Development and Validation of an APOS-Based Mathematics Mental Structure Scale

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Abstract

Action-Process-Object-Schema (APOS) theory emerges as an extension of constructivism, but it takes a more robust and targeted learner-centered approach to mathematics instruction. As a basic mathematics requirement, problem-solving skills and the development of mathematical mental structures are important. By aligning the scale's statements with the APOS theory, this study aims to bridge the gap between students' prior mathematics learning and their current educational needs. Therefore, given the critical need for a proper tool to measure the mathematical mental structure of students in the study's locale and mathematics education at large, this study provided a valuable scale. The scale development followed the process: (1) item generation, (2) item evaluation, (3) pilot-testing, (4) Exploratory Analysis, (5) final evaluation. Several studies and literature have been reviewed to generate the 62 items or statements for the scale. The items are evaluated, and only 54 of them are used for the draft scale for the pilot test. The scale was administered to 330 students for the pilot-test, and responses were screened and then subjected to Exploratory Factor Analysis (EFA). One factor was extracted. To establish internal consistency, the factor was correlated with other established constructs – motivation levels, with the scale being substantially correlated, and mathematics self-efficacy and anxiety, with the scale having predictive validity. Further tests were done, and the final scale consists of 40 items, which confirmed validity and reliability.

Keywords: mathematics proficiency, mental structure, problem-solving

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Introduction

Mathematical understanding is developed by a progressive structuring of mental structure, a hierarchical cognitive construction based on APOS Theory. Mental structures start with algorithmic, externally guided actions, which then undergo interiorization to become mentally coordinated, reversible processes; encapsulation then allows these processes to be viewed as objects that can be used for additional reasoning; and finally, a schema is created when actions, processes, and objects are logically connected into a single conceptual framework (Lande, 2021; Santos, 2021). Coordination and thematization of these systems, not just procedural execution, are essential to meaningful mathematical comprehension.

Mathematics proficiency is said to be one of the core skills an individual must have to survive in the career world. As a basic mathematics requirement, students are expected to acquire the necessary problem-solving skills and develop mathematical mental structures. Aligned with this, the APOS theory is a model of how students learn mathematical concepts in a mathematics classroom. APOS stands for “Action, Process, Object, and Schema,” and it describes the stages of mental development a learner goes through when understanding a new mathematical concept. The learner eventually sees the concept as a whole “object” within a broader schema after first performing external actions and then internalizing them as mental processes (Arnon et al., 2014). According to the APOS theory, the goal of teaching and studying mathematics should be to assist students in using their existing mental models and creating new, stronger ones so they can handle more complex mathematical problems (Borji et al., 2018; Ng & Chew, 2023).

Despite this ideal, challenges persist in many higher education institutions. Feedback from international assessments in which the country participated highlights low mathematics proficiency (OECD, 2023), and scored the lowest among Trends in International Mathematics and Science Study (TIMSS) participating countries in both mathematics and science (Mullis et al., 2019). This dilemma is said to be greatly affected by students' learning foundation, specifically the mathematical foundation (Deasmin & Paglinawan, 2024; Lapinid et al., 2022; Magas, 2023). This issue has been acknowledged in institutional reviews and is consistent with findings from the Second Congressional Commission on Education (EDCOM 2), which noted persistent gaps in learning foundation across tertiary institutions. Relatively, this predicament is often worsened by the dearth of a reliable and relevant instrument for measuring mathematical competency in the country (Aguhayon et al., 2023; Igarashi & Suryadarma, 2023). Additionally, the availability of a credible and valid mathematics mental structure measurement tool remains lacking.

To address these concerns, this study adopts and modifies established measurement tools concerning the mental structure in developing an APOS-based scale for college students. The APOS theory emerges as an extension of constructivism, but it takes a more robust and targeted learner-centered approach to mathematics instruction (Oktaç et al., 2019; Tsafe, 2024). It can and has been applied to a wide range of mathematical theories, and it serves as a language for exchanging learning-related ideas (Dubinsky & McDonald, 2001). Its application in higher education has proven beneficial in supporting mathematics curriculum (Arnon et al., 2014; Dubinsky & McDonald, 2001; Tall, 1999), mathematics instruction (Afgani et al., 2017; Arnawa et al., 2007; Borji et al., 2018; Chamberlain & Vidakovic, 2021; Hanifah, 2019; Istikomah & Jana, 2019; Leng et al., 2023; Nagle et al., 2019), and learning resources (Kintoko et al., 2021; Prasetyo et al., 2021; Samosir et al., 2020).

By aligning the scale's statements with the APOS theory, this study aims to bridge the gap between students' prior mathematics learning and their current educational needs. This study aimed to contribute to the existing scales in mathematics learning that could help improve the quality of mathematics learning by focusing on students' mental structure. The development and validation of a mathematics mental structure scale suitable for evaluating the process of mental structure of students during any mathematical problem-solving tasks. Given the critical need for a proper tool to measure the mathematical mental structure of students, this study provided a valuable scale. Specifically, it sought to (1) construct items/statements that measure mathematics mental structure aligned with APOS Theory; (2) describe the validity and reliability of the developed scale in terms of i) content validity of aligning the tool to the concept of Action-Process-Object-Schema in mathematics learning, ii) construct and predictive validity using the Metacognition Scale and the Mathematics Self-Efficacy & Anxiety Scale, iii) reliability using Cronbach's alpha; and (3) explore the underlying structure in the developed APOS-Based mathematics mental structure scale through exploratory factor analysis.

In essence, the development of this APOS-based mathematics mental structure scale responds both to EDCOM 2's calls for fixing the mathematical foundation and to local classroom challenges observed at the study's locale. Integrating APOS Theory into the creation of the scale provides an opportunity to accurately measure and improve students' mathematics mental structure and enrich their learning experience in mathematics.

Methodology

This study employed a developmental research design focused on the systematic construction and validation of an APOS-based measurement scale, following the procedure emphasizing iterative item development, validation, and refinement. The participants were 330 tertiary mathematics majors during the School Year 2024–2025, a sample size considered between fair and good for scale development. Item generation was grounded in an extensive literature review, followed by expert validation, pilot testing, and administration via Google Forms and in-person. Data were collected over two consecutive weeks, with respondents oriented on the study's objectives and required to provide informed consent in compliance with the Data Privacy Act of 2012. Statistical analyses included the computation of the Content Validity Index (CVI) based on expert ratings, Cronbach's alpha for internal consistency reliability, and Exploratory Factor Analysis (EFA) to determine underlying factor structure and refine scale items. Ethical considerations were strictly observed, including approval submission to the University Research Ethics Board of Saint Mary's University, protection of confidentiality and data security, avoidance of conflicts of interest, and assurance that participants—university students and instructors—were not from vulnerable populations.

Results and Discussion

Item Generation

Several studies and literature have been reviewed to generate the items or statements for the scale. These statements are believed to specifically measure the APOS (Action-Process-Object-Schema) Mathematics Mental Structure of students. Firstly, how should the items be aligned with the structure?

Action – is the alteration of things that the person perceives as fundamentally external and that needs explicit or implicit instructions on how to carry out the operation. Students may execute rote operations at this level; in other words, they are required to act on the knowledge that has been provided to them. Process – is an internal mental structure that an individual might create when they repeat an activity and reflect on it. Instead of actually carrying out a process, students might imagine executing it; thus, they can consider reversing it and combining it with other processes. Object – When students recognize the process as a whole and understand that changes may be made to it, they have created an object from a process. The ability to assemble transformations to generate cognitive objects through the application of activities at the action and process phases is a step in mathematical concept understanding. Schema – a particular mathematical idea is a person's collection of behaviors, procedures, items, and other schemas connected by general principles to create a mental framework that may be applied to a problem scenario involving that concept. A student's ability to arrange and understand novel and unfamiliar mathematical concepts results in a collection of processes and objects.

After careful consideration of these definitions of how mental structure is achieved in each phase, the items are appropriately structured. Some of the statements of the developed scale are adopted, and some of them are modified. Statements of the "Mathematical Problems Solving Conception (MaProSC) Scale" of Catindoy (2021); the scale of Gok (2011) about problem-solving strategy steps are modified and included in the construction. Additionally, some of the items are patterned after the scale "Mathematics Collaborative Problem-Solving Skills" of Medina et al. (2019) and "Problem-Solving Skill Scale" used in Bayarcal and Tan (2023). Most of the statements are adopted from Catindoy (2021) and Medina et al. (2019); hence, approval was sought and granted.

Following the creation of items from these studies and literature, a total of 62 items were created.

Table 1
Developed Items for the APOS-Based Mathematics Mental Structure Scale

Statements	4	3	2	1
	Strongly Agree	Agree	Disagree	Strongly Disagree
ACTION				
1. I always make sure that I have the goal in mind when solving.	○	○	○	○
2. I plan what I'm going to do before I start solving the problem	○	○	○	○
3. After understanding the problem, I will identify what given quantities first and identify the unknown to solve the problem.	○	○	○	○
4. I repeatedly read the problem if it is difficult.	○	○	○	○
5. I take note of important details in the problem to guide me in solving.	○	○	○	○

6. I make sure that I know the meaning of the words in the problem.	○	○	○	○
7. I try to figure out if the problem has already been discussed in class or not.	○	○	○	○
8. I recall my past lessons that I can apply to the given problem.	○	○	○	○
9. I can identify what are given conditions.	○	○	○	○
10. I can describe a mathematical condition.	○	○	○	○
11. I can produce assumptions.	○	○	○	○
12. I can analyze relevant data in a specific problem.	○	○	○	○
13. I can apply current knowledge to solve a problem.	○	○	○	○
14. I restate the problem in my own words.	○	○	○	○
15. I try to remember existing principles, strategies, equations, and concepts related to the problem.	○	○	○	○
16. I usually search for hints in a problem.	○	○	○	○
17. I limit the given data in the problem.	○	○	○	○
18. I usually do not know how to solve a problem.	○	○	○	○

PROCESS

1. I determine what series of operations I should use.	○	○	○	○
2. I solve step-by-step so that I can look back and know what I am doing.	○	○	○	○
3. I look for patterns in the problem so I to solve it.	○	○	○	○
4. I can propose a solution immediately.	○	○	○	○
5. I can directly create a potential solution.	○	○	○	○
6. I can easily identify the appropriate formula for the problem.	○	○	○	○
7. I can mathematically think about a situation to solve a problem.	○	○	○	○
8. I can easily pinpoint point unique method to solve a mathematical problem.	○	○	○	○
9. I use my own strategies to solve the problem.	○	○	○	○
10. I try to guess how it will work before I implement the solution.	○	○	○	○

11. Before I start to solve the problem, I can make a near-correct estimate.	○	○	○	○
12. I can easily test the solutions and think about the accuracy.	○	○	○	○
13. I divide the problem into sub-problems.	○	○	○	○
14. I put the given variables in the related equations.	○	○	○	○
15. I use the trial-and-error method to find a solution.	○	○	○	○
16. I can think of at least one way to begin to work on a problem that I've never seen before.	○	○	○	○
17. I do not use any strategy while solving a problem.	○	○	○	○
OBJECT				
1. I can generate a written record of the way/s I think.	○	○	○	○
2. I can find a solution to a mathematical problem by writing an equation.	○	○	○	○
3. I can create a plan to solve the problem by drawing pictures/illustrations.	○	○	○	○
4. I can solve a problem using tabular representations.	○	○	○	○
5. I can make an answer to a mathematical problem with the use of current knowledge.	○	○	○	○
6. I try to use short shortcut to solve a problem.	○	○	○	○
7. I try to illustrate the problem so that I could understand it better.	○	○	○	○
8. I use variables to represent quantities.	○	○	○	○
9. When solving the problem, I use visual elements such as shapes and schemes	○	○	○	○
10. I spend my spare time solving problems	○	○	○	○
11. I visualize the problem.	○	○	○	○
12. I focus on the problem's solution.	○	○	○	○
13. I try to solve the problem with a similar one.	○	○	○	○
SCHEMA				
1. I use assumptions in relation to my final solution.	○	○	○	○
2. I can create new techniques to solve a problem.	○	○	○	○

3. I do not implement a pre-learned process to solve it.	○	○	○	○
4. I can evidently explain the process in the given situation.	○	○	○	○
5. I can seek multiple solutions.	○	○	○	○
6. I check the answer to see if it is reasonable.	○	○	○	○
7. I make the dimension analysis for the solution.	○	○	○	○
8. I use a problem in solving a problem I use to solve other problems	○	○	○	○
9. I can create new problems that can be used to solve the problem	○	○	○	○
10. After finding an answer using one method, I use another method to confirm my answer.	○	○	○	○
11. I associate the given problem with the real world to come up with practical ways to solve the problem.	○	○	○	○
12. I manipulate the formula after determining what variables are missing.	○	○	○	○
13. After I've solved the problem, I'll review what I've done for the solution, and I'll check the transactions before making a definitive judgment	○	○	○	○
14. By trying to find solutions to the problems in mathematics, I think I will gain experience in solving real-life problems	○	○	○	○

Content Validity

Following the item generation, four content experts validated the scale. There are two high school mathematics teachers and two mathematics instructors who serve as validators. Two of them have completed their doctoral degrees, while the other two have earned master's degrees. They are all knowledgeable in scale construction since they are involved in research and have published their own scales. They used a Survey Instrument Validation Rating Scale (Oducado, 2020) with 13 elements to validate the instrument. Findings revealed that the validity rating is 4.77, which is an acceptably high rating.

Additionally, for the relevance and alignment of the items to the concept of Action-Process-Object-Schema in mathematics learning, further validation was done. Content validity is the extent to which components of an evaluation tool are pertinent to and reflective of the intended construct for a given assessment purpose, and is calculated and decided through the CVI. Before finalization of items to be included in the tool, the calculation of CVI, the relevance rating must be recoded as 1 (relevance scale of 4 or 5) or 0 (relevance scale of 1, 2, or 3). The index is 0.88, which is an acceptable threshold for CVI. Considering the findings of the content validation, there are 54 items included for the draft of the tool for pilot testing.

Table 2
Relevance Ratings on the Item Scale by Four Experts

Statement Code	Experts 1	2	3	4	Experts in Agreement	I-CVI	UA	Experts Remarks
A1	1	1	0	0	2	0.5	0	Revise/ Remove
A2	1	1	1	1	4	1	1	Revise
A3	1	1	1	1	4	1	1	Revise
A4	1	1	1	1	4	1	1	Revise
A5	1	1	1	1	4	1	1	
A6	1	1	1	1	4	1	1	
A7	1	1	1	1	4	1	1	
A8	1	1	0	1	3	0.75	0	Move to Process
A9	1	1	1	1	4	1	1	
A10	1	1	1	1	4	1	1	
A11	1	1	0	1	3	0.75	0	Move to Process
A12	1	1	1	1	4	1	1	
A13	1	1	0	1	3	0.75	0	Move to Process
A14	1	1	1	1	4	1	1	
A15	1	1	1	1	4	1	1	
A16	1	1	1	1	4	1	1	
A17	1	0	1	0	2	0.5	0	Remove
A18	0	0	0	0	0	0	0	Remove
P1	1	1	1	1	4	1	1	
P2	1	1	1	1	4	1	1	
P3	1	1	1	1	4	1	1	Revise
P4	1	1	1	1	4	1	1	
P5	1	1	1	1	4	1	1	
P6	1	1	1	1	4	1	1	
P7	1	1	1	1	4	1	1	
P8	1	1	1	1	4	1	1	Revise
P9	1	1	0	1	3	0.75	0	Move to Schema
P10	1	1	0	1	3	0.75	0	Move to Schema
P11	1	1	1	1	4	1	1	
P12	1	1	1	1	4	1	1	
P13	1	1	1	1	4	1	1	
P14	1	1	1	1	4	1	1	
P15	1	1	1	1	4	1	1	
P16	1	1	1	1	4	1	1	
P17	0	0	0	0	0	0	1	Remove
O1	1	1	1	1	4	1	1	
O2	1	1	1	1	4	1	1	
O3	1	1	1	1	4	1	1	
O4	1	1	1	1	4	1	1	
O5	1	1	1	1	4	1	1	
O6	1	1	1	1	4	1	1	
O7	1	1	1	1	4	1	1	
O8	1	1	1	1	4	1	1	
O9	1	1	1	1	4	1	1	

O10	1	1	0	0	2	0.5	0	Remove
O11	0	1	0	0	1	0.25	0	Remove
O12	1	1	1	1	4	1	1	Revise
O13	1	1	1	1	4	1	1	
S1	1	1	1	1	4	1	1	
S2	1	1	1	1	4	1	1	
S3	1	1	1	1	4	1	1	Revise
S4	1	1	1	1	4	1	1	
S5	1	1	1	1	4	1	1	Revise
S6	1	1	1	1	4	1	1	Revise
S7	0	1	1	0	2	0.5	0	Remove
S8	1	1	1	1	1	0.25	1	Revise
S9	1	0	1	0	2	0.5	0	Remove
S10	1	1	1	1	4	1	1	
S11	1	1	1	1	4	1	1	
S12	1	1	1	1	4	1	1	
S13	1	1	1	1	4	1	1	
S14	1	1	1	1	4	1	1	
Proportion Relevance	0.94	0.94	0.84	0.87		0.88	0.81	

Construct and Predictive Validity Using the Mathematics Self-Efficacy & Anxiety Scale and Metacognition Scale

Establishing construct and predictive validity of the Mathematics Self-Efficacy & Anxiety Scale (May, 2009) and the Metacognition Scale (Fergus & Bardeen, 2019) is essential to align these measures with APOS theory and students' problem-solving outcomes. Construct validity ensures that self-efficacy, anxiety, and metacognition accurately represent affective and regulatory conditions that support or hinder students' progression from APOS levels. Predictive validity further confirms that these constructs operate as theoretically expected: higher self-efficacy and metacognitive regulation predict more advanced APOS mental structures and stronger performance, while higher mathematics anxiety predicts difficulties in coordinating and internalizing mathematical processes. Together, these justify interpreting problem-solving outcomes as meaningfully linked to the developed scale.

The findings on the correlation of mathematics self-efficacy and anxiety, and motivation levels with the developed scale indicate a substantial positive correlation ($p < 0.05$). The obtained R-value of 0.314 indicates a positive correlation between the abovementioned variables. This means that when the level of mathematics self-efficacy increases while the level of anxiety decreases, their level of mathematics mental structure improves.

According to research by Akkan et al. (2019), Fatmasari et al. (2021), and Callan et al. (2021), there is a positive correlation between mathematics self-efficacy and mathematical abilities. Also, Listiawati et al. (2025) mentioned that a low math anxiety showed a good understanding of the mathematical mental structure. In contrast to this, Hay et al. (2022) and Donolato et al. (2019) imply an inverse relationship, linking lower student mathematical abilities with better mathematical self-efficacy.

Similarly, findings also show a statistically significant positive association ($p < 0.05$) between student motivation and the developed scale. Students' mental structures and motivation are positively correlated, as indicated by the computed R-value of 0.165. This suggests that there is a connection between students' level of motivation and their mental structure in mathematics.

These results are similar to studies by Baars et al. (2017), Fatimah et al. (2019), and Saadati and Celis (2023), which found that high motivation is linked to high problem-solving abilities. This is in contrast to research by Tran and Nguyen (2021) and Liu et al. (2020), which found that students with higher motivation had lower problem-solving abilities.

Along with these findings, the results of the regression analysis, examining mathematics self-efficacy and anxiety, and motivation as predictors of the developed scale. With a p-value of 0.000, the results show that all the factors analyzed predict students' mathematical mental structures. Furthermore, 10% of the variation in mathematics mental structure among students in higher education may be explained by the model, according to the coefficient of determination (R-squared) of 0.100. Meanwhile, the 90% coefficient indicates how much of the variation in tertiary education students' mathematical mental structures may be explained by other factors.

This finding is connected to the research of Suren and Kandemir (2020), who claimed that students' mathematics anxiety predicts their achievement in mathematics. This suggests that students' aptitude for solving mathematical problems is influenced by their level of mathematics self-efficacy and anxiety. Furthermore, it is in line with the results of Živković et al. (2023), who found a strong link between students' math self-efficacy and anxiety with mathematics performance.

Reliability Using Cronbach's Alpha

In terms of reliability, the developed scale was tested, and the computed Cronbach's alpha is 0.940, which is highly reliable. The result also suggested statements 3 and 4 under Action to be deleted, and reliability after deletion is 0.941, which is not a significant difference. For that reason, the statements weren't deleted. According to Knekta et al. (2019), before ruling out an item, it's critical to take the item's alignment with the proposed construct into consideration.

Refinement of the Instrument

To explore the underlying structure in the developed scale, EFA was administered. But before that, some important requirements need to be met to administer EFA – sample, scale of measurement, normality, linearity, outliers, factorability, and adequacy (Zeynivandnezhad et al., 2019).

For the ideal number of samples, Zeynivandnezhad et al. (2019) recommended that samples should be greater than 200, which was satisfied by this study, considering that it utilized 330 samples. The second requirement was also met since the study's data is in a "ratio" scale of measurement. Meanwhile, normality, linearity, and outliers were tested and analyzed. The normality of the data was examined using the values for Kurtosis (0.331) and Skewness (0.166), which implies not much deviation from the normal curve or approximately normally distributed. The linearity test confirmed this finding, and no noteworthy outliers were discovered.

Relevant to this study, before performing factor analysis, the “factorability” of the dataset should be evaluated first. There are two ways to determine the factorability or sampling adequacy: the Kaiser-Meyer-Olkin Test and Bartlett's Test. According to Schreiber (2021), by comparing the observed correlation matrix with the identity matrix, Bartlett's test of sphericity determines whether or not there is any intercorrelation between the observed variables. He further stated that factor analysis must not be performed if the outcome is statistically insignificant. Additionally, the adequacy of the data for factor analysis is evaluated using the Kaiser-Meyer-Olkin (KMO) test (Luong & Flake, 2023). It displays the sufficiency of every observed variable as well as the entire model. KMO took into account the percentage of variation across all observed variables, which fell between 0 and 1 (Muncer et al., 2022).

From the result, the Chi-Square for the construct was 5918.626 (df = 1378) ($p < 0.05$). It also displays a p-value for Bartlett's test is 0.000, which means that the test showed statistical significance. This suggests that the observed correlation matrix is not an identity matrix. Consequently, acceptable high results are shown by the study's Kaiser-Meyer-Olkin Measure of Sampling Adequacy (0.907). This value indicates that the tool can proceed with the factor analysis.

With all the indicators satisfied, EFA can be administered. The primary goal of this is to identify the fundamental structure that exists between the variables under study (Alavi et al., 2020; Huntley et al., 2022; Luong & Flake, 2023; Muncer et al., 2022; Nordahl et al., 2022; Retutas & Rubio, 2021; Schreiber, 2021).

The KMO value is 0.907 (cut-off = 0.5). Most of the loading factor was around 0.303 to 0.669 (cut-off = 0.3), establishing the suitability of the data for factor analysis. Several components have the items loaded after many rotations. Additionally, cross-loadings continue to occur in the rotated component matrices’ results. The scree plot result shows four factor loadings for the scale since there are four dots before the elbow; however, the scale has only one construct since it is developed to measure only the mathematical mental structure of students. Relatively, the highest loadings of the items are in the first construct. As a result, the scale’s last component that will characterize the APOS-Based Mathematics Mental Structure in general was determined to be the first construct.

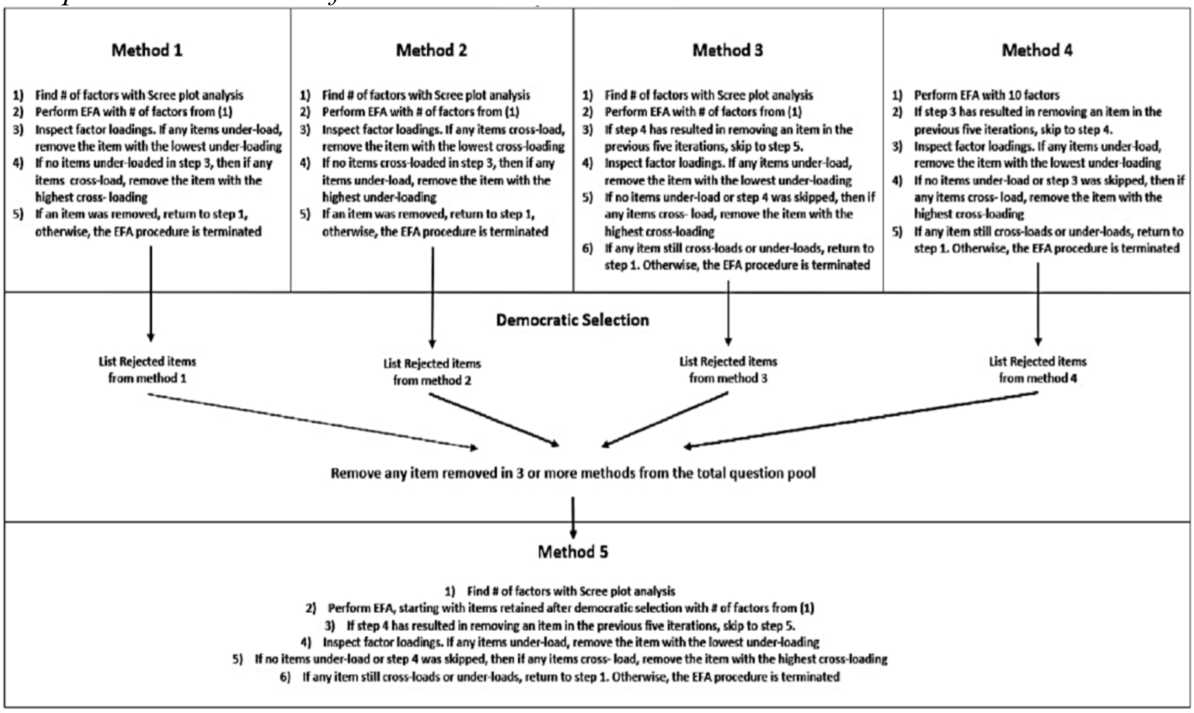
On the contrary, the study cannot overlook the items not loaded in the first construct (such as A3, A4, and O4) because of too low loading values. Also, some items cross-load to one or two other factors, most especially since they have high loadings in these factors. On this note, a thorough EFA was used to confirm that the tool will only display one construct. It is also used to further refine a produced instrument in order to guarantee its validity and reliability and to increase its overall usefulness. To investigate the dimensionality of the instrument and, eventually, improve its capacity to measure what it is intended to measure, EFA assists in locating and eliminating components that do not match the intended construct.

Generate the Final Form of the Instrument After EFA

To properly determine what items are loaded to the first construct, and further refinement of the developed tool, item removal using the results and procedures of EFA was done using the developed framework of Mirabelli et al. (2022).

Figure 1

Multiple EFA Procedures for Item Removal



Source: Mirabelli et al. (2022)

Figure 1 displays the five processes (Methods 1–5) of item removal of a developed scale. The first four methods are done as explained. After the four methods were complete, a democratically selected item deletion method was performed, where items that were not retained in at least three of the four methods were removed from the 54-item tool, and a new EFA procedure was conducted with the reduced item list and used to generate a final result. After items were removed, the fifth and final EFA method was conducted on the items that had not been removed due to the democratic selection process.

Table 3

Results of the Democratic Selection Process for Item Retention Methods

Statement Code	Democratic Selection Process					Final Remarks
	Method 1	Method 2	Method 3	Method 4	Method 5	
A1	Accept	Remove	Remove	Remove	Remove	Remove
A2	Accept	Accept	Accept	Accept	Accept	Accept
A3	Accept	Accept	Accept	Remove	Remove	Remove
A4	Accept	Accept	Accept	Remove	Remove	Remove
A5	Accept	Remove	Remove	Remove	Remove	Remove
A6	Accept	Accept	Accept	Accept	Accept	Accept
A7	Accept	Accept	Accept	Accept	Accept	Accept
A8	Accept	Accept	Accept	Accept	Accept	Accept

A9	Accept	Accept	Accept	Remove	Accept	Accept
A10	Accept	Accept	Accept	Accept	Accept	Accept
A11	Accept	Remove	Remove	Remove	Remove	Remove
A12	Accept	Accept	Accept	Accept	Accept	Accept
P1	Accept	Remove	Remove	Remove	Remove	Remove
P2	Accept	Accept	Accept	Accept	Accept	Accept
P3	Accept	Accept	Accept	Accept	Accept	Accept
P4	Accept	Accept	Accept	Accept	Accept	Accept
P5	Accept	Accept	Accept	Accept	Accept	Accept
P6	Accept	Accept	Accept	Accept	Accept	Accept
P7	Accept	Accept	Remove	Accept	Remove	Remove
P8	Accept	Accept	Accept	Accept	Accept	Accept
P9	Accept	Accept	Accept	Accept	Accept	Accept
P10	Accept	Accept	Accept	Accept	Accept	Accept
P11	Accept	Accept	Accept	Accept	Accept	Accept
P12	Accept	Accept	Accept	Accept	Accept	Accept
P13	Accept	Accept	Accept	Accept	Accept	Accept
P14	Accept	Accept	Accept	Accept	Accept	Accept
P15	Accept	Accept	Accept	Accept	Accept	Accept
P16	Accept	Accept	Accept	Accept	Accept	Accept
P17	Accept	Accept	Remove	Remove	Remove	Remove
O1	Accept	Accept	Accept	Accept	Accept	Accept
O2	Accept	Accept	Accept	Accept	Accept	Accept
O3	Accept	Accept	Accept	Accept	Accept	Accept
O4	Remove	Remove	Remove	Remove	Remove	Remove
O5	Accept	Accept	Accept	Accept	Accept	Accept
O6	Accept	Accept	Accept	Accept	Accept	Accept
O7	Accept	Accept	Accept	Accept	Accept	Accept
O8	Accept	Accept	Accept	Accept	Accept	Accept
O9	Accept	Accept	Accept	Remove	Remove	Remove
O10	Accept	Accept	Accept	Accept	Accept	Accept
O11	Accept	Remove	Remove	Remove	Remove	Remove
S1	Accept	Accept	Accept	Accept	Accept	Accept
S2	Accept	Accept	Accept	Accept	Accept	Accept
S3	Accept	Accept	Accept	Accept	Accept	Accept
S4	Accept	Accept	Accept	Accept	Accept	Accept
S5	Accept	Accept	Accept	Remove	Remove	Remove
S6	Accept	Accept	Accept	Accept	Accept	Accept
S7	Accept	Accept	Accept	Remove	Remove	Remove
S8	Accept	Accept	Accept	Accept	Accept	Accept
S9	Accept	Accept	Accept	Accept	Accept	Accept
S10	Accept	Accept	Accept	Accept	Accept	Accept
S11	Accept	Accept	Accept	Accept	Accept	Accept
S12	Accept	Accept	Accept	Accept	Accept	Accept
S13	Accept	Accept	Accept	Accept	Accept	Accept
S14	Accept	Remove	Remove	Remove	Remove	Remove

The democratic selection process omitted 7 items; thus, the fifth method began with 47 items, and iterations of that method removed another 7 items. It can be observed that for the democratic selection process, items were unanimously removed or accepted in only 40 out of 54 items. For the remaining 54 items, one or two methods demonstrated discrepancies in determining item retention.

Of note, for 8 items, one or two of the four methods rejected the item, and three or two kept the items, suggesting these items may have been more “borderline” items than other, more accepted items. For example, item number P17 had two votes to be rejected and was kept by democratic selection, but was among the four items deleted during the EFA iterations on the democratically selected items in Method 5.

After item removal, the new KMO and Bartlett's Test, and the component matrix test results are presented to show the change of values. The new KMO and Bartlett's Test revealed an increase in factorability, while the component matrix displays that all remaining 40 items are loaded to the first construct with minimal and low cross-loadings.

The result revealed a KMO value of 0.915, and most of the loading factors were around 0.502 to 0.699, establishing a greater suitability of the remaining items. Additionally, the reliability coefficient of the final scale is $\alpha = 0.948$. These findings indicate that there are 40 items in the final version of the APOS-based mathematics mental structure.

Conclusion and Recommendation

The scale APOS-based mathematics mental structure was primarily developed to support studies on mathematics achievement and performance, and to add to the scales that measure a positive construct among students’ mathematics proficiency. The scale that was created was determined to be reliable and valid. Only one component was discovered using exploratory factor analysis. With the extensive exploratory factor analysis, it is further suggested to use a framework to verify whether only one classification exists or can be reconstructed to Action-Process-Object-Schema factors using confirmatory factor analysis (CFA).

Further pilot testing is also encouraged, and tool refinement with the help of other experts. One of the study's limitations is that, although the scale may be applied to both secondary and postsecondary students, secondary students were not employed in the pilot testing. Nevertheless, additional reliability tests, further content validation (especially validation of experts in test/scale construction), and CFA can still be used for additional refining of this developed instrument.

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Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The author declares that Grammarly and QuillBot were used solely for proofreading, grammar correction, and language refinement to enhance clarity and accuracy. No other AI-assisted tools were employed in generating the manuscript's content. All ideas, research design, procedures, analyses, findings, and discussions are original and derived from the author's systematic conduct of the study.

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Appendix

APOS-Based Mathematics Mental Structure Scale

**Please provide the necessary information*

Instructions: Read each statement carefully. Place a ✓ mark on the choice corresponding to the degree of your agreement with the statement. Answer all items with honesty. There are four choices for each statement, which correspond to the following:

1 – Almost Always 2 – Often 3 – Rarely 4 – Almost Never

Statements	(*Check* ***one*** *box in each* ***row***)			
	Almost Always	**Often**	**Rarely**	**Almost Never**
ACTION				
1. After understanding the problem, I identify what given quantities there are and the unknowns to solve the problem.				
2. I try to figure out if the problem has already been discussed in class or not.				
3. I can identify what the given conditions are.				
4. I can describe a mathematical condition.				
5. I can analyze relevant data in a specific problem.				
6. I restate the problem in my own words.				
7. I usually search for hints in a problem.				
Process After obtaining the necessary information from the problem, …				
1. I determine what series of operations I should use.				
2. I solve step-by-step so that I can look back and know what I am doing.				
3. I look for patterns in the problem so I to solve it.				
4. I can apply current knowledge to solve a problem.				
5. I can propose a solution immediately.				
6. I can directly create a potential solution.				
7. I can easily identify the appropriate formula for the problem.				
8. I can mathematically think about a situation to solve a problem.				
9. I can easily pinpoint a unique method to solve a mathematical problem.				
10. I try to guess how it will work before I implement the solution.				
11. Before I start to solve the problem, I can make a near-correct estimate.				
12. I can easily test the solutions and think about the accuracy.				
13. I divide the problem into sub-problems.				
14. I put the given variables in the related equations.				
Object				
1. I can generate a written record of the way/s I think.				

2. I can find a solution to a mathematical problem by writing an equation.				
3. I can create a plan to solve the problem by drawing pictures/illustrations.				
4. I can make an answer to a mathematical problem with the use of current knowledge.				
5. I try to use a shortcut to solve a problem.				
6. I try to illustrate the problem so that I can understand it better.				
7. I use variables to represent quantities and solve for values.				
8. I focus more on the problem's solution by spending more time on the calculation.				
Schema				
1. I use assumptions in relation to my final solution.				
2. I can think of at least one way to begin to work on a problem that I have never seen before.				
3. I use my own strategies to solve the problem.				
4. I can create new techniques to solve a problem.				
5. I can evidently explain the process in the given situation.				
6. I check the answer to see if it is correct.				
7. I can create new, simpler problems that can be useful in solving the given problem.				
8. After finding an answer using one method, I use another method to confirm my answer.				
9. I associate the given problem with the real world to come up with practical ways to solve the problem.				
10. I manipulate the formula after determining what variables are missing.				
11. After I have solved the problem, I'll review what I have done for the solution, and I'll check the transactions before making a definitive judgment				

*************END*************