

Mining Complex Movement Sequences to Understand Cognitive Strategies in Puzzle Solving: Integrating Computing Analytics and Psychological Insights

Subeksha Shrestha, London Metropolitan University, United Kingdom

Wendy Ross, London Metropolitan University, United Kingdom

Arnabi Modak, London Metropolitan University, United Kingdom

Upendo Daudi Many, London Metropolitan University, United Kingdom

Ganegodage Chathuranga Madushan Dharmarathne, London Metropolitan University, United Kingdom

Prapa Rattadilok, London Metropolitan University, United Kingdom

The Paris Conference on Education 2026
Official Conference Proceedings

Abstract

Puzzle solving behaviour represents an important domain within educational research, offering insights into cognitive strategy formation, spatial reasoning, and decision making processes during interactive learning activities. This study investigates cognitive patterns underlying problem solving performance using trajectory based sequence analysis of a 10 coin triangular puzzle exhibited at a museum. Despite the growing use of computational analytics in education, understanding cognitive processes remains challenging as outcome evaluation alone cannot capture strategy. Using behavioural interaction data collected from puzzle solving sessions, the study applies data mining and sequence analysis techniques to examine movement trajectories, coin selection frequencies, and transition structures across solving attempts. The research focuses on identifying critical early stage decisions, high frequency coin usage patterns, and terminal sequence formations that influence solution success or failure. Each coin position is treated as a discrete cognitive decision node to extract dominant behavioural pathways associated with effective or ineffective problem solving strategies. Key research areas include computational detection of critical coin transitions for modelling puzzle solving dynamics, analysis of behavioural insights in psychology oriented puzzle tasks, temporal gap analysis and behavioural pause patterns as predictors of problem solving outcomes, and sequential decision behaviour modelling in grid based cognitive tasks. Temporal cognition is examined through pause interval analysis to evaluate cognitive load variation, uncertainty resolution, and strategic reflection during interaction sequences. This interdisciplinary study contributes to educational data mining and cognitive psychology by providing a computational framework for understanding insight related problem solving behaviour, with implications for learning analytics and cognitive assessment modelling.

Keywords: cognitive study, puzzle solving, data mining, sequence analysis, behavioural analytics

iafor

The International Academic Forum
www.iafor.org

Introduction

Problem solving unfolds through sequences of decisions and actions, yet it is often evaluated only through final outcomes, which obscures the cognitive processes through which solutions emerge (Duan et al., 2025; Kim et al., 2026). This sequential reasoning is especially important in structured tasks where each move changes the problem state and shapes subsequent choices. Understanding these step-by-step processes is therefore essential for explaining how people develop effective or ineffective problem-solving strategies (Wang et al., 2023; Wang & Wieman, 2022). Puzzle-based environments provide a useful setting for examining behaviours because they combine controlled task structures with rich interaction data. These environments capture fine-grained behavioural traces, making it possible to study not only what participants achieve, but how they explore, plan, hesitate, and adapt over time (Doorn et al., 2025; Kim et al., 2026; Pusey et al., 2021). However, most existing analyses still rely on aggregate measures such as correctness or completion time, which overlook the temporal, sequential, and spatial organisation of behaviour.

Recent advances in educational data mining and learning analytics have enabled process-oriented approaches that analyse temporally ordered interaction data to reveal behavioural patterns (Duan et al., 2025; Torre et al., 2024). Sequence-based methods, including sequential pattern mining, can identify recurring action structures and transition pathways in problem-solving data (Wang et al., 2024). Despite this progress, their application to spatially constrained puzzle environments remains limited, particularly in studies that aim to link movement patterns to underlying cognitive strategies (Lieb et al., 2025).

To address this gap, this study examines problem-solving behaviour through trajectory based sequence analysis of a ten-coin triangular puzzle using interaction data collected from visitors at the Copernicus Science Centre in Warsaw. The puzzle provides a controlled environment for studying decision making, as each move alters the system state and requires sequential reasoning, spatial planning, and strategy refinement. Its constrained yet challenging structure enables systematic analysis of movement trajectories while maintaining clear interpretation of behavioural patterns.

The study contributes a computational framework that integrates data mining techniques with cognitive and psychological interpretation. By representing actions as trajectories across decision points, the framework captures the temporal, sequential, and spatial characteristics of problem-solving behaviour. This enables the identification of movement patterns, key transitions, and behaviours associated with successful and unsuccessful attempts. From an educational perspective, the approach supports deeper understanding of strategy development and provides potential applications for learning analytics, targeted interventions, and adaptive learning environments.

Literature Review

The increasing availability of digitally recorded behavioural data has led to substantial growth in computational approaches for analysing interaction sequences in problem-solving and learning environments. Rather than relying solely on outcome-based indicators such as correctness or completion time, recent research has shifted toward examining temporally ordered behavioural traces to uncover strategic regularities, transition dynamics, and latent decision pathways embedded within interaction data. This shift reflects a broader movement in learning analytics and educational data mining toward process-oriented analysis of complex

behavioural data. Within this context, sequence-based computational techniques have emerged as powerful tools for analysing ordered interaction traces. Sequential pattern mining (SPM) enables the identification of recurring behavioural structures embedded within action sequences. Unlike aggregate indicators such as action frequency or total duration, SPM preserves temporal ordering and captures local structural regularities, allowing researchers to uncover strategic pathways and behavioural motifs that characterise problem-solving processes (Lu et al., 2025). Such approaches have proven particularly valuable in examining higher-order competencies, including problem-solving and strategic reasoning.

In digital puzzle and game-based learning environments, sequence-based methods have been applied to differentiate effective and ineffective problem solvers. Wang et al. (2024) analysed ordered action sequences within a rule-based puzzle game and identified distinct behavioural patterns associated with successful performance. Their findings demonstrate that structured interaction traces can reveal meaningful differences in strategic organisation that are not observable from final outcomes alone. Similarly, Acosta et al. (2025) examined trace log data from a collaborative game-based learning environment and identified behavioural transition patterns associated with collaborative problem-solving practices and learning gains. By modelling transitions between behavioural states, their study highlights the importance of representing interaction data as temporally structured sequences when analysing problem-solving dynamics.

Sequence-based analytics have also been explored in educational puzzle contexts to support instructional insight. Gomez, et al., (2021) transformed raw gameplay logs from an educational puzzle game into interpretable behavioural sequences and developed process-oriented metrics to support classroom monitoring and feedback. While their work demonstrates the pedagogical value of sequence mining, the focus remained primarily on aggregated process indicators and error patterns rather than detailed structural movement transitions within constrained spatial configurations. Beyond game-based learning environments, computational analysis of temporally ordered behavioural data has been applied in large-scale assessment contexts. Jerez et al. (2026) employed sequential process analysis on log data from complex problem-solving assessments to identify distinct behavioural subtypes among slow responders. Their findings illustrate how analysing ordered action and pause patterns can reveal process-level differences that are not captured by aggregate performance measures. This work reinforces the importance of examining fine-grained interaction sequences to better understand heterogeneous cognitive strategies.

Process mining approaches have similarly been used to reconstruct behavioural flows from sparse event logs in training environments. Becker et al. (2022) applied process mining techniques to puzzle-based cybersecurity training data to derive behavioural process models from interaction traces. Their study highlights both the potential and the challenges of transforming low-level event data into interpretable behavioural representations in process-oriented learning tasks. Additional research has demonstrated how data mining techniques can extract semantically meaningful indicators from interactive puzzle environments. Wang et al. (2021) showed that engineered features derived from interaction log data in an interactive puzzle simulation can automate the assessment of problem-solving practices. Likewise, Liu et al. (2023) analysed log data from the puzzle-based game *Baba Is You* to characterise learning progression and extract behavioural metrics predictive of future performance. However, their approach focused on feature engineering and predictive modelling rather than examining structural movement transitions or identifying critical state transformations within constrained spatial puzzles.

Similarly, Liu & Maya (2022) integrated sequence mining and probabilistic modelling to uncover students' problem-solving phases and transition dynamics in a puzzle-based learning environment. Their findings demonstrate that temporally ordered interaction traces can reveal strategic shifts and behavioural transitions that are not visible through outcome measures alone. While these studies underscore the value of computational sequence analysis for understanding problem-solving processes, they largely emphasise phase detection, clustering, or predictive modelling rather than detailed structural analysis of movement dynamics within spatially constrained puzzles.

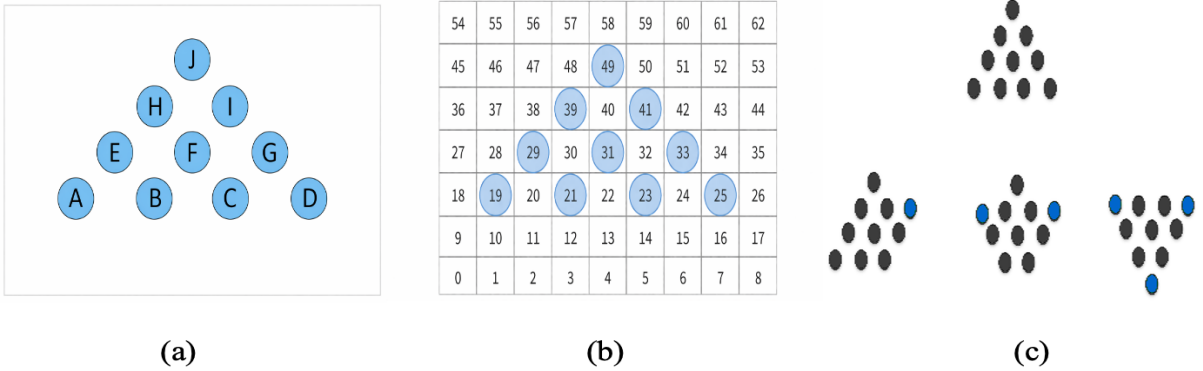
Spatial coin-movement puzzles such as the triangular coin puzzle have been explored primarily from a mathematical and combinatorial perspectives. (McCaffrey & Atwill, 2018) derived a general formula for determining the minimum number of coin movements required to invert triangular and related geometric arrangements. Although this study provides insight into optimal structural solutions, it does not examine the behavioural processes through which individuals attempt to solve such puzzles. The dynamic movement sequences, decision pathways, and cognitive strategies underlying human performance in spatial coin puzzles remain largely unexplored from a computational and psychological standpoint.

Although prior research demonstrates the value of sequence mining, process mining, and computational analytics for analysing behavioural traces in digital learning and puzzle environments, most existing studies focus on phase identification, predictive modelling, clustering, or aggregated behavioural metrics. While these approaches reveal broad behavioural patterns, they provide limited insight into the fine-grained spatial dynamics that emerge during step-by-step puzzle solving. Relatively little attention has been given to analysing detailed movement trajectories within spatially constrained puzzles, where individual actions transform the configuration of the puzzle state space. Existing studies rarely examine how the selection of specific objects, the order of movements, and spatial transitions between positions collectively shape solution pathways. Furthermore, the influence of early decision behaviour, such as first-move selection, and the role of key interactive elements in determining successful outcomes remain underexplored.

To address these limitations, the present study investigates puzzle-solving behaviour through movement sequence analysis in a triangular coin puzzle. Specifically, the study analyses individual movement trajectories, identifies common movement patterns across participants, examines first-move decision behaviour, evaluates the relationship between initial actions and problem-solving outcomes, detects key coins associated with successful solution pathways, and explores temporal gaps between movements. By integrating spatial movement analysis with behavioural and temporal insights, this work provides a deeper understanding of human problem-solving dynamics in spatial coin puzzles.

Methodology

Figure 1
Illustrates the Puzzle Representation Framework: (a) Coin Labels (A–J), (b) Grid Indexing, and (c) Movement Encoding Across Successive Puzzle



Panel (a) presents the coin labelling scheme applied to the triangular coin puzzle, in which ten coin positions are assigned alphabetical identifiers (A–J). The coins are arranged in an inverted triangular hierarchy: Coin A occupies the apex of the base row alongside Coins B, C, and D; the second row contains Coins E, F, and G; the third row contains Coins H and I; and Coin J occupies the single apex position at the top. This labelling scheme provides a standardised referencing system for recording and analysing individual coin selections across participant sessions.

Panel (b) presents the grid indexing system used to encode spatial positions within the puzzle state space. The physical board is represented as a two-dimensional integer grid spanning cells 0 to 62, structured across nine columns and seven rows. Each coin occupies a specific grid cell corresponding to its physical position on the board, for example, Coin A maps to cell 19, Coin D to cell 25, and Coin J to cell 49. This grid representation enables all coin movements to be encoded as ordered cell-to-cell transitions (e.g., 49→13), forming the basis for trajectory extraction, temporal gap computation, and sequential pattern mining conducted in this study.

Panel (c) provides a visual example of a three-step movement sequence that satisfies the puzzle objective of being solved in exactly three moves, illustrating how coin transitions are recorded across successive puzzle states. The blue-highlighted coins in each state representation denote the positions actively displaced at each step, while the remaining black coins retain their prior positions. This encoding captures both the spatial displacement and the sequential ordering of moves, enabling the reconstruction of complete participant trajectories for downstream computational analysis.

Together, these three components define the representational framework through which raw interaction data are transformed into structured movement sequences suitable for frequency mining, temporal pattern analysis, and spatial trajectory modelling.

Dataset and Preprocessing

This study draws on behavioural and self-report data generated through the Coin Puzzle Project, a ten-position triangular coin arrangement task (positions A–J). Each participant session produced a timestamped sequence of coin moves, an outcome label per move (success/unsuccessful), the elapsed time between consecutive moves, and a set of seven self-

report psychological responses: Companionship, EnjoysPuzzles, FeelInsight, FeelStuck, WayOfSolvingTheProblem, GaveUpReason, and TerminationType. Gender was also recorded across four categories. Sessions were assigned anonymised identifiers (format YYYYMMDD-XXXX). Prior to analysis, records with missing outcome labels or incomplete move sequences were excluded. All categorical psychological variables were encoded as ordinal or nominal integers appropriate to each downstream mining task.

Data Mining Methods

Frequency Mining and Descriptive Pattern Extraction

Frequency distributions were calculated across coin positions and transition pairs to identify the most common movement patterns. Consecutive moves were represented as bigrams (e.g., $D \rightarrow A$, $A \rightarrow D$), and the top 15 transitions were ranked by frequency to reveal dominant action sequences.

The frequent occurrence of transitions such as $D \rightarrow A$ and $A \rightarrow D$ indicates repeated movement between key positions, suggesting local search behaviour and attentional anchoring rather than globally planned strategies. This pattern aligns with cognitive load theory (Sweller, Merriënboer, & Paas, 2019), where limited working memory capacity may encourage shorter, repetitive actions instead of long term planning.

Association Rule Mining via Correlation Analysis

Pearson correlation coefficients were computed across the seven psychological self-report variables from the dataset to identify pairwise associations (Boslaugh, 2012). The correlation matrix was visualised using an interactive heatmap to explore underlying associations.

The analysis showed moderate correlations between FeelStuck and WayOfSolvingTheProblem ($r = 0.36$) and TerminationType ($r = 0.31$), suggesting that greater difficulty was linked to specific solving behaviours and unsuccessful task completion. The weak correlation between EnjoysPuzzles and FeelStuck ($r = 0.11$) indicates that puzzle enjoyment had limited influence on reducing perceived difficulty. These findings reflect psychological patterns related to fixation, persistence, and disengagement during problem solving.

Temporal Pattern Mining via TimeGap Analysis

Inter move response time (TimeGap, seconds) was analysed as a behavioural signal across coin transition pairs and visualised through a 10×10 transition heatmap. This temporal data mining approach identified timing patterns within sequential move logs by comparing successful and unsuccessful transitions.

Longer unsuccessful TimeGaps, such as $B \rightarrow J$ (9.07s) and $C \rightarrow J$ (6.25s), indicate points where participants spent more time before making incorrect moves, suggesting hesitation or misdirected reasoning. In contrast, longer successful transitions reflect deliberate processing before correct decisions, highlighting the role of reflective thinking in problem solving.

Spatial Pattern Mining via Movement Visualisation

Participant movement trajectories were mapped onto a two-dimensional coordinate space representing the triangular coin layout. Animated sequences were used to identify spatial patterns, preferred movement paths, and unusual trajectories. Key coin analysis identified the three positions most frequently linked to successful moves (1020, 1017, and 1009 occurrences), highlighting important nodes within the movement network.

The spatial analysis reflects differences in problem-solving strategies. Structured trajectories suggest stronger spatial reasoning and mental representation, while fragmented paths indicate trial and error behaviour. The concentration of successful moves around specific positions suggests that highly connected spatial locations act as intuitive anchors during puzzle solving.

Participant Profiling and Segmentation

The Movement Analysis module included a Participant Analysis view for profiling individual move sequences, outcomes, and psychological responses. Participants could be grouped by TerminationType (completed vs. abandoned) or FeelStuck ratings, enabling comparison of behavioural patterns across different solver profiles.

This data driven segmentation aligns with psychological typology research by identifying solvers with high FeelStuck scores and early termination as a distinct group characterised by lower frustration tolerance and more constrained problem-solving approaches. Mining behavioural signatures such as shorter sessions, longer per move TimeGaps, and repeated low success transitions provides empirical support for psychological profiles previously inferred only from self-reported data.

Connecting Data Mining to Psychological Analysis

The five mining operations form an integrated analytical pipeline that combines computational pattern discovery with psychological interpretation. Together, they capture multiple dimensions of problem-solving behaviour, including frequency, association, temporality, spatiality, and individual differences.

Frequency mining identifies common actions, association analysis links behaviours with psychological characteristics, temporal analysis reveals hesitation and errors, spatial analysis examines movement strategies, and segmentation highlights differences between participants. Collectively, these methods transform raw interaction logs into measurable indicators of cognitive load, fixation, insight, spatial reasoning, and frustration tolerance. The Copernicus Dashboard we have created provides the analytical interface that connects behavioural data with psychological theory, enabling a psycho-computational understanding of problem-solving processes.

Results and Discussion

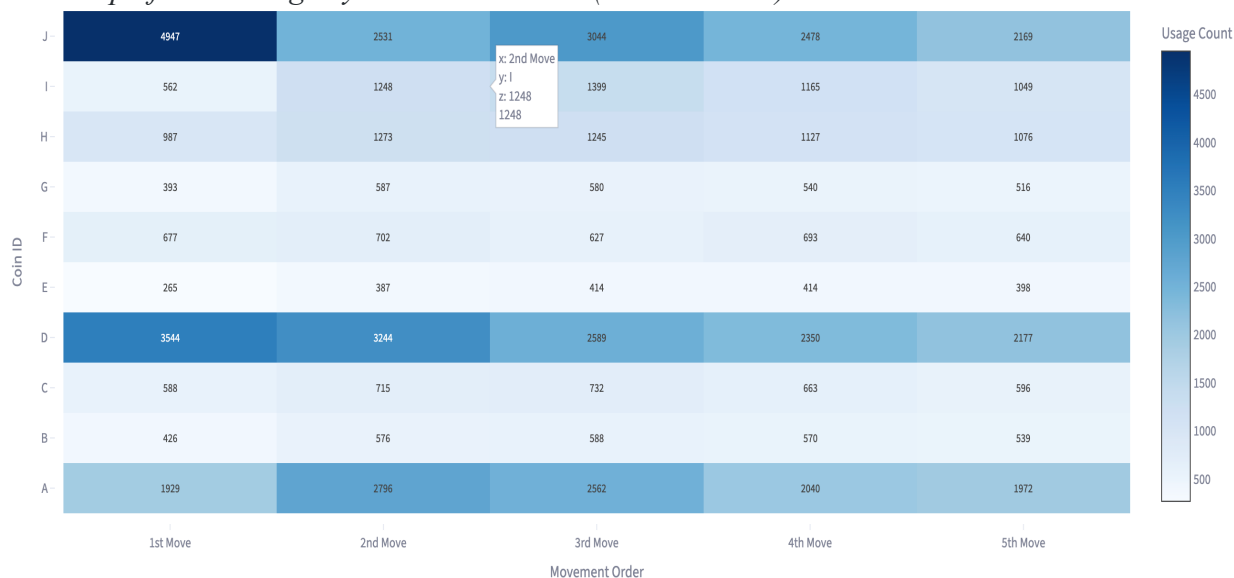
Puzzle solving behaviour is shaped not only by the frequency of individual moves, but also by the structural organisation of movement trajectories, the role of specific positions within sequences, and the relationship between spatial decisions and solution outcomes. Together, these findings provide a deeper computational understanding of how participants navigate the constrained state space of the triangular coin puzzle.

Positional Frequency Patterns in Move Sequences

Figure 2 illustrates the selection frequency of the ten coin positions (A to J) across the first five moves, with colour intensity indicating usage frequency.

Figure 2

Heatmap of Coin Usage by Movement Order (1st–5th Move)



Coin J was the most frequently selected position in Move 1 ($n = 4,947$) and remained highly used throughout, suggesting strong attentional focus due to its apex location in the triangular layout. Coin D showed consistently high engagement across all moves ($3,544 \rightarrow 2,177$), indicating its role as a key navigational point in participants' solving strategies. Coin A displayed a peak at Move 2 ($n = 2,796$), supporting the frequent $D \rightarrow A$ transition observed in sequence analysis.

Conversely, Coins E, G, and B showed consistently low selection frequencies, suggesting reduced strategic importance during solving. Overall, the heatmap reveals a concentrated selection pattern where participants focused on a small number of high salience positions rather than exploring the puzzle space uniformly.

Per-Coin Success vs. Failure Rate Analysis

Figure 3

Success vs. Failure

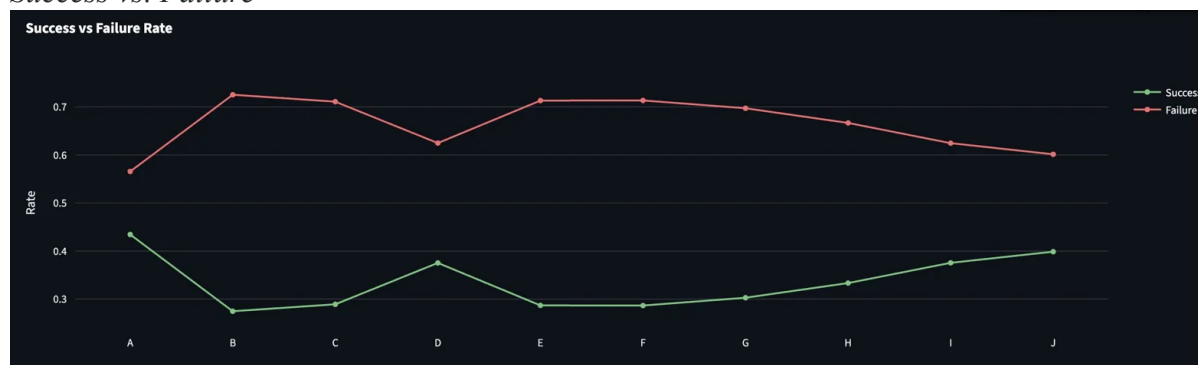


Figure 3 presents success and failure rates across the ten coin positions (A–J) using a dual line chart in the Copernicus Dashboard. Failure rates were consistently higher than success rates across all positions (approximately 0.60–0.72), indicating that unsuccessful moves were more common regardless of coin selection. Coin B showed the highest failure rate (~ 0.72) and lowest success rate (~ 0.28), while Coins A and E also demonstrated high failure rates despite frequent selection.

Coin J, the most frequently selected position (40,443 moves), achieved the highest success rate (~ 0.40) and lowest failure rate (~ 0.60), highlighting its importance in both successful and unsuccessful paths. Coins H and I showed relatively higher success rates (~ 0.33) despite lower usage, suggesting more deliberate selection. Overall, the results indicate that frequently selected positions did not always lead to successful outcomes, reflecting local heuristic search behaviour and aligning with participants' reported feelings of being stuck and trial and error strategies.

Overall Coin Movement Patterns Across Complete Puzzle Trajectories

Figure 4

Overall Frequency of Coin Movements Across Complete Puzzle Trajectories

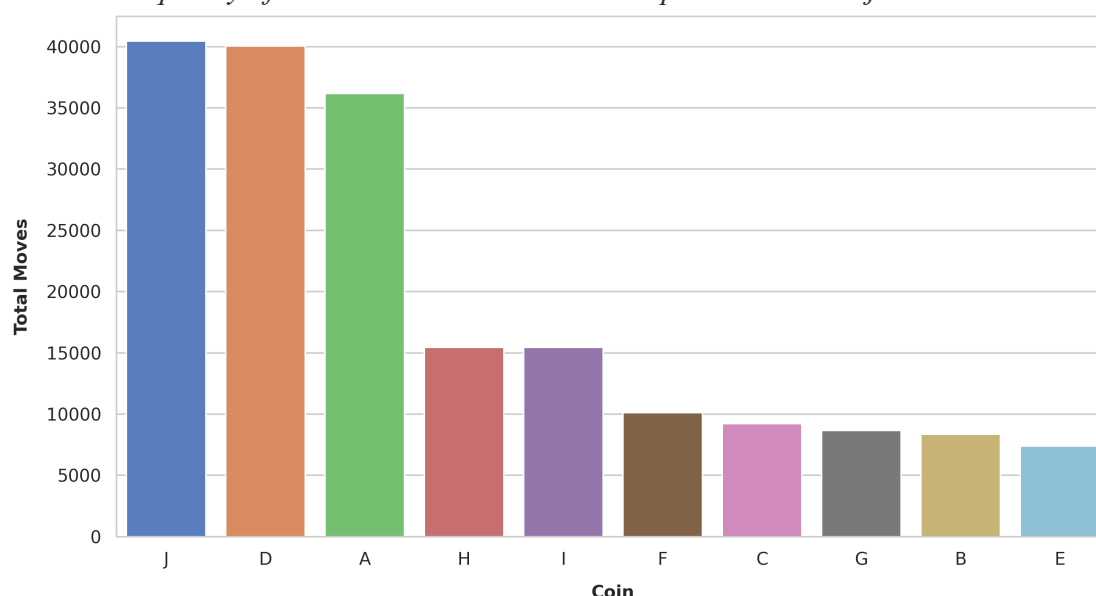


Figure 4 presents the movement frequency of each coin across all recorded puzzle trajectories. Each movement event was analysed to identify the selected coin, and occurrence counts were calculated across all sessions.

Coin J was the most frequently moved coin (40,443), followed by Coin D (40,046) and Coin A (36,169). Together, these three coins accounted for approximately 61.3% of all movements, despite representing only three of the ten positions. Coin H (15,447) and Coin I (15,443) exhibited almost identical movement frequencies, whereas Coins F (10,134), C (9,236), G (8,669), B (8,349), and E (7,399) were moved less frequently.

The concentration of movements around Coins J, D, and A indicates that participants focused on a small subset of positions during problem solving rather than exploring the puzzle space uniformly.

Strategic Solution Patterns Identified From Frequent Movement Sequences

Figure 5

Top Ten Movement Sequences Identified Through Frequency Mining, Showing Successful and Unsuccessful Puzzle Outcomes

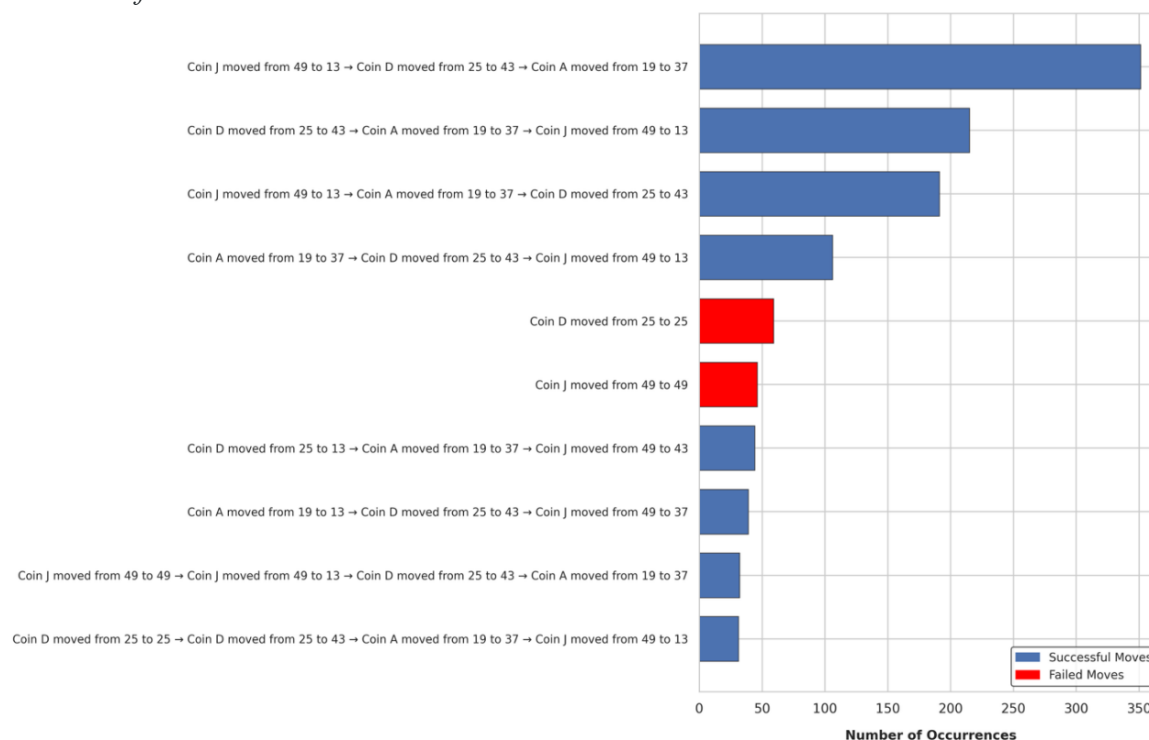


Figure 5 presents the ten most frequently occurring movement sequences identified through frequency mining of the reconstructed participant trajectories. The complete movement sequences were ranked according to their frequency of occurrence and compared across successful and unsuccessful puzzle outcomes. The final three moves within each movement sequence were then interpreted against the three predefined strategic solution patterns specified in the puzzle framework: Movement Across the Median, Rosette A, and Rosette B (Repeated Spatial Trajectories A and B).

The first four sequences followed the predefined Movement Across the Median strategy. The most frequent sequence consisted of Coin J (49→13), Coin D (25→43), and Coin A (19→37), occurring 351 times. The remaining variations of this strategy in the second, third and fourth movement sequences occurred 215, 191, and 106 times, together accounting for 863 occurrences.

The fifth and sixth sequences represented unsuccessful same position movements: Coin D (25→25), occurring 59 times, and Coin J (49→49), occurring 46 times, where no change occurred in the puzzle state. The seventh and eighth sequences matched Rosette B and Rosette A, consisting of Coin D (25→13), Coin A (19→37), and Coin J (49→43), occurring 44 times, and Coin A (19→13), Coin D (25→43), and Coin J (49→37), occurring 39 times, respectively. The ninth and tenth sequences began with same position movements before transitioning into the Movement Across the Median strategy: Coin J (49→49), occurring 32 times, and Coin D (25→25), occurring 31 times. Overall, frequency mining showed that successful movement sequences largely followed predefined strategic patterns, with movement across the median being the most dominant, followed by Rosette B and Rosette A.

Animation Example of a Successful Puzzle Solution

Figure 6

Animation of a Successful Puzzle Solution Reconstructed From Participant 20230623-0069, Showing the Sequential Coin Movements From Start to Final Solution

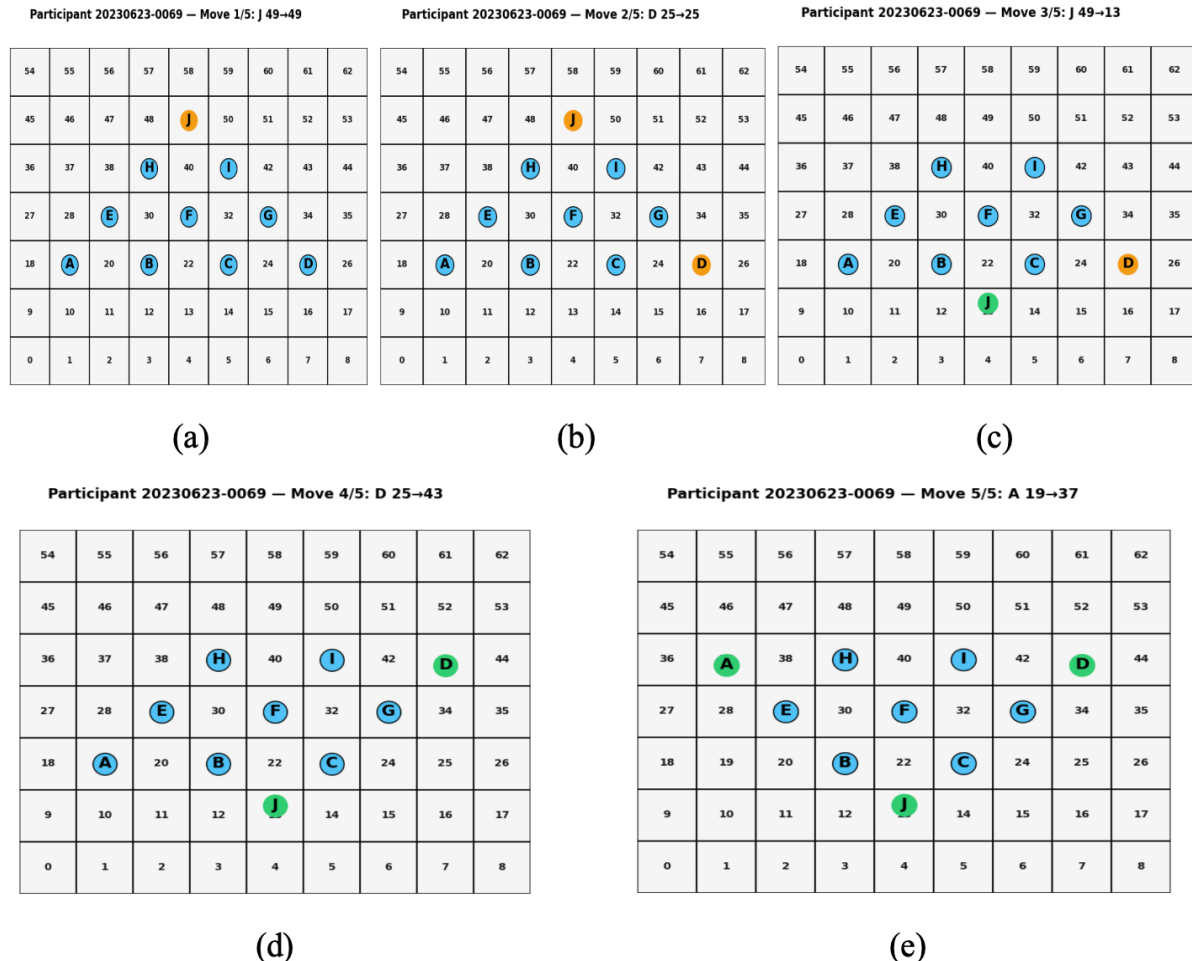


Figure 6 presents an animation example of a successful puzzle solution for participant **20230623-0069**. The animation was generated by reconstructing the participant's recorded movement trajectory and replaying each recorded coin movement sequentially to illustrate the transition from the initial board state to the successful solution. This provides an illustrative example of how a strategic solution pattern identified through the frequency mining analysis is demonstrated through an individual participant's movement trajectory.

The first two movements consist of Coin J (49→49) and Coin D (25→25). Although these movements do not alter the board configuration, they form part of the participant's complete movement trajectory before the successful solution is achieved. The remaining three movements consist of Coin J (49→13), Coin D (25→43), and Coin A (19→37). These movements correspond to the predefined Movement Across the Median strategy and result in a successful puzzle solution. The animation illustrates how the participant's trajectory progresses from the initial board configuration to the successful execution of the Movement Across the Median strategy.

Animation Example of an Unsuccessful Puzzle Solution

Figure 7

Animation of an Unsuccessful Puzzle Solution Reconstructed From Participant 20230605-0000, Showing the Sequential Coin Movements From Start to Unsuccessful Completion

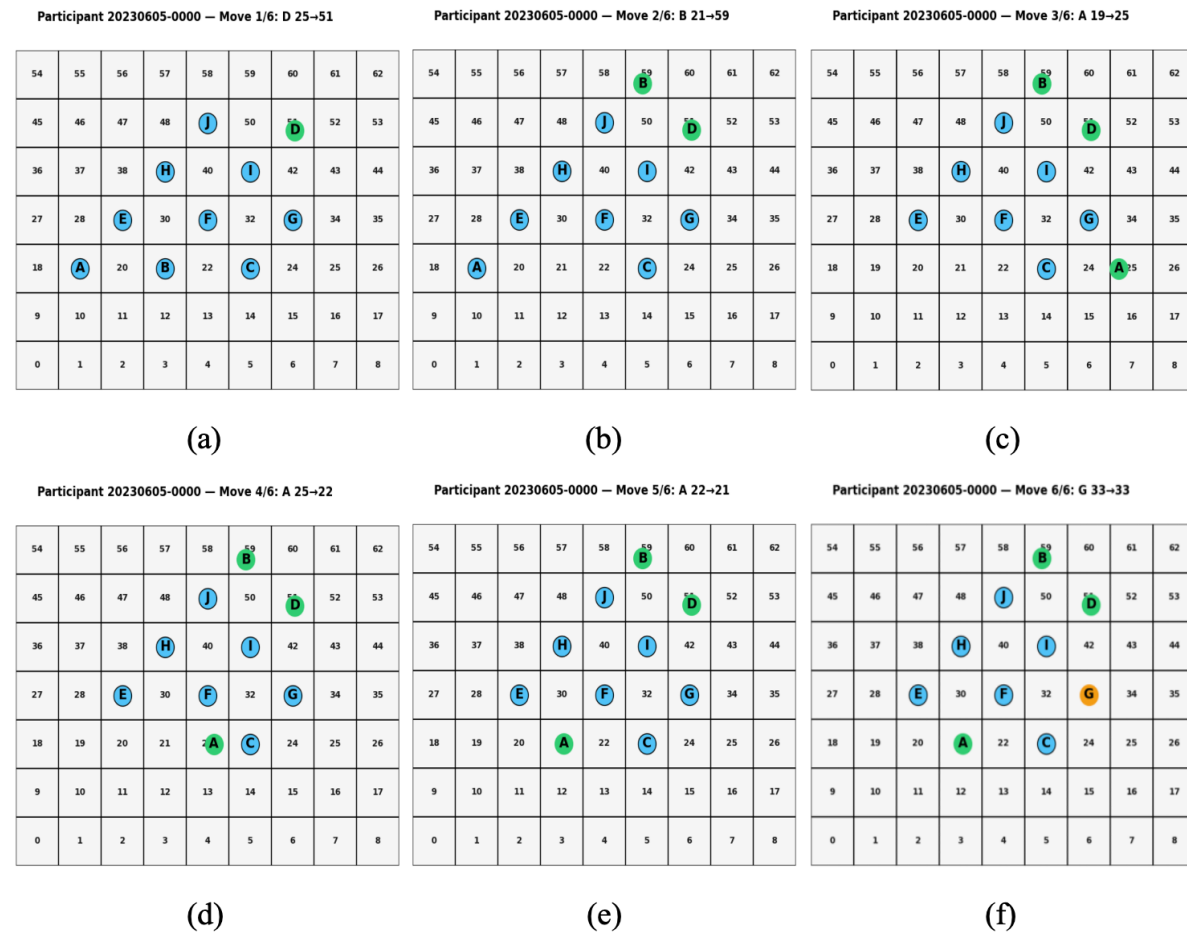


Figure 7 presents an animated reconstruction of an unsuccessful puzzle attempt by participant 20230605-0000. The recorded coin movements are replayed sequentially to illustrate the progression of the solution. Unlike successful trajectories, this movement sequence does not converge towards the strategic solution patterns identified through frequency analysis.

The participant performed six recorded movements involving Coins D, B, A, and G. The movement trajectory began with Coin D (25→51), followed by Coin B (21→59). Coin A was subsequently moved three consecutive times: Coin A (19→25), Coin A (25→22), and Coin A (22→21). The final movement involved Coin G (33→33), which did not result in any change to the board configuration.

Unlike the successful example presented in Figure 6, the movement trajectory did not terminate in any of the predefined strategic solution patterns (Movement Across the Median, Rosette A, or Rosette B) described in the puzzle framework. This animation complements the frequency mining analysis by providing a visual example of an unsuccessful movement trajectory that does not conclude with any of the recognised strategic solution patterns identified within the puzzle framework.

Figure 8
Frequency Distribution of Coin Movements Across the First Five Movement Stages

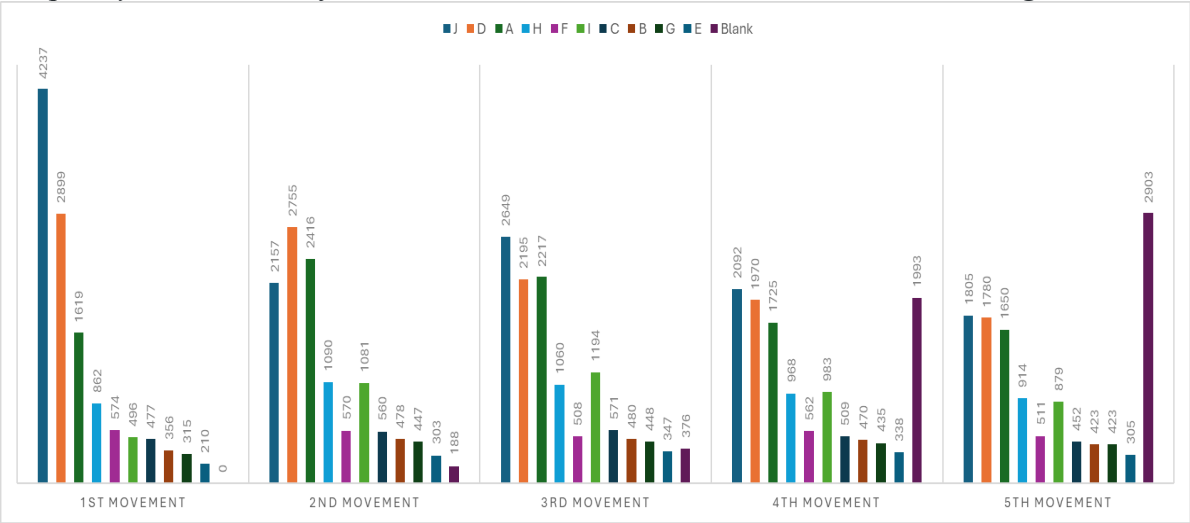


Figure 8 shows the frequency distribution of coin movements across the first five puzzle stages, highlighting participants’ movement preferences and strategic choices. Coin J was consistently the most frequently moved coin, followed by Coins D and A, indicating their importance in early and intermediate solving strategies. In contrast, Coins E, G, and B showed lower movement frequencies, suggesting limited involvement during the solving process. The Blank position increased notably in later stages, particularly at Move 5, indicating a greater reliance on empty space transitions as participants explored more complex configurations. The results show a non-uniform movement pattern, with participants focusing on key coin positions while gradually shifting towards more strategic repositioning as the puzzle progressed.

The Time Gap (Pausing Time or Thinking Time) for the Coin Movements

Figure 9
The Heatmaps Compare Average Time Gaps Between Coin Movements in Unsuccessful and Successful Puzzle Attempts

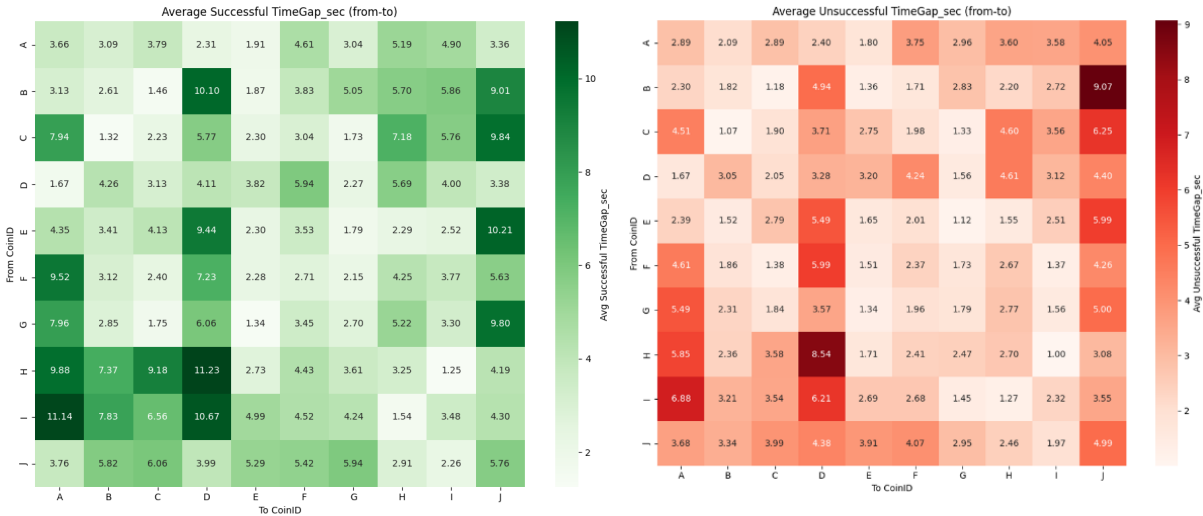


Figure 9 presents heatmaps of the average time gaps between consecutive coin movements for successful and unsuccessful sequences, with darker shades indicating longer decision times.

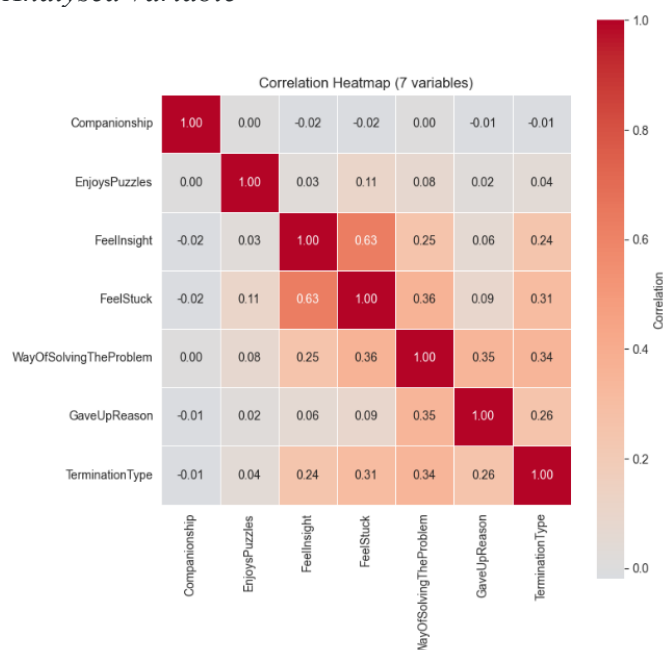
Successful movements (left green heatmap) show longer pauses for transitions such as I→A (11.14s), H→D (11.23s), and E→J (10.21s), suggesting greater cognitive consideration, while shorter gaps indicate more immediate decisions. Unsuccessful movements (right reddish orange heatmap) show longer pauses for transitions including B→J (9.07s), H→D (8.54s), and I→A (6.88s), reflecting increased uncertainty or hesitation.

Overall, the patterns suggest that pause duration provides insight into decision making processes. Rather than representing simple delays, longer time gaps may indicate deeper cognitive evaluation, while inconsistent pauses in unsuccessful attempts may reflect confusion or trial and error behaviour. These transition level patterns demonstrate how temporal analysis can reveal behavioural indicators of problem solving strategies.

The Relationships Among Seven Categorical Variables

Figure 10

Correlation Heatmap Showing the Strength and Direction of Relationships Between the Seven Analysed Variable



The correlation heatmap illustrates relationships among seven variables related to participants' puzzle solving experiences and outcomes. Most variables show weak correlations, indicating limited direct associations. The strongest positive relationship is observed between Feeling Insight and Feel Stuck ($r = 0.63$), suggesting that participants who experienced moments of insight also tended to report periods of difficulty. Moderate correlations were found between Feel Stuck and Way of Solving the Problem ($r = 0.36$), Way of Solving the Problem and Gave Up Reason ($r = 0.35$), and Way of Solving the Problem and Termination Type ($r = 0.34$), highlighting connections between emotional states, problem solving strategies, and task outcomes.

Several weaker relationships were also identified. Feeling Insight showed low correlations with Way of Solving the Problem ($r = 0.25$) and Termination Type ($r = 0.24$), suggesting that insight experiences were only partially related to strategy choice and final outcomes. Gave Up Reason showed minimal associations with Feel Stuck ($r = 0.09$) and Feeling Insight ($r = 0.06$),

indicating that reasons for stopping were not strongly linked to reported cognitive experiences. Enjoys Puzzles demonstrated consistently minimal correlations with all variables ($r = 0.00$ to 0.11), suggesting that general puzzle enjoyment had little influence on behaviour in this specific task. Similarly, Companionship showed negligible relationships across variables (approximately $r = -0.02$ to 0.00), indicating no meaningful association with puzzle solving experiences or outcomes.

Overall, the heatmap shows that while some cognitive and behavioural factors are interconnected, several personal characteristics have limited influence on solving behaviour. Combining correlation analysis with behavioural data mining provides a broader understanding of problem solving processes beyond final performance outcomes.

Conclusion

The findings highlight the potential of movement sequence analysis as an effective approach for uncovering behavioural patterns underlying problem-solving processes in spatial coin puzzles. By analysing movement trajectories, timing patterns, and decision sequences, this study demonstrates how data driven approaches can provide deeper insights beyond final solution outcomes. However, further research is required to enhance the applicability, interpretation, and educational value of this approach.

A key direction for future research is the development of real-time adaptive learning systems that can respond to learners' behaviours while a task is in progress. The identified movement and pause patterns could be used to dynamically adjust puzzle difficulty, provide personalised hints, and recommend suitable learning strategies, transforming behavioural analysis into immediate instructional support.

Future studies should also validate the approach across different puzzle types, learner populations, and educational contexts. As the current research focuses on a specific triangular coin puzzle, further investigation is needed to determine whether the observed movement patterns, first move effects, and timing behaviours represent general problem-solving strategies or are influenced by the structure of a particular task.

Additionally, future work should explore the cognitive meaning behind these behavioural indicators by examining their relationship with processes such as planning, hesitation, insight, trial and error, and strategy revision. Combining sequence analysis with complementary methods, such as eye tracking or physiological measurements, could provide deeper insights into the relationship between observable behaviours and underlying cognitive processes.

Finally, longitudinal research could investigate whether these behavioural measures can predict learning progress and the development of problem-solving skills over time. Tracking participants across multiple sessions would help determine whether improvements in movement efficiency, pause duration, and solution pathways reflect enhanced reasoning abilities and the transfer of skills to new tasks. Such developments would further establish movement sequence analysis as a valuable tool for educational assessment and personalised learning support.

Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The author declares that no AI or AI-assisted technologies have been used to generate, refine, or correct the content in the manuscript. The ideas, design, procedures, findings, analyses, and discussion are originally written and derived from careful and systematic conduct of the research.

References

- Acosta, H., Hong, D., Lee, S. Y., Min, W., Mott, B. W., & Hmelo-Silver, C. E. (2025). Collaborative Game-based Learning Analytics: Predicting Learning Outcomes from Game-based Collaborative Problem Solving Behaviors. *Proceedings of the 15th International Learning Analytics and Knowledge Conference* (pp. 429–438). Association for Computing Machinery.
- Becker, B. A., Quille, K., Laakso, M. J., Barendsen, E., & Simon. (2022). Process Mining Analysis of Puzzle-Based Cybersecurity Training. *Proceedings of the 27th ACM Conference on on Innovation and Technology in Computer Science Education. 1*. Association for Computing Machinery.
- Boslaugh, S. (2012). *Statistics in a Nutshell, 2nd Edition*. O'Reilly Media, Inc.
- Doorn, N., Vos, T. E., Marin, B., & van Diggelen, M. (2025). Puzzle-based learning for developing software testing skills. *Proceedings of the 2025 29th International Conference on Evaluation and Assessment in Software Engineering Companion* (pp. 202–212). Association for Computing Machinery.
- Duan, Z., Yang, Z., Welles, B. F., & Radke, R. (2025). Beyond Utterance: Understanding Group Problem Solving through Discussion Sequences. *Proceedings of the 27th International Conference on Multimodal Interaction* (pp. 284–293). Association for Computing Machinery.
- Gomez, M. J., Ruipérez-Valiente, J. A., Martinez, P. A., & Kim, Y. J. (2021). Exploring the Affordances of Sequence Mining in Educational Games. *Eighth International Conference on Technological Ecosystems for Enhancing Multiculturality Pages* (pp. 648–654). Association for Computing Machinery.
- Jerez, D., Mazzullo, E., & Bulut, O. (2026). Exploring Slow Responses in International Large-Scale Assessments Using Sequential Process Analysis. *Computers*, 15(1), 64.
- Kim, S., Choi, H., Lee, S., & Kim, S. (2026). ARCTraj: A Dataset and Benchmark of Human Reasoning Trajectories for Abstract Problem Solving. *Proceedings of the 32nd ACM SIGKDD Conference on Knowledge Discovery and Data. 1*, 2687–2698. Association for Computing Machinery.
- Lieb, S., Mazrek, F., & Thormählen, T. (2025). Using Interlocking Building Blocks as a Game Interface. *ACM Games: Research and Practice*, 3(3), 1–18.
- Liu, H., Sun, F.-Y., Rong, F., Nakajima, K., Haber, N., & Salehi, S. (2023). Characterizing Learning Progress of Problem-Solvers Using Puzzle-Solving Log Data. *Proceedings of the 16th International Conference on Educational Data Mining* (pp. 529–533). Educational Data Mining.
- Liu, T., & Maya, I. (2022). Uncovering students' problem-solving processes in game-based learning environments. *Computers & Education*, 182, 1–14.

- Lu, W., Griffin, J., Laffey, J., Sadler, T. D., & Goggins, S. (2025). From In-Game Behaviors to Learning Gains: Constructing Bayesian Networks for Stealth Assessment. *Proceedings of the ACM on Human-Computer Interaction*, 9(6), 566–605.
- McCaffrey, T., & Atwill, O. (2018). Inverting the 10-Coin Triangle Puzzle and Other Shapes: A New General Solution. *arXiv preprint arXiv:1810.02202*.
- Pusey, M., Wong, K. W., & Rappa, N. A. (2021). The puzzle challenge analysis tool. A tool for analysing the cognitive challenge level of puzzles in video games. *Proceedings of the ACM on Human-Computer Interaction*, 5, 1–27. Association for Computing Machinery.
- Torre, M. V., Oertel, C., & Specht, M. (2024). The Sequence Matters in Learning - A Systematic Literature Review. *Proceedings of the 14th Learning Analytics and Knowledge Conference* (pp. 263–272). Association for Computing Machinery.
- Wang, K. D., Cock, J. M., Käser, T., & Bumbacher, E. (2023). A systematic review of empirical studies using log data from open-ended learning environments to measure science and engineering practice. *British Journal of Educational Technology*, 54(1), 192–221.
- Wang, K. D., Liu, H., DeLiema, D., Haber, N., & Salehi, S. (2024). Discovering Players' Problem-Solving Behavioral Characteristics in a Puzzle Game through Sequence Mining. *Proceedings of the 14th Learning Analytics and Knowledge Conference* (pp. 498–506). Association for Computing Machinery.
- Wang, K. D., Salehi, S., Arseneault, M., Nair, K., & Wieman, C. (2021). Automating the Assessment of Problem-solving Practices Using Log Data and Data Mining Techniques. *Proceedings of the Eighth ACM Conference on Learning* (pp. 69–76). Association for Computing Machinery.
- Wang, K. D., & Wieman, C. E. (2022). Applying Sequence Mining to Explore Students' Problem-solving Practices Using an Interactive Simulation-based Task. *Proceedings of the 16th International Conference of the Learning Sciences* (pp. 433–4439). International Society of the Learning Sciences.
- Zhang, C., Kemp, C., & Lipovetzky, N. (2024). Human Goal Recognition as Bayesian Inference: Investigating the Impact of Actions, Timing, and Goal Solvability. *arXiv preprint arXiv:2402.10510*.