Investigating Pre-service Secondary Mathematics Teachers' Mathematics Proficiency and Critical Citizenship in the Design of Social-Issue-Themed Mathematics Online Lessons

Art Walden A. Dejoras, Ateneo de Manila University, Philippines
Catherine P. Vistro-Yu, Ateneo de Manila University, Philippines

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Abstract
Conditions that recently threatened the breakdown of social order led the authors to question the role of mathematics in addressing these conditions. In the light of these conditions, this study aimed to explore the tension between the need for pre-service secondary mathematics teachers (PSMTs) to be mathematically proficient and society's expectations of them to develop skills for critical citizenship. Twenty randomly selected PSMTs from an intact class in a Philippine university participated in three researcher-designed online mathematics lessons covering descriptive statistics, logic and reasoning, and linear functions that addressed the country's issues which include the government-declared war on drugs, proliferation of fake news, and armed conflicts, respectively. Drawing on online ethnographic practitioner research approach, both quantitative and qualitative data-gathering methods were employed. In the quantitative phase, pre-test-and-post-test single-group design was employed to determine the effectiveness of the lessons in improving PSMTs' mathematical proficiency. In the qualitative phase, deductive qualitative content analysis was used to determine and categorize the PSMTs' skills for critical citizenship before and after exposure to the lessons and determine if there was a difference in their skills. Seven categories of critical citizenship were theoretically deduced, though only five were pertinent, to consider PSMTs' communication of critical citizenship. The findings showed a significant difference in the PSMTs' mathematical proficiency score before and after the lessons. Moreover, improvement of PSMTs' skills for critical citizenship were shown. The activities created a communicative space that helped PSMTs develop their critical citizenship without sacrificing their mathematical proficiency.

Keywords: Critical Citizenship, Mathematics Proficiency, Pre-service Secondary Mathematics Teachers

Introduction

History has shown that problems of social order arise from difficulties like environmental disasters, plagues and pandemics. Problems of social order are exhibited by the following: (1) resistance to measures from authorities (Reicher & Stott, 2020) and (2) emergence of open conflict resulting in threats to security, to property and even to life itself (Jamrozik & Nocella, 2011). These problems have been seen to appear during certain periods. In the Philippines, for example, problems of social order (or unrest) came about during the Martial Law era.

Problems of social order became prevalent once more in recent years, most especially during the Covid 19 pandemic which exacerbated the situation. An example of a situation that posed problems of social order to the Filipino public was the government-led war-on-drugs initiated by Rodrigo Duterte when he assumed office as the Philippine president on June 30, 2016 (Warburg & Jensen, 2020). Another equally alarming situation was the widespread fake news, particularly through social media. Disinformation has been problematic in the Philippines with social media playing a critical role in influencing discourse ranging from politics to health, beliefs, religion, current events, and others (Bringula et al., 2022). Armed conflicts have been regularly problematic. An example is the Marawi Siege which was a five-month-long armed conflict in Mindanao, Philippines, between government security forces and militants affiliated with the Islamic State (IS), including the Maute and Abu Sayyaf groups (Sandongdong et al., 2020).

In the light of these situations that threaten the breakdown of social order, we question how mathematics could become an instrument for developing students' understanding of the social degradation that is happening in the country and in the world. In this study, our goal was two-fold. We wanted to first, challenge the persisting view of mathematics that is objective, value-free, decontextualized, and non-political and argue that mathematics has a role to play in maintaining an orderly society and, second, find ways to develop "critical" and engaged citizens who are motivated to change and improve society. These dual goals are in line with Skovsmose's (1994) principle regarding mathematics education. According to him, "The purpose of mathematical education should be to enable students to realize, understand, judge, utilize and also to perform the application of mathematics in society, in particular in situations which are of significance to their private, social and professional life" (p.55).

To realize the dual goals, this study focused on investigating pre-service secondary mathematics teachers' (PSMTs') development of mathematics proficiency and skills for critical citizenship. A number of scholars have argued for PSMTs to strengthen their mathematics proficiency while they are still students in a teacher-training institution (e.g., Bosica et al., 2021). However, the development of PSMTs' mathematics proficiency should never be independent from their development of critical citizenship (Freire, 2000).

Theoretical Perspectives

This study was anchored on the theories of Kilpatrick et al.'s (2001) mathematics proficiency and Skovsmose's (1992) critical citizenship.

Kilpatrick et al's. (2001) mathematical proficiency consists of five intertwined and overlapping strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Conceptual understanding refers to the

comprehension of mathematical concepts, operations and relations. Procedural fluency covers the skills in carrying out procedures flexibly, accurately, efficiently and appropriately. Strategic competence is the ability to formulate, represent, and solve mathematical problems. Adaptive reasoning is the capacity for logical thought, reflection, explanation and justification. Productive disposition is the habitual inclination to see mathematics as sensible, useful, and worthwhile.

A number of mathematical assessments both local and international have included mathematical tasks that measure various strands of mathematical proficiency due to their usefulness in leveraging students' mathematics learning (Kepner & Huinker, 2012, p. 28 as cited in Groves, 2012). Many studies have designed and administered mathematics proficiency tests to students assessing only three strands, namely, procedural fluency, conceptual understanding, and strategic competence (e.g., Khairani & Nordin, 2011; Groves, 2012; Usman, 2020) as these have been identified in the National Assessment of Educational Progress (NAEP) and the National Council of Teachers of Mathematics (NCTM) as the main strands in developing students' procedural knowledge, conceptual knowledge and problem-solving skills – mathematical abilities students needed to be mathematically proficient (Al-Mutawah et al., 2019). Thus, the present study focused only on the three strands.

Skovmose (1992) defined critical citizenship as a quality of thought that supports one "to be critical citizens who can challenge and believe that their actions will make differences in society" (p. 2) According to him, critical citizenship is needed to promote democratic ideals. He wrote, "Democracy can be destroyed if a critical citizenship cannot be brought to life." (p.5) Furthermore, he elucidated that "being critical" involves "(1) investigation of conditions for obtaining knowledge, (2) identification of social problems and evaluation making, and (3) a reaction to problematic social situations" (p. 37).

Research Questions

In this paper, we sought to determine if there was a significant difference in the PSMTs' mathematical proficiency score before and after going through the social-issue-themed mathematics online lessons, and how prevalent and to what extent critical citizenship was evident in the PSMTs articulation about critical citizenship before and after going through the social-issue-themed mathematics online lessons.

Methodology

The study used online ethnographic practitioner research employing both quantitative and qualitative data-gathering methods (Barton, T.D., 2008; Skageby, 2011; Eisenhart, 1988). In the quantitative phase, pre-test-and-post-test single-group design was employed to determine PSMTs' level of mathematics proficiency. In the qualitative phase, deductive qualitative content analysis (Bikner-Ahsbahs et al., 2015) was used to determine the prevalence and extent of PSMTs' skills for critical citizenship.

The research subjects were 20 PSMTs who were 3rd Year Bachelor of Secondary Education major in Mathematics (BSEd-Math) students at a university in Mindanao during 1st semester of SY 2022-2023.Random sampling technique was used in determining the participants.

In this study, three lessons covering different mathematics contents incorporating different Philippine issues as themes were developed by the first author (See Table 1).

Lesson No.	Mathematics Content	Theme
1	Organization and Presentation of Data & Measures of Central Tendency and Variation	Drug-related Fatalities/ Human Rights Violations
2	Logic and Reasoning	Fake News/Leadership and Governance
3	Linear Function and Simple Linear Regression Analysis (SLRA)	Safety and Security/ Civil Conflict

Table 1 The Three Social-Issue-Themed Lessons

The social issue themes were chosen based on Skovsmose's (1985) two criteria: They must be closely connected to the experiences or interest of students and they must be closely related to existing social problems. The first author of the article played dual roles as researcher-instructor. The researcher-instructor implemented the lessons following the *Launch-Explore-Summarize* (LES) instructional model (Schroyer & Fitzgerald, 1986) that emphasizes dialogic teaching and learning. It consists of forms of interactions in the classroom where both teacher and students are involved in the co-creation of knowledge. During the *Launch* phase, the researcher-instructor uploaded mathematics content reviewers and links to a relevant social issue, and introduced the social issue. During the *Explore* stage, students performed the group task in self-selected groups of 4 to 5 members per group. They were given real-life data with instructions to express their thoughts and questions about the issue, apply the mathematics content in addressing the issue, and finally propose some actions that will address the issue. During the *Summarize* stage, the PSMTs identified and discussed their common mistakes, challenges, and difficulties in a whole-class discussion facilitated by the researcher-instructor.

In this study, data collected were the pre-test and post-test scores in the Mathematics Proficiency Test (MPT) and the prevalence and extent of PSMTs' Critical Citizenship as reflected in their Critical Citizenship Test (CCT) administered before and after the lessons.

The MPT is a 14 - item, 42-point, open-ended, and researcher-developed test which PSMTs were required to answer individually for 2 hours. The test was validated by a pool of 5 expert raters composed of 4 PhD Mathematics Education and 1 MA Education major in Mathematics degree holders. The choice of 5 expert raters for content validation is the minimum acceptable number (Ayre & Scally, 2013) for this type of study. The MPT covered descriptive statistics, logic and reasoning, and linear functions. A 3-point scale was used to score each item in the MPT (Refer to Table 2). The scoring guide was a modification based on the works of Usman (2020) and Cartwright (2020). The score of each PSMT from the MPT was obtained by adding all the points obtained from the 14 items in the test. The lowest possible score is 0 and the highest possible score is 42. The PSMTs' MPT scores in the pre-test and post-test were then analyzed using paired samples t-test.

Point	Procedural Fluency	Conceptual Understanding	Strategic Competence
3	Procedures are used accurately and efficiently; Correct formulas and calculations	Explanation exhibits complete knowledge and understanding of the concepts or relationships taken into consideration	Able to solve problem with accurate details, shows knowledge and understanding of concepts and relationships and process, and complete solutions
2	Minor mistake in procedure due to carelessness; minor error due to lack of understanding	Minor mistakes in the explanation; explanation lack minor supporting details	Acceptable strategy for solving a problem; shows knowledge and understanding of concepts and relationships, but lacking accuracy in the details/solutions due to carelessness; not able to solve the problem
1	Major mistake in procedure due to carelessness; major error due to lack of understanding	Major errors in explanation due to lack of knowledge or understanding	Acceptable choice of strategy, exhibit some knowledge and understanding with some errors and misconceptions; unable to solve the problem
0	Inappropriate/ wrong procedures due to lack of knowledge/ understanding of concepts; inappropriate formula used, or no response.	Inappropriate explanation; explanation does not connect/ correspond to the concept or relationships discussed; Not following instruction; No explanation.	Inappropriate choice of strategy for solving a problem; major mistakes in the process, or wrong process; unable to solve the problem

Table 2. Scoring Guide for Mathematics Proficiency Test

The CCT is a three-item, open-ended, researcher-developed test which PSMTs were required to answer individually for 20 minutes. The test asked PSMTs to draw out their feelings and thoughts about each of the three social-order related issues in the Philippines and to share how they could address these issues as Filipino citizens. The test was also validated by a pool of 5 expert raters composed of 2 high school Social Studies teachers, 1 university instructor with a degree in Sociology and 2 university instructors with a degree in Political Science.

In the study, deductive qualitative content analysis (QCA) (Bikner-Ahsbahs et al., 2015) was used by employing predetermined structures to organize narrative data in categories. Seven categories of critical citizenship were deduced from Skovsmose's notion of critique (see Table 3).

Category	Definition
Critical Competence	Ability to critique traditional notion that teacher is always in control of educational process (Skovsmose, 1985)
Critical Distance	Ability to critique the applicability of subject matter (Skovsmose, 1985)
Critical Engagement	Ability to critique the applicability of the problem selected (Skovsmose, 1985)
Knowledge of the Social Issue	Ability to articulate ideas concerning the causes, processes, and expected outcomes of the social issue (Hong, 2018).
Application of the Social Issue to Broader Context	Expanding of one's understanding of the social issue by connecting it to broader social issue (Hong, 2018; Oh & Kwon, 2014)
Taking Personal Stance (Passive Stance)	Ability to express personal stance and feelings for the social issue (Hong, 2018; Oh & Kwon, 2014)
Taking Active Stance via Collective Social Action for Social Change	Ability to suggest community engagement that allows individual role to become active contributor and participant of individual community beyond observing passively (Hong, 2018; Oh & Kwon, 2014)

Table 3. Seven Categories of Critical Citizenship and Corresponding Definitions

However, in this study, only five were pertinent in taking into account the PSMTs' communication of critical citizenship. These were all the categories except *critical competence* and *critical engagement*.

All text components of the PSMTs' responses in each of the three social issues in the CCT during the pre-test that fit into each of the five categories were extracted and coded. A text component ranged from a string of words to a sequence of sentences. Each PSMT was coded at most once per category. If two or more text components belonging to the same PSMT fit into one category of critical citizenship, then the PSMT was given a single code only in that category. However, a text component was coded as one or more categories if deemed applicable. The same procedure was done for the PSMTs' responses in each of the three social issues in the CCT during the post-test.

Results

Figure 1 shows the distribution of the scores of 20 PSMTs' pre-test and post-test scores in the MPT. The graph indicates an increasing trend of the scores of the PSMTs from pre-test to post-test.

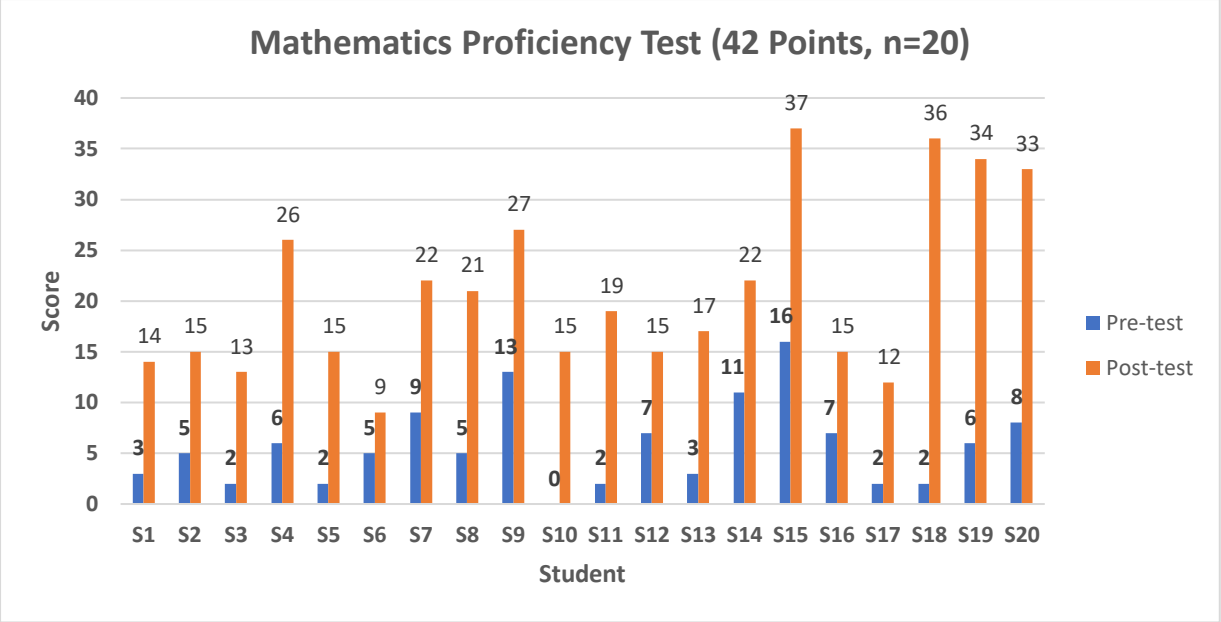

Figure 1: Distribution of PMSTs' Mathematics Proficiency Test Pre-Test and Post-Test Scores

Table 4 shows the scores of the PSMTs in the MPT before and after exposure to the lessons. Their pre-test mean score, 5.70 with a standard deviation of 4.13, was interpreted as low test performance. Their post-test mean score, 20.85 with a standard deviation of 8.57, was interpreted as average test performance. The paired samples t-test revealed a critical value of 9.236 with 19 degrees of freedom (t (19) = 9.236, p <0.05) indicating a significant difference of 15.15 between the pre-test and post-test means. Such differential effect was large (Cohen's d = 2.07).

Group	Mean	SD	Mean Difference	SD Difference	t	df	p-Value
Pre-Test	5.7	4.13	15.15	7.336	9.236	19	0.000*
Post-Test	20.85	8.57					

*: Significant at p < 0.05

Table 4. Test of Difference in the Mathematics Proficiency Test Mean Scores

Figure 2 shows the prevalence and extent of PSMTs' critical citizenship with respect to the war-on-drugs issue in the CCT before and after the implementation of the lessons as reflected in their pre-test and post-test results, respectively.

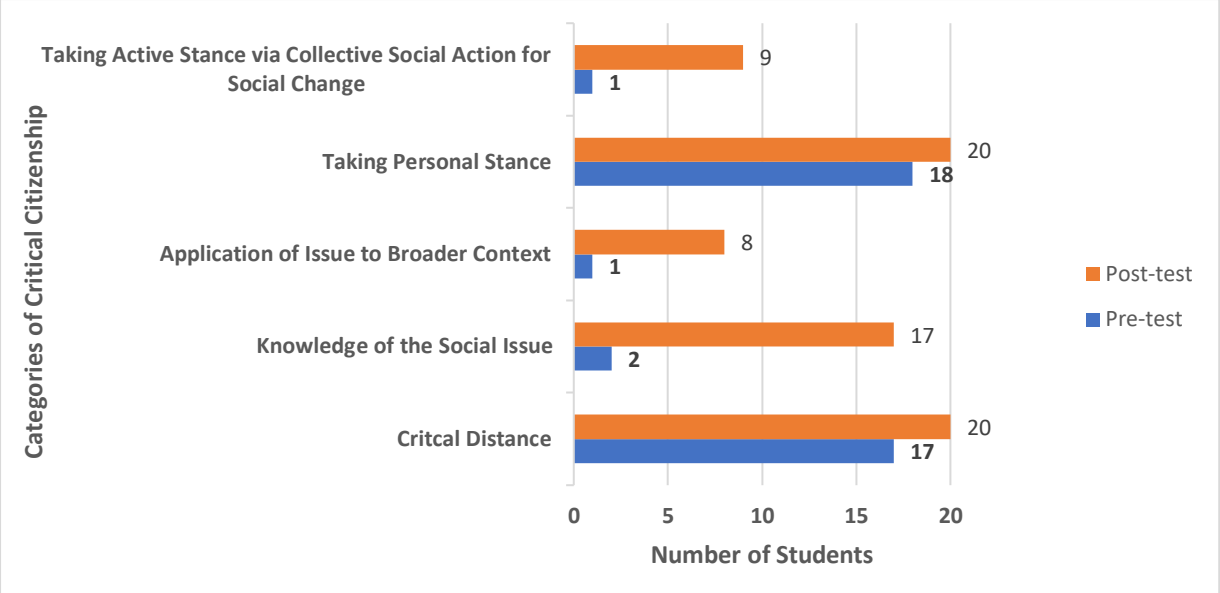

Figure 2: Prevalence and Extent of Critical Citizenship in Issue #1: War on Drugs

In the pre-test, most of the PSMTs gave statements categorized as *taking personal stance* (18 out of 20). In general, the PSMTs expressed concern about the high cases of prohibited drug use in the country and the effect of drug use in the safety of the citizens. They urged the government to conduct awareness programs, campaigns, education and strict implementation of the law prohibiting drug use. There were 17 out of 20 PSMTs whose statements fall under the *critical distance* category. They stated the use of statistics, data-gathering techniques, and graphs as appropriate tools to promote understanding of the issue. There were also a few who mentioned the applicability of calculus. There were only two out of 20 PSMTs who articulated ideas indicating *knowledge of the social issue*. One mentioned that most of the crime suspects usually use illegal drugs. Another stated that through the implementation of the war-on-drugs in the country, there were now fewer drug addicts and crime offenders. Only one PSMT spoke about the issue that indicated as *application of the issue to a broader context* citing the effect of drug use towards health, emotion and mental well-being. Also, only one PSMT gave statements that indicated *taking active stance via collective social action for social change* advocating the need to create activities that can help raise awareness of the consequences of using illegal drugs.

The post-test showed an increase in the number of PSMTs articulating each category in the CCT. Also, in general, the PSMTs were more expressive of their thoughts, ideas and feelings in the post-test. In the *taking personal stance* category, all 20 PSMTs resonated with the need to be concerned like those in the pre-test. But there were some PSMTs who showed concern about the alarming number of deaths although decreasing, the extrajudicial killings and the need to strengthen the country's law and justice system. In the *critical distance* category, all 20 PSMTs expressed appreciation for descriptive statistics as a tool in addressing the issue which was the same as in their pre-test. However, in the post-test many more were able to explain further how descriptive statistics can be used. In the *knowledge of social issue* category (17 out of 20), most PSMTs have expressed more ideas about the issue compared to the pre-test. Some mentioned the President Duterte's successful implementation of the campaign to lessen drug problem in the county. Others mentioned the weak judicial system of our country and the prevalence of injustices. Many admitted that the use of statistics has promoted better understanding of the scale the issue has taken. In the *application of the issue to a broader context* category (8 out of 20 PSMTs), some PSMTs expressed the need to abide

by the law, to be good and responsible citizens. Others emphasized the need to strengthen the human rights advocacy, the discipline from authorities and the country's justice system. In the *taking active stance via collective social action for social change* (9 out of 20) category, there were more PSMTs who showed increased knowledge on how to act with regards to the issue. They mentioned the need for them to participate in activities to raise awareness in the social media using mathematics to justify the scale of the issue.

Figure 3 shows the prevalence and extent of PSMTs' critical citizenship with respect to the issue of fake news in the CCT pre-test and post-test.

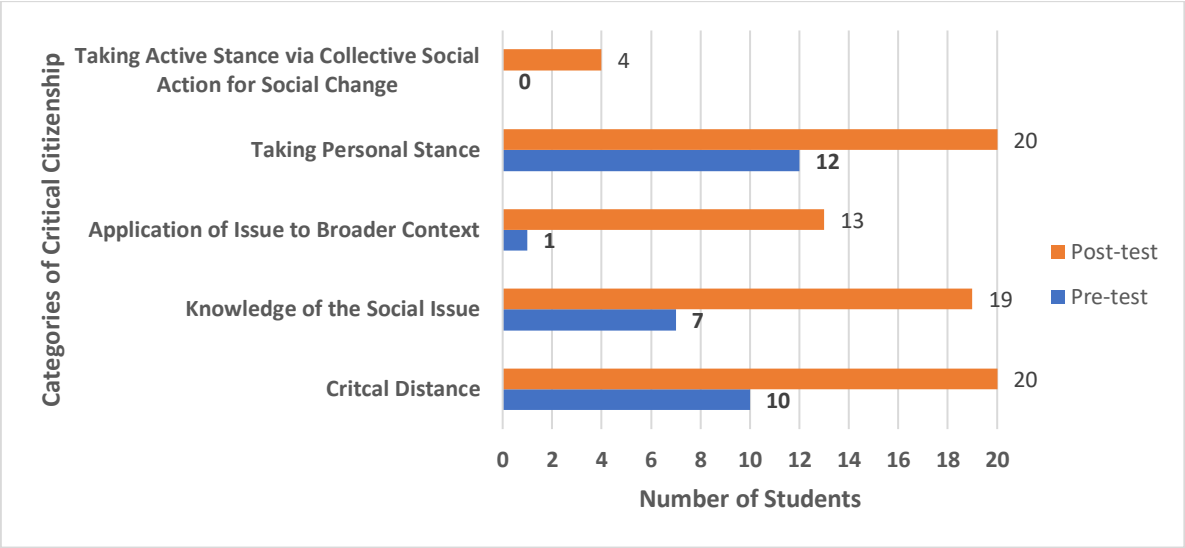

Figure 3: Prevalence and Extent of Critical Citizenship in Issue #2: Fake News

In the pre-test result, most of the PSMTs (12 out of 20) gave statements consistent with the *taking personal stance* category. A number of PSMTs expressed the need to be concerned about the injustices, abuses and atrocities that took place during the Martial Law era, to learn from history, and to make sure this part of our history will not repeat. However, there were some PSMTs who cited some "benefits" of President Ferdinand Marcos, Sr.'s Martial Law, such as maintaining order during that era as imposed by the government. Others argued to leave the negativities behind since these do not affect their future or they were not yet born during that time. There were 10 out of 20 PSMTs who maintained *critical distance*. Generally, they stated the use of statistics and data-gathering to promote understanding of the issue. Some mentioned the applicability of other mathematics contents like arithmetic, algebra and calculus. There were few who thought that mathematics cannot be applied. There were 7 out of 20 PSMTs who manifested *knowledge of the social issue*. Most of them argued that there were legitimate documents proving Ferdinand Marcos Sr's Martial Law was full of police atrocities. Others argued that only those against the government were punished and that the posts in social media showing negative views about Martial Law only wanted to create pressure on media to make people mad at the government. Only one PSMT exhibited *application of the issue to a broader context* by mentioning the need to be knowledgeable about fake news and to spread only authentic information on social media.

In the post-test, most of the PSMTs gave statements consistent with both the *taking personal stance* and *critical distance* categories (all PSMTs). In the *taking personal stance* category, those who had positive views about martial law during the pre-test expressed the same in the post-test. However, in the post-test, more PSMTs articulated the need to be concerned

because there are still injustices happening at present and that they or their loved ones are not immune to these injustices. More PSMTs also mentioned the need to protect themselves from fake news, misinformation and disinformation especially on social media, as well as the need to know the truth. In the *critical distance* category, the PSMTs realized the applicability of logic and reasoning in evaluating the veracity of information especially on social media. The use of statistics was also mentioned by a number of PSMTs to promote better understanding of the issue. There were 13 PSMTs who gave statements within the category of *application of the issue to broader context*. More PSMTs argued for the need to value human rights, to be critical about the information on social media, to be informed about fake news, misinformation and disinformation and to participate in spreading the truth. There were also some who stressed the need to be responsible citizens and to follow the government to avoid punishments. Lastly, there were four PSMTs who expressed *taking active stance via collective social action for social change.* More PSMTs advocated the need for them to help spread awareness about the issue of fake news, misinformation and disinformation through social media. The need to share factual information to friends and family about Martial Law was also mentioned.

Figure 4 shows the prevalence and extent of PSMTs' critical citizenship with respect to the issue of civil conflict in the CCT pre-test and post-test.

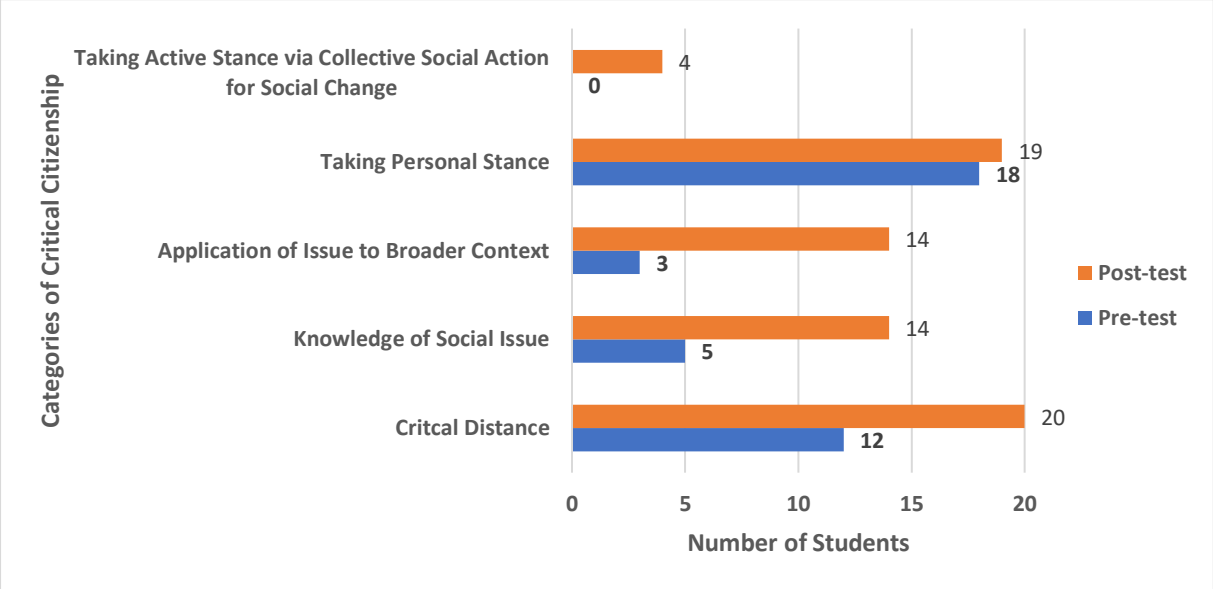

Figure 4: Prevalence and Extent of Critical Citizenship in Issue #3: Civil Conflict

In the pre-test result, 18 of the PSMTs gave statements consistent with the *taking personal stance* category. In general, the PSMTs expressed concern about the need to maintain peace, safety, and security for the welfare of everyone, their loved ones and fellow citizens. They indicated the need for everyone to show concern about what is happening and be involved in finding solutions. They also mentioned the need for the government to step up and eradicate acts of violence and terrorism. There were 12 PSMTs who demonstrated a *critical distance*. Many of them stated the relevance of mathematics to address the issue like using arithmetic, calculus, and descriptive statistics. However, there were some who mentioned that mathematics cannot be used to address the issue. There were only five PSMTs who articulated *knowledge of the social issue*. These PSMTs noted that the existence of terrorists in some parts of the country can really affect the country's progress. They acknowledged the effort of the government and articulated the need to continue the peace talks. Only three

PSMTs exhibited *application of the issue to a broader context* stating the importance of being a law-abiding citizen.

In the post-test result, all 20 PSMTs articulated the *critical distance* category. They emphasized the relevance of mathematics to address the issue of armed conflict. Many realized the applicability of linear functions, simple linear regression analysis and correlation analysis to address this. Nineteen PSMTs articulated the *taking personal stance*. In generally, the PSMTs expressed the same concerns as in the pre-test. They specified the need for the government to make safety and security a top priority. There were 14 PSMTs who expressed *knowledge of the social issue* and the *application of the issue to a broader context*. The sentiments expressed by the PSMTs were just the same as the ones they mentioned in the pre-test. There were four PSMTs who articulated *taking active stance via collective social action for social change* unlike in the pre-test where there was none. The PSMTs argued for the need to raise awareness through social media.

Discussion

The findings of the study showed an improvement in the PSMTs' mathematical proficiency after the PSMTs went through the social-issue-themed mathematics online lessons. The findings also showed increase of the number of PSMTs expressing each category of critical citizenship about the three social issues, namely, government-led war on drugs, widespread fake news, and civil conflicts, after going through the social-issue-themed mathematics online lessons. Before and after being exposed in the lessons, it was found that most PSMTs were comfortable in expressing thoughts which resonated with *critical distance* and *taking personal stance (passive stance)* categories of critical citizenship in each of the social issues.

Most PSMTs expressed critical distance in both CCT pre-test and post-test. Their expressions suggest appreciation and seeing a positive view of mathematics contents such as descriptive statistics, logic and reasoning, and linear functions when used as a tool to address social issues. This corroborates with the finding across studies in the literature. Previous studies showed positive change in most students' perceptions of the utility of mathematics when they have engaged in social inquiry with mathematics in the classroom (e.g., Brelias, 2015; Goodson-Espy et al., 2016; Turner et al., 2009; Verzosa, 2015).

In terms of *knowledge of the social issues*, the PSMTs realized that they were more knowledgeable about the issues and more appreciative of mathematics as a tool to promote understanding of the issues than before being exposed to the lessons. However, they all agreed also that there is still more to learn. Their communications of feelings about the issues suggested only some limited awareness. Hong (2018) argued that awareness of social problems, requires one to be able to exhibit knowledge and skills to understand causes, processes, and expected outcomes of the social issues and create solutions for them. Furthermore, in order for PSMTs to *apply an issue to broader contexts*, they must be at least knowledgeable about the issue at hand. According to Oh and Kwon (2014), one must imbibe a certain depth of understanding of the issue under scrutiny to be able to expand and comment to broader ones such as issues related to fairness, inequality, human rights, poverty, and/or social justice, to name a few.

Findings showed that the PSMTs were, generally, least confident in expressing thoughts which resonated with *taking active stance via collective social action for social change* category of the critical citizenship. Perhaps the PSMTs' sociocultural background could be

one of the factors that limit the possible avenues for the development of critical citizenship. In their study, Oh and Kwon (2014) pointed out that PSMTs must understand the prerequisite conditions to develop critical citizenship and believe in the need for the development of critical citizenship. In the present study, the PSMTs expressed their values, feelings, and emotions regarding problems of social order. Some PSMTs expressed thoughts about social-order issues applied in broader contexts of justice and the need for discipline. However, in general, few PSMTs took active stance about their individual role in implementing social change. Participants did not have a common position on social problems, and their awareness were varied. Also, PSMTs are in an environment where participation in social movements is discouraged. For example, some confessed that their parents disallow them from joining street protests.

Conclusion

To conclude, our analysis showed that the view of mathematics as objective, value-free, decontextualized, and non-political can no longer hold. Mathematics has a role to play in maintaining an orderly society. This role is to provide and ensure the acquisition of needed mathematical skills and proficiencies that enable learners to process numerical data associated with problems of social order. Moreover, social-issue-themed mathematics lessons are ways that can help develop "critical" citizens who are motivated to address some of these problems that threaten the breakdown of social order.

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Contact email: waldem.dejoras@obf.ateneo.edu