Teaching Students to Prove: An International Comparative Analysis of Geometry and Algebra Textbooks

Andrey Koptelov, Sam Houston State University, United States
Alexander Mironychev, Union College of New Jersey, United States

The Paris Conference on Education 2026
Official Conference Proceedings

Abstract

This paper presents findings from an international comparative study of secondary-school mathematics textbooks, with a particular focus on Geometry and Algebra. Given the broad scope of such a comparison, the analysis is limited to two representative topics: proofs related to triangle congruence in Geometry and proofs of selected algebraic formulas. The study examines textbooks from four countries and publishers: the United States (McDougal Littell), France (Hatier), Germany (Ernst Klett Verlag), and Italy (Atlas). The comparison identifies similarities and differences in content organization, instructional design, and pedagogical approaches, with particular emphasis on the presentation, purpose, and role of mathematical proof. In addition, the paper explores opportunities to integrate traditional printed textbooks with online lessons and other digital learning resources to enhance mathematics instruction. Based on the comparative analysis, recommendations are proposed for the development of future secondary-school mathematics textbooks and instructional materials. The study is further informed by the authors' more than twenty years of combined experience teaching secondary mathematics in the Houston Independent School District (HISD), USA, providing practical insights into the role and implementation of mathematical proofs in high-school mathematics courses.

Keywords: mathematics education, mathematical proof, textbook analysis, comparative education, secondary-school mathematics

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Introduction and Theoretical Framework

In recent years, numerous studies have undertaken international comparisons of mathematics education and instructional materials, examining a variety of curricular and pedagogical perspectives. One area of research has focused on cultural differences in the implementation of similar mathematical tasks across countries. For example, Clivaz and Miyakawa (2020) compared classroom practices in Switzerland and Japan, demonstrating how similar mathematical activities may be presented and implemented differently across educational contexts.

Another important line of research concerns the role of mathematical proof in contemporary school mathematics (Miyakawa, 2012; Miyakawa & Herbst, 2007). These studies show that, in both France and Japan, formal mathematical proofs are often replaced by empirical verification, discussions of evidence, or activities intended to convince students of the validity of a statement. Rather than proving fundamental mathematical properties, students are frequently asked to justify simple consequences of definitions or previously established theorems.

The teaching of mathematical proof has also been investigated from the perspective of teachers' mathematical knowledge and classroom practices. Knuth (2002) and Wiworo (2021) reported that some teachers experience difficulties constructing formal proofs or avoid presenting proofs directly in classroom instruction. Other comparative studies have examined the treatment of triangle congruence in Chinese and U.S. textbooks (Lo et al., 2021). These studies describe approaches for helping students recognize and justify triangle congruence through interactive discussions and validation activities; however, explicit formal proofs of congruence theorems are often limited or absent.

Comparative textbook research has also explored the organization and sequencing of mathematical content. Mironychev (2015) compared geometry textbooks from the United States, Mexico, and Russia, finding that courses may be organized either around the properties of geometric objects (e.g., lines, segments, triangles, and circles) or around the logical relationships among concepts and theorems. Fujita and Jones (2008) conducted a comprehensive analysis of Japanese mathematics textbooks and observed that formal proofs are often presented with limited explanatory support, which may create challenges for students' understanding of mathematical reasoning. Similarly, Prytz (2007) examined Scandinavian mathematics textbooks and discussed the connections among educational goals, curriculum requirements, textbook content, and the role of mathematical proof.

These studies indicate a broader trend in contemporary mathematics education: formal proofs of fundamental geometric properties and algebraic formulas are increasingly supplemented or replaced by empirical verification and intuitive justification. While these approaches may support accessibility and conceptual exploration, they may also limit students' opportunities to develop rigorous mathematical reasoning and proof-writing skills, which are essential for advanced study in mathematics, science, engineering, and related fields.

The present study investigates the treatment of mathematical proof in secondary-school mathematics textbooks from the United States, France, Germany, and Italy. Specifically, it examines how proofs of fundamental geometric theorems and selected algebraic formulas are presented and the pedagogical roles they serve within each textbook. The study uses comparative textbook analysis as its methodological framework, focusing on representative topics in Geometry and Algebra. One textbook from each country was selected for analysis. Although this sample is not intended to be statistically representative, it provides an initial basis for identifying similarities and differences among national approaches and establishes a foundation for future, more comprehensive comparative studies.

Curriculum and Textbooks for Mathematics Courses

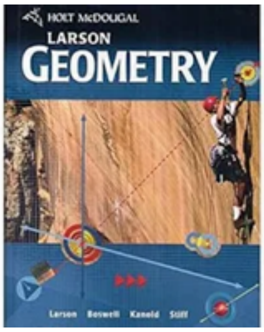
Despite the wide variety of educational and instructional materials available through the Internet, school districts, and educational organizations, these resources are generally grounded in the theoretical foundations and curricular expectations of national education systems. These foundations are reflected in mathematics textbooks developed by different publishers, which serve as important resources for classroom instruction.

For the analysis of mathematical proofs, the present study selected representative secondary-school mathematics textbooks from four countries: the United States (Larson et al., 2017), France (Chapiron et al., 2012), Italy (Vacca et al., 2021), and Germany (Lambacher & Schweizer, 2020). Table 1 provides a brief summary of the selected textbooks and the corresponding mathematics courses in each country.

Table 1
Study of Geometry and Math Curriculum in the High School of Countries

USA: McDougal Littell	**France**: Hatier	**Italy**: Atlas	**Germany**: Klett Verlag
Algebra I – 9 grade	Triangles 3 – 50/320 p.	Geometria 1 – 384 p.	Mathematik 7 – 30/291 p.
Geometry – 10 grade, 900 p.	Triangles 4 – 100/300 p.	Geometria 2 – 336 p.	Mathematik 8 – 30/200 p.
Algebra II – 11 grade	Triangles 5 – 108/256 p.	Geometria 3 – 363 p	Mathematik 9 – 94/250 p.

Figure 1
American Textbook on Geometry Published by McDougal Littell, 2007



In the United States, a high-school Geometry course (see the Fig. 1) is typically taught as a one-year course in Grade 10, following Algebra I. Students generally receive instruction through either two 90-minute lessons or four 45-minute lessons per week. Due to the flexibility of the U.S. education system, some students may complete Geometry in Grade 9 if they have already earned credit for Algebra I during middle school.

Despite the rapid expansion of online educational resources, the structure and content of Geometry curricula in the United States continue to reflect many features of traditional pre-COVID textbooks, including those published by McDougal Littell, Holt, Pearson, and Glencoe. The present analysis is based on the *McDougal Littell Geometry* textbook (Larson et al., 2011). The textbook contains approximately 900 pages and covers 12 chapters addressing major topics in plane and solid Geometry. These topics include fundamental definitions, proofs, parallel and perpendicular lines, triangles, circles, three-dimensional figures, and area and volume. In addition to Geometry. After Geometry, in grade 11, students study Algebra II.

In France, Geometry is not taught as a separate subject (see the Fig. 2); instead, students study a unified mathematics course (*Mathématiques*) that includes different modules of Geometry integrated throughout the curriculum. The organization and emphasis of Geometry topics vary by grade level and textbook. For example, in the *Mathématiques* textbook series (see Table 1) published by Hatier (Chapiron et al., 2012), the Grade 5 volume contains 256 pages, of which approximately 107 pages (two chapters) are devoted to Geometry. The *Triangle 4* textbook contains 304 pages, with approximately 100 pages dedicated to Geometry, while *Triangle 3* includes 63 Geometry pages out of a total of 300 pages. The distribution of Geometry content reflects the integrated structure of the French mathematics curriculum, in which geometric concepts are developed alongside other areas of mathematics rather than organized as a separate course.

Figure 2
French Textbook on Geometry Published by Hatier

The Italian Geometry curriculum (Vacca et al., 2017) is presented through a three-volume textbook series (see the Fig. 3). The first volume (approximately 380 pages; 800 g) covers measurements of length, fundamental geometric concepts, parallel and perpendicular lines, and the properties of triangles and polygons (see Table 1). The second volume (approximately 330 pages; 700 g) focuses on circles and inscribed and circumscribed polygons. The third volume (approximately 260 pages; 590 g) addresses three-dimensional figures and introduces the use of software for constructing and exploring geometric objects in both the plane and space.

Figure 3
Italian Textbook on Geometry Published by Atlas

In Germany, the mathematics curriculum is presented through the textbook series *Mathematik für Gymnasien 7–10* (see Fig. 4), published by Ernst Klett Verlag (Lambacher & Schweizer, 2020). Each volume contains selected chapters devoted to Geometry rather than a separate Geometry course (see Table 1). The Grade 7 textbook includes a chapter on triangles (30 pages out of 290 pages).

Figure 4
German Textbook on Math Published by Klett Verlag

The Grade 8 textbook contains a chapter on similar figures (30 pages out of 200 pages). The Grade 9 textbook devotes 74 pages (out of 250 pages) to circles and right triangles, including the Pythagorean Theorem. The Grade 10 textbook includes a chapter on algebraic methods in Geometry, comprising 20 pages out of 180 pages. The majority of the content in these textbooks is devoted to Algebra and related mathematical topics.

Types of Proofs in High School Curricula

Several proof formats are commonly used in high-school Geometry. One method establishes the validity of a statement by demonstrating that the conclusion follows logically from given conditions and previously established results. This approach is known as the direct method of proof. Another method begins by assuming that the statement to be proven is false and then showing that this assumption leads to a contradiction. This approach is referred to as the indirect method of proof.

Within these general approaches, several formats are commonly used to represent mathematical arguments. The first is the paragraph (or narrative) proof, which presents a sequence of sentences describing the logical steps of an argument. This format resembles an essay in which reasoning is expressed through connected explanations. Many classical proofs from antiquity follow this structure. For example, Euclid (2016, p. 247) presents a proof of the Side-Angle-Side (SAS) congruence theorem for triangles (circa 300 BC) using a narrative format. The second format is the two-column proof, in which mathematical statements are presented in one column and their corresponding justifications are provided in another. This format emphasizes the relationship between claims and reasons and is widely used in secondary mathematics education in the United States (McDougal Littell, 2017).

A third format is the flow-chart proof, which represents the logical structure of an argument through connected statements and implications (McDougal Littell, 2017, p. 250). This format can support classroom discussions, collaborative work, and visual representation of mathematical reasoning.

In contemporary classroom practice, formal geometric proofs and algebraic derivations are sometimes supplemented by activities designed to verify the validity of mathematical statements. Such activities may involve measurement, hands-on exploration, or numerical examples. While these approaches can support conceptual understanding and student engagement, they do not replace formal mathematical justification. Effective instruction requires a balance among exploration, discussion, and rigorous proof.

Contemporary Classroom Practice

Current educational expectations emphasize several principles of effective classroom instruction, including efficient use of instructional time, active student engagement, student-centered learning, and opportunities for collaborative work. These principles are frequently included in teacher evaluation frameworks and classroom observation criteria used by schools and educational organizations.

Another important expectation of contemporary mathematics education is the connection between classroom learning and real-world applications. Students are encouraged to understand how mathematical concepts can be applied to authentic situations and practical problems.

These instructional priorities, combined with limited classroom time and increasing curricular demands, may influence the extent to which teachers include formal proofs of theorems in Geometry and derivations of formulas in Algebra. As a result, some mathematical justifications may be replaced by applications, examples, or problem-solving activities. The present study examines ways to reintegrate proofs and formula derivations into regular classroom practice while maintaining student engagement and conceptual understanding.

Proof of Congruent Triangles

The study of congruent triangles is a fundamental component of secondary-school Geometry curricula. Most high-school Geometry courses introduce several major congruence criteria for triangles, including Side-Angle-Side (SAS), Angle-Side-Angle (ASA), Angle-Angle-Side (AAS), Side-Side-Side (SSS), and Hypotenuse-Leg (HL) for right triangles. Additional congruence relationships involve special cases of isosceles and equilateral triangles, as well as configurations involving altitudes, angle bisectors, and medians.

Curricula in different countries approach the teaching and justification of triangle congruence in different ways. Some emphasize formal proofs and logical deduction, whereas others focus more heavily on exploration, visualization, and verification activities. The present study compares how congruence proofs are presented in secondary-school mathematics textbooks from different countries and examines the role assigned to formal mathematical reasoning.

Table 2 below summarizes approaches adopted in different countries.

Table 2
Statements About Congruent Triangles

Statement	**USA**: McDougal Littell	**France**: Hatier	**Italy**: Atlas	**Germany**: Klett Verlag
SAS	Postulate	No topic	Criterio, not proved	Statement
ASA	Postulate	No topic	Criterio, not proved	Statement
AAS	Proved theorem	No topic	Criterio, not proved	Statement
SSA	skipped	skipped	skipped	Statement

In the *McDougal Littell Geometry* textbook (Larson et al., 2017), the statements concerning triangle congruence, including SAS, ASA, and SSS, are presented as postulates. The AAS statement is introduced as a theorem; however, its proof is not developed in the textbook. Instead, students are asked to complete the proof as an example of a flow-chart proof (Larson et al., 2017, p. 250). It is worth to mention that in younger generation of American textbooks, statements SAS, ASA, and SSS were considered as theorems with proofs (Seymour, 1925).

In the set of French textbooks published by Hatier (Volumes 3, 4, and 5, including sections related to triangles), available to the authors, statements about triangle congruence are not explicitly included. It may be assumed that this content is addressed through supplementary instructional materials or other curriculum resources.

The Italian textbook series published by Atlas presents the four major triangle congruence statements in Book 1 (Vacca et al., 2017, p. 224). The SAS, ASA, and SSS conditions are described as criteria that can be mathematically justified. However, complete proofs are not provided in the textbook; instead, students are encouraged to construct the proofs themselves.

In the German textbook series, statements about triangle congruence are discussed in Chapter V4 of Book 7 (*Mathematik für Gymnasien 7–10*). Four conditions are presented: SAS, ASA, SSS, and even the ambiguous case of SSA. These statements are introduced without formal proofs. The textbook briefly discusses the surprising validity of the SSA case under specific conditions. A more detailed proof of a related case, based on simpler assumptions, was presented by Mironychev (2015). The comparative analysis of Geometry textbooks reveals several common trends in contemporary secondary-school Geometry education. First, the examined textbooks contain relatively few formal proofs of fundamental geometric properties. This pattern is observed not only in the treatment of triangle congruence but also in the presentation of theorems related to parallel lines, parallelograms, and other geometric relationships. In addition, mathematical statements are sometimes presented without clearly identifying whether they represent postulates, theorems, or consequences of previously established results. Similar tendencies are observed in the treatment of algebraic and geometric formulas, including relationships involving triangle segments, angle properties of polygons and circles, and area formulas.

A notable tension emerges between school mathematics and advanced mathematical practice. While formal proof receives limited attention in many secondary-school courses in Geometry, Algebra, and Calculus, proof remains a central component of university mathematics and mathematical research. At the school level, formal proofs are often supplemented by exploratory activities, examples, and applications designed to help students develop confidence in the validity of mathematical statements (Miyakawa & Herbst, 2007). The present study argues for maintaining an appropriate balance between exploration and rigorous mathematical justification in secondary mathematics education.

Proof of the Pythagorean Theorem

The Pythagorean Theorem is a traditional and fundamental component of secondary-school Geometry curricula. Throughout the history of mathematics, numerous proofs of this theorem have been developed, and the related literature is extensive (Gerwig, 2026). The two primary approaches to proving the theorem are based on the concepts of area and similarity.

The theorem can be represented geometrically as a relationship among the areas of squares constructed on the sides of a right triangle. Alternatively, it can be expressed algebraically through the relationship among the squares of the side lengths. These different perspectives are presented and emphasized in various ways across the textbooks analyzed in this study. Table 3 provides a brief summary of how the Pythagorean Theorem is introduced and justified in the selected textbooks.

Table 3

The Pythagorean Theorem Presented in Different Textbooks

Statement	**USA**: McDougal Littell	**France**: Hatier	**Italy**: Atlas	**Germany**: Klett Verlag
Area	There is the explanation of the concept, p.49.	Discussion about the length of sides in 4, 159	Definition with extensive explanations and numerous examples in 2, p.102	Discussion in 8, p.190
Pythagorean Theorem	–Proof with area, p. 434 –Proof with the similarity, p.455	Demonstrations and exercises in 4, p. 159–162	Detailed explanation and proof of the theorem in 2, p.194	Discussion of the theorem in 9, p.161

The American textbook published by McDougal Littell presents two approaches to proving the Pythagorean Theorem. The first proof is based on the concept of area (Larson et al., 2017, p. 434), although a formal discussion of area appears later in the textbook (Larson et al., 2017, p. 720). The authors rely on students' prior knowledge of area concepts developed in middle school. The second proof of the theorem is based on the concept of similarity and similar right triangles (Larson et al., 2017, p. 455). Thus, students have the opportunity to explore different approaches for establishing this fundamental result.

In the French Hatier textbook, the primary emphasis of the Pythagorean Theorem is placed on the relationship between the length of the hypotenuse and the lengths of the two legs of a right triangle (*Triangle 4*, p. 162). Several examples involving different types of triangles demonstrate how the theorem can be applied to determine unknown side lengths in right triangles. However, an explicit proof is not provided and is left as a possible activity for students. The chapter devoted to the Pythagorean Theorem does not discuss the relationship between the areas of squares constructed on the sides of a right triangle.

In the Italian Atlas textbook, the Pythagorean Theorem is introduced through the concept of area (*Geometria 2*, p. 193). Area is defined as a measurable quantity (*misura della sua superficie*), and units of area are discussed before introducing the theorem. The theorem is presented with an explicit proof, and its algebraic form is also provided. The textbook also describes numerous applications of the theorem to polygons and other geometric figures.

The German Geometry textbooks published by Klett Verlag present the Pythagorean Theorem as a property (*der Satz*) without an explicit proof (*Mathematik für Gymnasien 9*, p. 161). The emphasis is placed on the meaning of the theorem, its applications, and its converse statement.

Overall, the primary focus of the contemporary textbooks analyzed in this study is on the application of geometric statements rather than on their formal justification. In the case of triangle congruence, students are often encouraged to recognize, memorize, and apply congruence criteria through diagrams and carefully structured exercises. A similar emphasis on application rather than proof can be observed in many online educational resources, which frequently support procedural problem solving rather than the development of deductive reasoning.

Findings and Discussion

The analysis of the selected textbooks allows several preliminary generalizations regarding current educational materials used for teaching Mathematics and Geometry.

First, in some countries, particularly France and Germany, the Geometry course is integrated into a broader mathematics curriculum rather than taught as a separate subject. Within this structure, students encounter geometric concepts in smaller instructional units distributed throughout the school year. Such an organization may provide limited opportunities for developing formal mathematical reasoning and proving fundamental statements, including triangle congruence criteria, the Pythagorean Theorem, conditions for parallel lines, and formulas for areas and volumes of geometric solids.

Second, contemporary classroom practices often place less emphasis on direct and explicit proofs of fundamental theorems and instead incorporate demonstrations, exploration, and activities designed to verify the validity of mathematical statements. These approaches are frequently combined with small-group discussions, collaborative projects, and student presentations. Such strategies create more opportunities for active student participation, increase students' confidence, and support engagement in the learning process. However, a significant concern is that this approach may reduce the level of mathematical rigor, the depth of understanding, and decrease the college readiness if formal justification is not sufficiently emphasized.

The limited number of formal proofs presented in the analyzed textbooks are often simple consequences of previously established postulates or definitions. For example, the AAS theorem in the McDougal Littell textbook is derived from previously introduced congruence conditions (Larson et al., 2017, p. 250), while some geometric properties in the Hatier textbook are consequences of definitions (Hatier, 2012, Vol. 4, p. 147). As a result, some textbooks may appear primarily as collections of established facts accompanied by procedures and examples demonstrating how these facts can be applied to solve problems. As Miyakawa (2012) noted, many textbook proofs are presented as examples or solutions to exercises rather than as central components of mathematical learning. Consequently, students may experience difficulties constructing proofs independently without teacher guidance or additional support.

This study argues that formal proofs should remain an essential component of contemporary Geometry courses; however, they should be introduced through pedagogically effective and student-centered approaches. A possible instructional model combines short teacher-led demonstrations, structured student activities, and collaborative learning. The instructional sequence may begin with a brief teacher presentation of a proof, either through a short explanation or a guided classroom discussion. Such demonstrations should be limited in length to maintain students' cognitive engagement. During this stage, students can take notes, ask questions, and predict subsequent steps in the argument. To support active learning, students may also use proof templates that provide the structure of an argument while allowing space for completing reasoning and justification.

At present, most textbooks have both student editions and teacher editions. Teacher editions typically include answers to exercises, lesson plans, expanded instructional objectives, and classroom activity suggestions. However, these resources often do not provide complete proofs of fundamental theorems, properties, and formulas. A teacher resource containing a systematic collection of mathematical proofs could provide valuable support for preparing well-qualified mathematics teachers and future generations of educators.

Third, the analysis indicates that some textbooks are not always structured according to the logical organization of mathematics. In particular, they do not consistently distinguish among undefined terms, definitions, postulates, and theorems. For example, discussions of triangle congruence may appear without an explicit definition of congruence (Lambacher & Schweizer, 2020, p. 180), and discussions of parallel lines or rhombuses may present properties without clearly introducing the underlying definitions (Hatier, 2012, Vol. 4, p. 145).

In addition, textbooks frequently use related terms such as "property," "statement," and "conjecture" without clearly explaining the distinctions among these concepts. The presentation often assumes that students already understand the mathematical meaning and relationship of these terms.

Fourth, the limited role of proofs in Geometry courses and the limited derivation of formulas in Algebra appear to be connected to broader curriculum priorities emphasizing applications and real-world problem solving. For example, learning objectives in some Grade 10 mathematics textbooks emphasize applications through statements such as "students will describe," "students will demonstrate," and "students will formulate," while providing less emphasis on constructing proofs of theorems or deriving formulas (Larson et al., 2017, p. TX4).

These preliminary findings and trends provide a basis for developing recommendations for improving current and future mathematics educational materials.

Conclusions and Recommendations

The review of selected Geometry textbooks, together with the authors' teaching experience, indicates that in recent years the role of theorem proofs and the derivation of algebraic formulas has become less central in secondary mathematics education. The emphasis has increasingly shifted toward activities and applications designed to help students verify the validity of mathematical statements and apply established properties to solve problems. However, it is important to recognize that proofs of fundamental geometric properties and derivations of mathematical formulas play a role in education that is equally important to their applications.

A curriculum based primarily on mathematical facts and procedures without sufficient attention to formal justification may limit students' opportunities to develop deeper mathematical understanding, particularly in Geometry and Algebra at the high school level. In addition, limited exposure to mathematical proof may create challenges in preparing future generations of mathematics educators who need a strong understanding of deductive reasoning.

Based on the findings of this comparative study, the following recommendations are proposed for educators, publishers, and textbook designers to improve secondary-school Geometry and Algebra textbooks.

1. Explanations in textbooks should be more mathematically structured. Fundamental mathematical elements, including undefined terms, definitions of mathematical objects, postulates, and theorems, should be explicitly identified and distinguished. The use of related terms such as “properties,” “statements,” and “conjectures” should be limited and clearly explained to readers.
2. Proofs of theorems concerning fundamental geometric properties should be included in the main body of textbooks. An alternative approach would be to provide comprehensive proofs in teacher editions to support instructors. Important algebraic formulas should also be derived and explained rather than presented only as computational tools. Textbooks and curricula should include assignments and activities that engage students in constructing and explaining proofs of fundamental properties.
3. Learning objectives for Geometry and Algebra courses should include mathematical justification as an explicit goal. For example, objectives should state that students understand the proofs of important theorems and the derivations of major formulas and can explain the underlying reasoning.
4. Geometry should be considered as a separate subject within the high school mathematics curriculum, with sufficient instructional time to develop both conceptual understanding and formal reasoning.

The international comparison of mathematics textbooks should continue in future research. Further collaboration among educators, school administrators, publishers, and other professionals can help identify important topics for comparing educational materials and support the development of more effective resources for students. Such efforts may contribute to improving students’ college readiness and preparing future generations of qualified mathematics educators.

Acknowledgements

The authors are grateful to many colleagues who contributed substantially to this research. Important support was provided by Dr. Andrey Sarychev (University of Florence, Italy), Dr. Ercole Suppa (Ministero dell’Istruzione, dell’Università e della Ricerca, Italy), Mr. Michael Halfter (Hohenlimburg-Hagen, Germany), and Dr. Anne Papakonstantinou (Rice University School Mathematics Project, USA).

The authors also acknowledge the financial support provided by the National Science Foundation through Grant No. 1556006.

Any opinions, findings, conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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The authors declares that ChatGPT, an AI-generative software, was used in proofreading and refining the language used in the manuscript. The usage was limited to correcting grammatical and spelling errors and rephrasing statements for accuracy and clarity. The author further declares that, apart from ChatGPT, no other AI or AI-assisted technologies have been used to generate content in writing the manuscript. The ideas, design, procedures, findings, analyses, and discussion are originally written and derived from careful and systematic conduct of the research.

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Contact email: amironit@yahoo.com