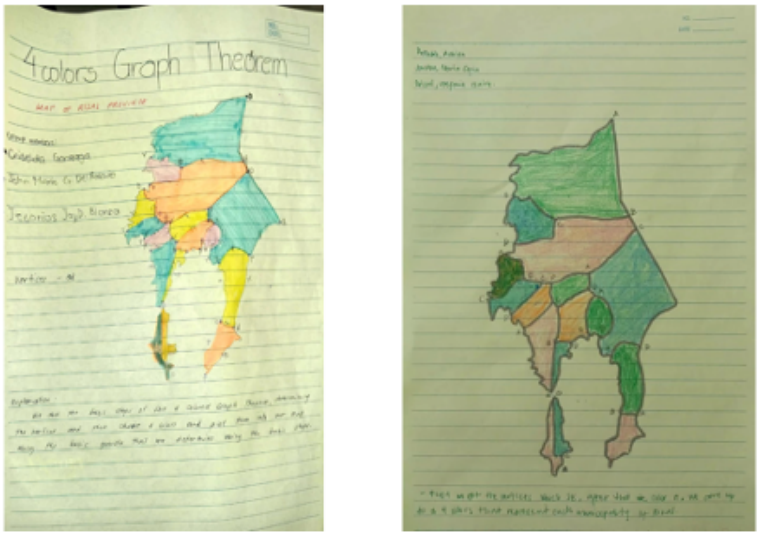
Fibonacci Sequences Identified in Campus Flora



Note. Documentation of spiral patterns and petal counts (e.g., 5-petal flowers) found at the Antipolo Institute of Technology.

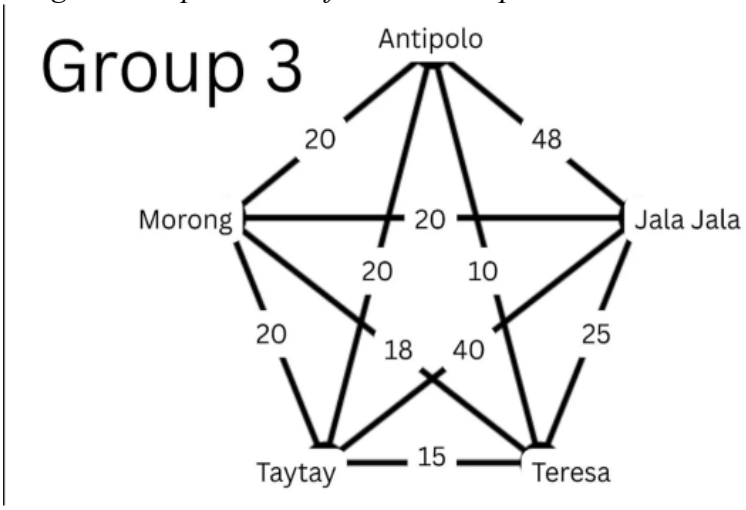
Figure 3

Application of the Four Color Theorem to Rizal Province



Note. Student-generated map demonstrating valid vertex coloring where no adjacent municipalities share the same color.

Figure 4
Weighted Graph Model of Five Municipalities



Note. Optimization model created by Group 3 showing edge weights based on approximate road distances between towns.

Discussion

The findings of this study illustrate the pedagogical value of the Concrete–Representational–Abstract (CRA) framework in facilitating conceptual knowledge, learner autonomy, and ethical engagement within Mathematics in the Modern World (MMW). As the only general education mathematics course in the Philippine K–12 curriculum, MMW provides a critical avenue for developing mathematical literacy across diverse academic tracks. The activities featured in this study demonstrate how CRA-based instruction can transform abstract mathematical ideas into meaningful experiences through contextual inquiry and experiential design.

Activity on Golden Ratio Exploration

Exploring the concept of the Golden Ratio involved students' engagement with proportionality using real-life materials within the school premises and the community at large. Students took measurements of hollow blocks (W = 32 cm; L = 19 cm; ratio ≈ 1.68), lavatory tiles (W = 10 cm; L = 16 cm; ratio ≈ 1.6), and motorcycle number plates (W = 14 cm; L = 23 cm; ratio ≈ 1.64). These activities enabled the learners to link the idea of proportional thinking with that of irrational numbers through taking pictures and making notes on the various instances observed. In effect, it is true according to Boaler (2016) that mathematics takes power through connections to the environment. Ethical authorship was also evident, as students responsibly attributed sources in their digital artifacts.

Activity on Fibonacci Sequence Investigation

The study of the Fibonacci sequence helped to reveal the way students engaged with recursive sequences in nature. The evidence gathered from the shape of flowers resembling trumpets, bunches of long petals, and the positioning of leaves showed the occurrence of Fibonacci sequences within nature's growth. The sketches and photos taken served as a representation for moving from concrete experience to abstract thought. It was clear to the students that these patterns ultimately led to the Golden Ratio. This activity validated

Toong's (2020) claim that CRA scaffolding bridges conceptual gaps, particularly when learners transition from representational to abstract reasoning.

Activity on Map Coloring of Rizal Province

Through the mapping activity, the students were able to learn graph theory from a civic and spatial perspective. The Four-Color Theorem was used by the learners to mark Rizal Province maps with colors such that adjacent municipalities would not have the same colors. This helped in developing spatial skills as well as mathematical combinatorics skills among the students. The maps were then transformed into graphs and the students were able to further learn about chromatic numbers. According to Marikyan (2023), visual techniques play an important role in teaching mathematics. Furthermore, Furner (2024) stressed on manipulative and digital resources as tools for gaining mathematical confidence. Similarly, Ramos-Miguel (2024) described how graph coloring is useful for scheduling.

Activity on Weighted Graph of Five Municipalities

Through the use of graphs, learners learned how to apply concepts like shortest path and spanning tree in finding optimal solutions. This activity provided learners with the ability to analyze data in an ethical manner and to make decisions based on real-life experiences. According to Beltran and Bartican (2025), adult learning principles must be adhered to when implementing andragogy, which stresses the learner's responsibility in the learning process. Bautista et al. (2015) also support the idea of using graphs to develop analytical skills. This is further proven by Zhang et al. (2024), who believe that simulations of life experiences promote problem-solving and creativity.

Integrative Insights

Throughout all activities, the students showed good documentation, digital literacy, and interpretation skills, which were indicative of good ethical writing and outcomes-based learning. The CRA model offered an effective yet flexible approach to inquiry, enabling students to participate emotionally and socially while making cognitive progress. Such observations indicate that mathematics learning through contextual inquiry is not only effective academically but also socially and personally.

Conclusion

The current research proves that the application of contextually relevant, inquiry-based CRA pedagogies can make the MMW course much more effective, engaging, and liberating for college students, helping them turn a merely formal educational obligation into something exciting and meaningful. Indeed, by involving students in mathematical inquiries that have something to do with their immediate environment, researchers managed to help students change from passive consumers of information into active explorers and creators of knowledge. As a result, students gained not just conceptual competencies, but a civic mindset as well.

Apart from the specific tasks performed by the students of the Antipolo Institute of Technology, the results have broader implications as a model for General Education mathematics instruction throughout the Philippines. In particular, the CRA method does not involve any expensive technology, but instead involves the deliberate development of

experiential tasks connecting theories with concrete experiences. This research therefore clearly demonstrates the ability of mathematics courses to fulfill the outcomes-based goals set out by the Commission on Higher Education (CHED).

In terms of future developments within the MMW curriculum, further formalization of mentorship systems, as well as the application of mathematical concepts in other disciplines, could further ensure continued benefits for the wider community. At the same time, this research has proven the ability of mathematics to reveal the harmony and beauty in real-life surroundings through its interpretation of the world.

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Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The author would like to emphasize that Duplichecker, a tool based on AI for writing, was used solely in detecting any potential plagiarism as well as grammatical and spelling mistakes. The author declares that apart from Duplichecker, any other AI tools were not used for the generation or corrections of any material contained in this paper. The ideas, design, procedures, findings, analyses, and discussion are originally written and derived from careful and systematic conduct of the research.

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Contact email: aileen.diansuy@aitech.edu.ph