Linking Mathematical Reasoning to Physics Understanding

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Abstract

Physics instruction traditionally presents projectile motion as a parabolic trajectory described by a quadratic equation. Although mathematically correct, this approach often emphasizes the completed path rather than the simultaneous evolution of two independent functions of time. We argue that students develop deeper conceptual understanding when they construct parametric models of motion from empirical observations instead of beginning with symbolic equations. Although parametric equations provide a natural framework for representing two-dimensional motion, they remain underused in physics instruction. Unlike previous studies that used interactive simulations primarily for visualization, this study employed the PhET Projectile Motion simulation as a source of empirical data from which students developed parametric models. The research investigated the effectiveness of an inquiry-based instructional intervention designed to promote conceptual understanding of projectile motion through mathematical modeling. Participants were 24 advanced high school students enrolled in an AP Physics 1 course. The intervention consisted of three 45-minute instructional units that guided students from constructing independent horizontal and vertical functions, x(t) and y(t), to developing a complete parametric model of projectile motion. Guided inquiry activities integrated simulation-based investigations with mathematical modeling tasks focused on interpreting position, velocity, and acceleration as vector-valued functions. Student learning was evaluated using a mixed-methods design that combined survey responses with descriptive analyses of conceptual kinematics tasks. Results indicate that the intervention improved students' ability to interpret projectile motion as the simultaneous evolution of two independent functions linked by the common parameter of time. These findings suggest that simulation-supported parametric modeling provides an effective framework for strengthening conceptual understanding and connecting mathematics with physics.

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Introduction

Projectile motion is one of the most conceptually challenging topics in upper secondary and introductory undergraduate physics courses because it requires students to integrate concepts from kinematics, vectors, algebra, functions, and graphical representations into a coherent two-dimensional model of motion. Students are expected to understand that the horizontal and vertical components of projectile motion are independent yet together determine the characteristic parabolic trajectory. Despite this conceptual framework, instruction frequently reduces projectile motion to a collection of kinematic equations that students apply algorithmically. Consequently, many students can solve textbook problems procedurally but struggle to explain the underlying physical principles or interpret mathematical equations as meaningful representations of physical phenomena (Eichenlaub & Redish, 2019; Kuo et al., 2019). Traditional instruction often emphasizes algebraic manipulation and numerical problem solving while providing limited opportunities for students to construct conceptual links among position, velocity, acceleration, and time or to interpret mathematical functions as models of physical phenomena.

Recent research in physics and mathematics education advocates instructional approaches that emphasize mathematical reasoning rather than formula manipulation. Mathematical reasoning enables students to recognize structural relationships among physical quantities, formulate covariational relationships, and develop mathematical models that explain physical phenomena (Sokolowski, 2021). By integrating mathematical reasoning with scientific inquiry, students develop a deeper understanding of both the algebraic representations that underline physical laws and the physical meaning of these representations. This perspective aligns with research on multiple representations, which suggests that conceptual understanding is strengthened when learners coordinate symbolic, graphical, numerical, and contextual representations of the same phenomenon (Lesh & Doerr, 2003). Within this framework, mathematical functions become more than computational tools; they serve as models that describe continuous relationships among changing quantities and provide a foundation for explaining physical behavior.

Parametric functions provide a natural mathematical framework for modeling projectile motion by expressing the horizontal and vertical coordinates as separate functions of time. Unlike the Cartesian equation y(x) of a parabola, parametric functions explicitly describe an object's movement, allowing students to examine the horizontal and vertical components simultaneously while preserving their independence. This representation supports students in connecting symbolic equations with graphical and physical representations of motion while interpreting how each coordinate changes continuously over time. Research in mathematics education has consistently demonstrated that conceptual understanding is strengthened when students coordinate multiple representations, including symbolic, graphical, numerical, and contextual formsand use these representations to construct and interpret mathematical meaning (Ainsworth, 2006; Jäder & Johansson, 2025; Munfaridah et al., 2021). Although parametric functions constitute a natural mathematical language for representing multidimensional physical phenomena, they remain underutilized in physics education, where instruction continues to rely predominantly on Cartesian representations and formula-based problem solving (Sokolowski, 2023). This underrepresentation is noteworthy because parametric function modeling promotes covariational reasoning and empirical–mathematical modeling, enabling students to interpret motion as the simultaneous variation of multiple quantities (Sokolowski, 2021, 2023). From this perspective, parametric functions serve not merely as computational tools but as mathematical models that describe the coordinated variation of horizontal and vertical position with time.

Interactive computer simulations can further enhance students' understanding by enabling them to manipulate variables, observe immediate feedback, and effectively compare mathematical representations with physical phenomena. The PhET Interactive Simulations developed at the University of Colorado Boulder provide an inquiry-based learning environment in which students can generate data describing horizontal and vertical position as functions of time, examine the corresponding velocity and acceleration functions, and explore how changes in initial conditions affect the resulting trajectory. Such interactive environments promote active learning, scientific inquiry, and conceptual understanding when combined with guided instructional activities (Wieman et al., 2008).

The instructional approach presented in this paper is grounded in constructivist learning theory, which views knowledge as actively constructed through exploration, reflection, and interaction with meaningful tasks (Piaget, 1970; Vygotsky, 1978). It also builds on the empirical–mathematical modeling framework proposed by Sokolowski (2021), in which students formulate mathematical models directly from empirical observations, analyze relationships among variables, and validate those models against physical behavior. In this study, students use the PhET Projectile Motion simulation to collect and analyze empirical data, construct parametric functions that model the horizontal and vertical components of projectile motion, interpret the physical meaning of these functions and their parameters, validate the resulting models against simulated motion, and coordinate graphical, numerical, symbolic, and physical representations of projectile motion.

Literature Review

Physics education research has documented persistent misconceptions about projectile motion and proposed a variety of instructional approaches to address them. Early studies focused on helping students develop a more coherent understanding of the vector nature of the parameters describing the motion. For example, Mihas and Gemousakakis (2007) emphasized that an object's velocity vector is always tangent to its trajectory and that, under constant speed, centripetal acceleration is perpendicular to the tangential velocity. Similarly, Springuel et al. (2007) used cluster analysis to examine how students conceptualize vectors representing acceleration, revealing persistent difficulties in interpreting vector quantities during two-dimensional motion. While these ideas target the vectorial nature of kinematic quantities embedded in the analysis, they assume a deep understanding of vector components, which may pose an additional pedagogical obstacle.

Other researchers have introduced alternative instructional contexts to strengthen students' conceptual understanding of projectile motion. Rather than analyzing projectile motion from a stationary frame of reference, Millar and Kragh (1994) and Klein et al. (2014) situated the launch platform on a moving vehicle. This approach was supposed to help students recognize that a released object retains the horizontal velocity of its carrier and illustrates the principle of relative motion. Although this context encourages inquiry and conceptual exploration, it simultaneously introduces the additional complexity of relative motion, potentially diverting attention from the fundamental goal of understanding the independence of horizontal and vertical motion. Consequently, successful implementation of this approach presupposes that students already possess a solid understanding of one- and two- dimensional kinematics.

Another line of research has demonstrated the benefits of technology-enhanced instruction. Wee et al. (2012) employed computer software to trace projectile trajectories, allowing students to visualize the resulting parabolic path. Jimoyiannis and Komis (2001) integrated computer simulations with vector analysis to improve students' conceptual understanding of motion in a gravitational field. Likewise, Klein et al. (2014) used tablet-based video analysis to generate position data, graph motion components, and derive mathematical functions describing projectile motion. Aslan (2021) reported that GeoGebra-assisted instruction reduced misconceptions among pre-service science teachers by enabling them to compare dynamic visualizations with their own predictions. Together, these studies demonstrate that interactive technologies facilitate conceptual learning by linking physical motion with graphical and mathematical representations.

Several studies have also examined conceptual understanding from broader perspectives. Das and Roy (2014) extended traditional projectile motion by incorporating air resistance proportional to the square of velocity, allowing students to investigate more realistic trajectories. Espinoza (2004) connected projectile motion to momentum, finding that students who studied momentum before forces demonstrated a stronger understanding of the tangential nature of velocity. Similarly, Gamble (1989) argued that reconstructing students' conceptual frameworks leads to more meaningful learning in mechanics than emphasizing procedural problem solving alone.

More recent research has focused on scientific reasoning and the coordination of multiple representations. Hidayatulloh et al. (2021) reported that students experience substantial difficulties interpreting graphs and vector representations of projectile motion, particularly those involving velocity and acceleration. The authors concluded that instruction should explicitly help students translate among symbolic, graphical, and vector representations. Jamaludin et al. (2025) found that inquiry-oriented experiments combined with peer discussion promoted deeper conceptual understanding by encouraging students to predict outcomes, interpret evidence, and revise their reasoning. Likewise, Shofiyah et al. (2024) demonstrated that students with stronger scientific reasoning exhibited a better understanding of projectile motion concepts, suggesting that instruction should emphasize hypothesis generation, evidence-based reasoning, and explanation rather than routine algebraic manipulation.

Collectively, these studies indicate that the most effective instructional approaches combine inquiry-based experimentation, interactive technologies, multiple representations, and scientific reasoning. Existing research on teaching projectile motion has largely emphasized conceptual change, multiple representations, computer simulations, and inquiry-based learning, whereas comparatively little attention has been devoted to using parametric equations as a central instructional representation. Most of the studies continue to rely on Cartesian equations, vector diagrams, or graphical analyses without explicitly engaging students in modeling two-dimensional motion through time-dependent functions as the initial quantitative tools for representing motion. Because parametric equations naturally represent multidimensional phenomena, they provide opportunities to strengthen students' covariational reasoning as well as the quality of vectorial analysis (Sokolowski, 2021, 2023, 2025). This study builds on earlier work that introduced the parametrization of projectile motion as an instructional framework for undergraduate physics students (Sokolowski, 2021). Whereas the earlier work focused primarily on the mathematical formulation of parametric models at the college level, the present study extends this approach to secondary physics education by integrating inquiry-based instruction, empirical data collection, and simulation-supported mathematical modeling. The present study addresses this shift by integrating parametric equations with the PhET Projectile Motion simulation within a constructivist, inquiry-based instructional framework.

Theoretical Framework and Instructional Rationale

The present study is grounded in the premise that conceptual understanding in physics develops through the construction of mathematical models rather than through the application of memorized formulas. Within the empirical–mathematical modeling framework (Sokolowski, 2018, 2020, 2023), mathematical functions are viewed as representations of observed physical relationships rather than symbolic expressions to be manipulated independently of their physical meaning. Students therefore develop conceptual understanding by identifying relationships among physical quantities, expressing these relationships through mathematical functions, and interpreting the resulting models in terms of the behavior of physical phenomena.

A central feature of this framework in this study is parametric modeling, in which a physical phenomenon is represented by two or more functions connected through a common independent parameter of time. Rather than treating equations as isolated mathematical objects, parametric models describe how multiple physical quantities evolve simultaneously as functions of the same time variable. In kinematics, time serves as the common parameter linking the horizontal and vertical components of motion, allowing students to interpret two-dimensional motion as the coordinated evolution of independent yet complementary functional relationships. From this perspective, parametric equations are not merely an alternative mathematical notation but a modeling framework that reveals the structure of physical phenomena.

Projectile motion provides an ideal context for developing this type of reasoning because it requires students to coordinate the behavior of multiple representations to describe the phenomenon. Parametric modeling emphasizes that the horizontal and vertical coordinates are generated independently while remaining synchronized through the common parameter of time. Consequently, the familiar parabolic trajectory emerges as a consequence of these coordinated functional relationships rather than as the primary object of study. This shift encourages students to focus on the underlying physical processes instead of memorizing equations that describe the resulting path.

This perspective is consistent with constructivist theories of learning (Piaget, 1970; Vygotsky, 1978), which suggest that knowledge is actively constructed through meaningful experiences. Interactive simulations provide an environment in which these representations can be explored simultaneously while allowing students to generate, test, refine, and validate parametric models through empirical observations.

Unlike traditional instruction, which typically derives kinematic equations before asking students to apply them to projectile motion, the instructional sequence presented in this study deliberately postpones symbolic equations until students have accumulated sufficient empirical and graphical evidence to formulate their own mathematical models. The PhET Projectile Motion simulation serves not merely as a visualization tool but as a source of empirical data from which students construct parametric models describing two-dimensional motion. Consequently, equations become explanations of observed physical behavior rather than formulas to be memorized, reinforcing the view that mathematical models emerge from empirical evidence.

The central hypothesis underlying this instructional design is that students who construct parametric models from empirical observations develop a more coherent understanding of projectile motion than students who learn the topic primarily through symbolic manipulation. Specifically, the instructional sequence was designed to help students recognize projectile motion as the simultaneous evolution of two independent functions linked by the common parameter of time. By constructing, interpreting, and validating these parametric models, students develop stronger connections between mathematical reasoning and physical interpretation, leading to a deeper conceptual understanding of projectile motion and greater success in problem solving.

Methodology

Research Design

Following the experimental design taxonomy of Shadish et al. (2002), this study employed a single-group posttest design to investigate the effects of a constructivist instructional intervention on students' conceptual understanding of projectile motion. Because the purpose of the study was exploratory, a mixed-methods approach was adopted, combining descriptive quantitative data with qualitative analyses of students' written responses.

The instructional intervention was motivated by well-documented findings that many students can solve projectile-motion problems procedurally while remaining unable to explain the underlying physics. Students frequently struggle to answer questions such as:

- Why does the horizontal velocity remain constant?
- Why does the vertical velocity change?
- How can two independent functions describe a single motion?

These persistent conceptual difficulties motivated the following research question:

Research Question

How does constructing and interpreting parametric functions influence students' conceptual understanding of projectile motion?

To investigate this question, students completed a post-instruction survey and were asked to justify their responses verbally to assess their conceptual understanding of two-dimensional motion.

Which learning experience most helped you understand projectile motion as the combination of two algebraic functions?

(A) Observing real-world demonstrations
(B) Experiences outside the classroom
(C) The three-unit laboratory sequence

Participants

Participants were 24 students enrolled in an Advanced Placement (AP) Physics 1 course at a suburban high school. None had received formal instruction in physics prior to the study. The sample consisted of 18 seniors and 6 juniors. At the time of the intervention, 16 students were enrolled in Calculus, whereas 8 were enrolled in Precalculus.

Instructional Intervention

The instructional intervention was designed in response to previous research indicating that students have trouble interpreting algebraic equations in physical contexts, decomposing projectile motion into independent horizontal and vertical components, and coordinating multiple mathematical representations of motion.

Grounded in constructivist learning theory and empirical–mathematical modeling (Sokolowski, 2021), the intervention consisted of three 45-minute instructional units in which students progressively constructed parametric models of projectile motion from observations obtained with the PhET simulation. Working collaboratively, they formulated hypotheses, compared graphical, symbolic, and physical representations, tested their models against simulated motion, and refined their explanations. Learning therefore emphasized the construction of conceptual understanding rather than the application of memorized formulas.

Introducing parametric equations before formal instruction on projectile motion served two complementary purposes. First, it reduced cognitive load by separating the learning of a new mathematical representation from the simultaneous learning of a new physical phenomenon (Clark & Mayer, 2016). Second, it supported constructivist learning by allowing students to build progressively richer mental models of motion through increasingly complex applications of the same mathematical structure.

Throughout the activity, students coordinated graphical, symbolic, and physical representations while interpreting the mathematical properties of parametric functions in terms of the corresponding physical quantities. This integration of modeling and conceptual reasoning reflects the study's constructivist framework.

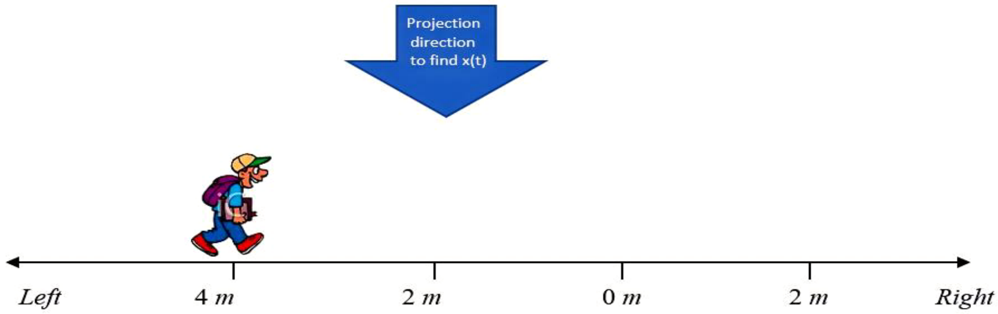
Unit 1. Developing Component Functions

The instructional goal of Unit 1 was to develop students' representational fluency by extending one-dimensional motion into two-dimensional component functions through inquiry-based analysis of uniform motion.

Part A. Uniform Motion

Students were presented with a diagram (see Figure 1) of a person walking. The person's horizontal position at selected times was provided.

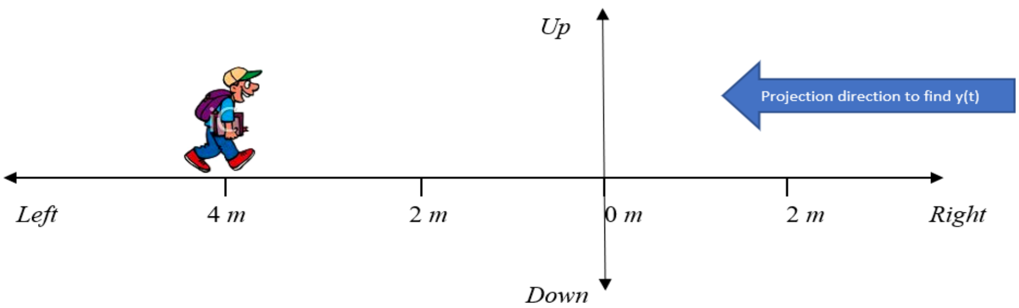
Figure 1
Reference Line to Formulate Top View of the Function



Using the top-view representation, students determined the horizontal position function, $x(t) = 2t - 4$.

To complete the motion description, students' attention was redirected to the side view (see Figure 2) to determine the vertical position function of the walking person.

Figure 2
Reference System to Formulate Side View for the Parametric Function



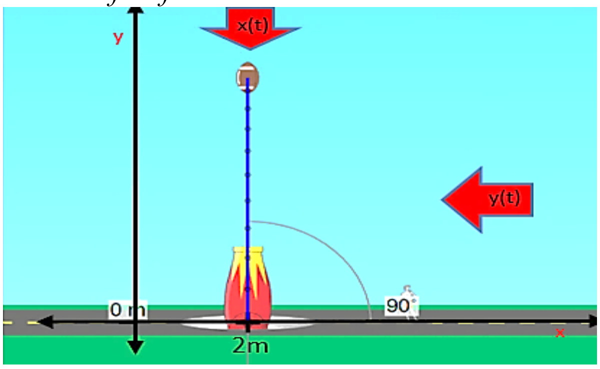
Because the side view did not indicate any movements of the person, students concluded that $y(t) = 0$. Thus the parametric functions that modelled the person's position were $x(t) = 2t - 4$ and $y(t) = 0$.

Although the motion was one-dimensional, students represented it using both horizontal and vertical component functions. This low-cognitive-load activity established the representational framework needed for later analyses of projectile motion.

Part B. Accelerated Motion

This activity introduced the key idea that different motion components may be described by different functions while sharing a common time parameter. The frame of reference was consistently provided (see Figure 3).

Figure 3
Frame of Reference to Write Parametric Functions for Free Fall



Students determined that the side view $y(t) = 30t - \frac{9.8t^2}{2}$ represented the vertical position function, while the top view represented the constant horizontal position, $x(t) = 2$.

This activity introduced the important conceptual idea that different components of the same motion may be described by different types of functions while sharing a common time parameter. Students therefore began coordinating multiple mathematical representations of a single physical event, an ability that is central to the study's theoretical framework.

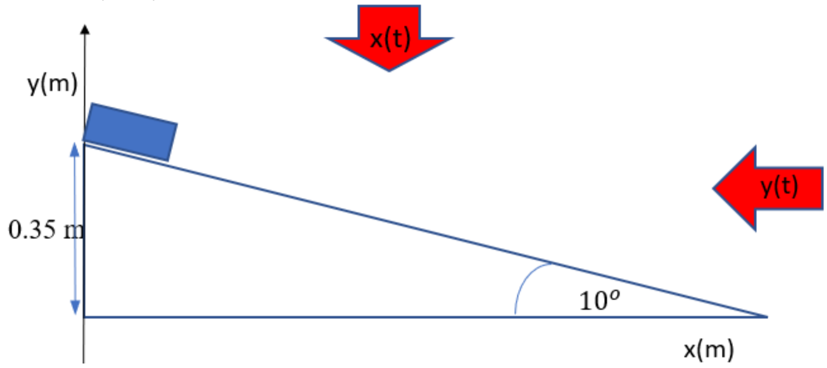
The primary conceptual insight from this activity was that each component of motion must be analyzed independently. Once students became comfortable constructing component functions in familiar situations, they extended this reasoning to motion on an inclined plane.

Unit 2. Motion on an Inclined Plane

The instructional goal of Unit 2 was to extend parametric modeling by using trigonometric ratios to determine vector components in a laboratory investigation of motion on an inclined plane (see Figure 4).

Traditionally, motion on an inclined plane is analyzed using coordinate axes parallel and perpendicular to the inclined plane. Students began with this conventional representation. Using a photogate, they measured the object's position at specified time intervals, constructed a data table and, from the quadratic form of the generated position-time graph, concluded that the object's acceleration was $a = 1.70\ \frac{m}{s^2}$.

Figure 4
Frame of Reference to Parametrize Motion on an Inclined Plane



The coordinate system was then redefined according to the side view (y-axis) and top view (x-axis). Students recognized that acceleration, velocity, and position each possess horizontal and vertical vector components. They found the components of the acceleration along the newly established views and concluded that $a_x = 1.7\cos(10^o) = 1.67\,\frac{m}{s^2}$, and $a_y = 1.7\sin(10^o) = 0.30\,\frac{m}{s^2}$. Finding the components of the acceleration initiated the modeling process for the parametric position functions.

$$y(t) = 0.35 - \frac{0.30t^2}{2} \text{ (side view)} \quad (1)$$

$$x(t) = \frac{1.67t^2}{2} \text{ (top view)} \quad (2)$$

In this activity, students derived the parametric equations directly from experimental evidence rather than receiving symbolic equations from the instructor. This represented the study's key conceptual transition.

Resolving motion into orthogonal components helped students coordinate vector, graphical, and algebraic representations. After transforming the coordinate system to the standard x- and y-axes, they formulated and sketched the corresponding position, velocity, and acceleration functions.

This activity completed the transition from one-dimensional reasoning to two-dimensional parametric modeling, preparing students to analyze projectile motion.

Unit 3. Resolving Parabolic Path Into Component Functions

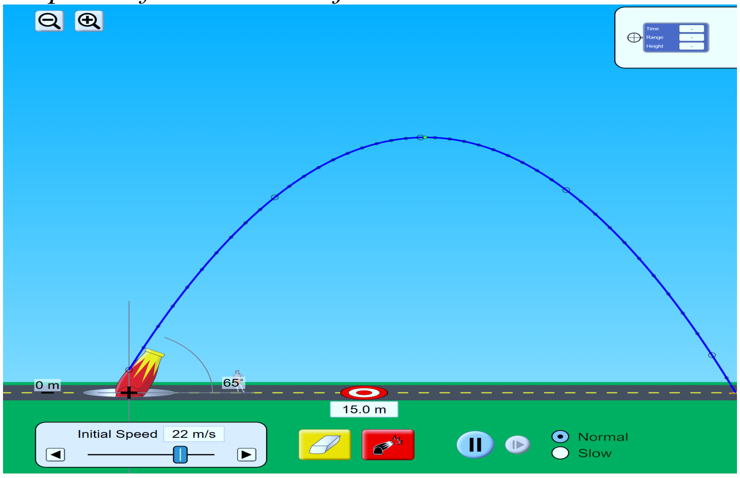
Building on Units 1 and 2, this final instructional sequence extended component analysis to full projectile motion. Students integrated conceptual understanding, mathematical modeling, and scientific reasoning to derive and justify parametric models from empirical evidence.

Students investigated projectile motion using the PhET Projectile Motion simulation (PhET Interactive Simulations, n.d.) which served as a source of empirical data. By recording successive coordinates of the projectile, students generated horizontal and vertical position data and constructed the corresponding parametric functions x(t) and y(t).

The central objective was to help students recognize that projectile motion emerges from the simultaneous evolution of two independent functions linked by time rather than from a single quadratic equation. Students therefore constructed parametric models directly from empirical observations.

Figure 5

Snapshot of the PhET Projectile Motion Simulation



Students received instructional support containing a snapshot of the trajectory, along with guidance on collecting data and formulating the object's position functions.

Following the established top and side views, students completed a table containing time, horizontal position, and vertical position. Using a single table emphasized that time is the common parameter shared by both motion components.

Results

Part A. Generating Data for Modeling Parametric Equations

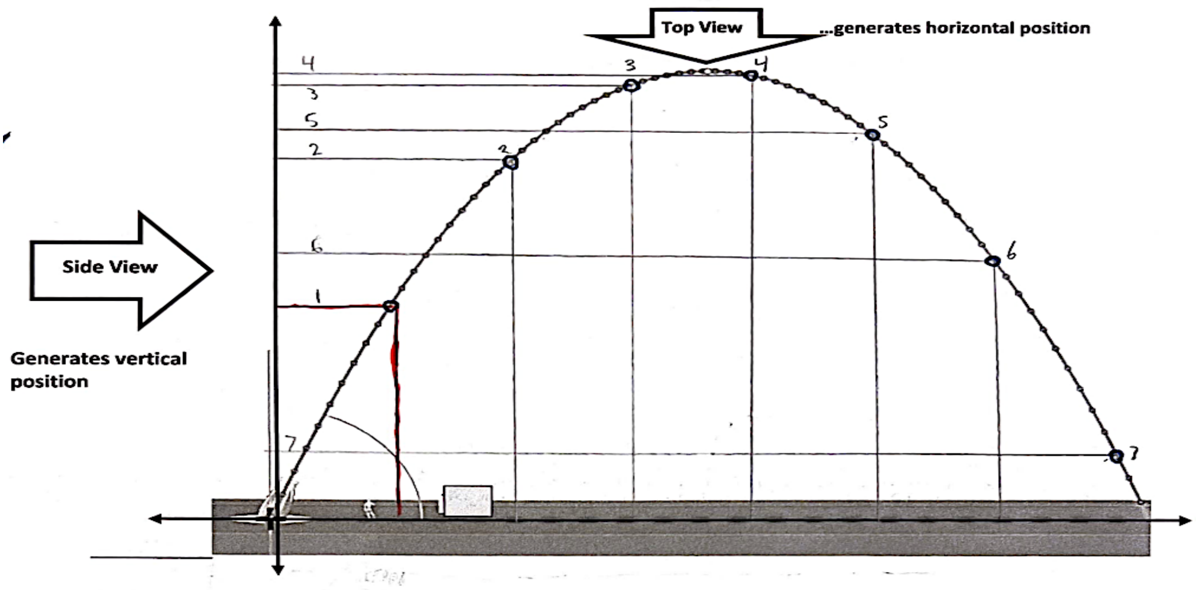
Using the trajectory, students projected the object’s position at each second on the y-axis using the side view and the x-axis using the top view (see Figure 6). They measured the coordinates and generated the data table.

Figure 6

Sample of Student's Work on Generating Data Table for Parametric Equations

The diagram depicts the path of a projected object.

- **The larger circular dots on the trajectory denote the object's position after every *second* of motion.**
- **The smaller circles show the position after every 0.1 sec. Ignore the angle of projection.**



- **Highlight all larger circles on the motion trajectory and draw vertical and horizontal axillary lines to be able to measure, using a ruler, the object's position in vertical and horizontal directions after each second of motion. You can label them e.g., 1, 2, etc. The lines in red are an example.**
- **Starting from the position (0,0) measure, using a metric ruler, the object's positions along the x and y directions separately. E.g. the position after 2 seconds in x,-the direction is the separation between x=0 xm and the object's position after 2 sec in the x-direction.**
- **Generate a table with the x and y positions. Express the positions to the nearest decimal place (e.g., 3.1 cm)**

Table 1

1 Time (s)	0	1	2	3	4	5	6	7
2 x-position	0	2	4	6	8	10	12	14
3 y-position	0	4.5	7.5	9	9.25	8	5.25	2.5

The views to gather the data followed the previously established convention.

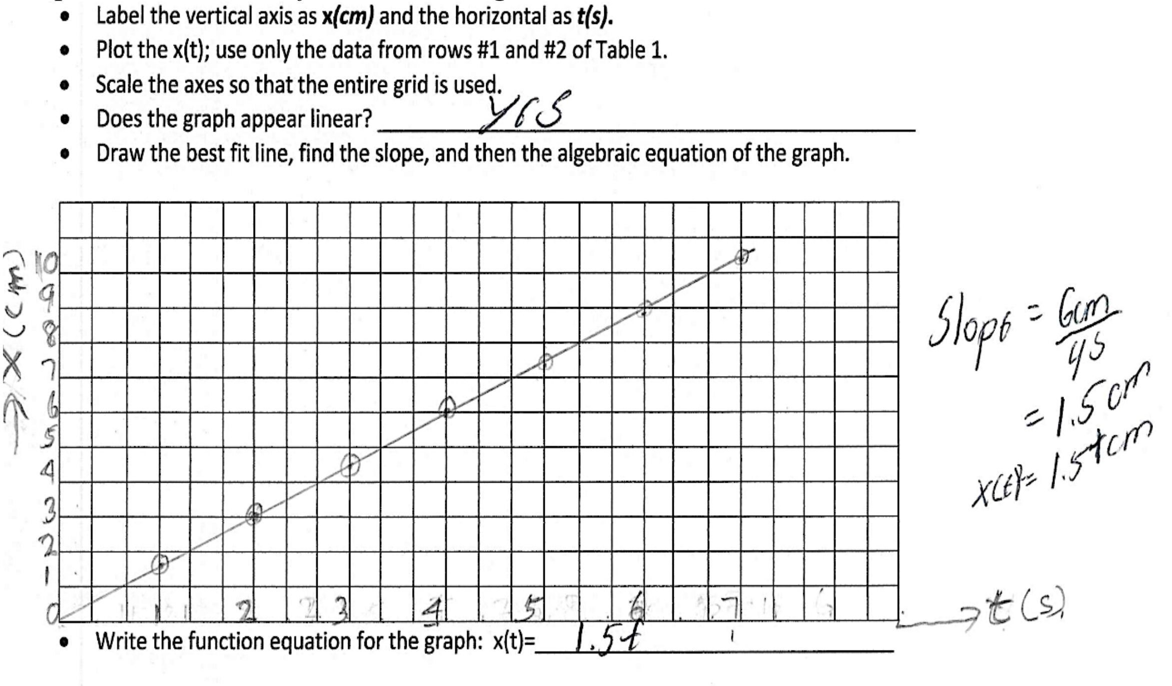
Students completed Table 1 and used the data to construct the graphs and functions $x(t)$ and $y(t)$. The table consisted of three rows: time, the x-position, and the y-position of the object from t = 0 s to t = 7 s.

Part B. Analysis of Motion Horizontal Component

Following the data collection phase, students analyzed the horizontal component of projectile motion to determine its mathematical characteristics. Using the time and horizontal-position data, students constructed *x(t),* identified its constant slope, and concluded that the horizontal velocity remained constant while the horizontal acceleration was zero (see Figure 7). They then sketched the corresponding velocity and acceleration functions.

Figure 7

Sample Student's Work for Formulating Horizontal Position Function



Using the generated graph, students identified a constant slope for the horizontal position function. From this observation, they concluded that:

- The horizontal velocity remained constant throughout the motion.
- Optional precision: The horizontal position function is linear.

Students also formulated and sketched the corresponding velocity and acceleration functions, recognizing that the velocity remained constant while the horizontal acceleration was zero throughout the motion.

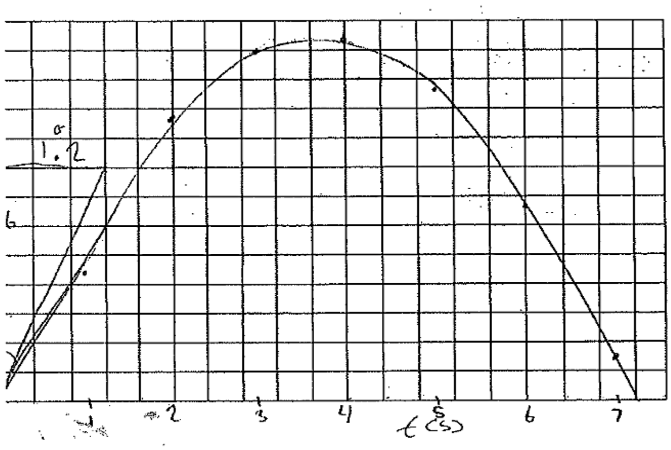
Part C. Analysis of the Vertical Component

After examining the horizontal component, students analyzed the vertical component to investigate the effects of constant gravitational acceleration on the motion.

Using the time and vertical-position data from Table 1, students generated the vertical position function, $y(t)$, and determined that it was parabolic.

Figure 8

Student's Work for Formulating Vertical Position Function



Label y(cm) on the vertical axis and t(s) on the horizontal. Plot the points using rows 1 and 3 from the data table.
Using a French curve, draw the best-fit curve.

$y_1 = 0$

$V_{y_1} = 5 \frac{m}{s}$

$\frac{6}{1.2} = 5 = V_{y_1}$

$y(t) = 0 + 5t + \frac{at^2}{2}$

$y(3) = 9.0$

$9.0 = 5(3) + \frac{a(3)^2}{2}$

$9.0 = 15 + \frac{a9}{2}$

$-6 = a\frac{9}{2}$

$-6(\frac{2}{9}) = \boxed{a = -\frac{4}{3}}$

- Find the function equation of the graph using:
  - $y(t) = y_1 + v_1 t + \frac{at^2}{2}$ where y_1 and v_1 are initial motion parameters. Use an additional coordinate and solve for a
    v_1 = the initial velocity that equals the slope of a tangent line to the graph at t=0s.
    y_1 = Initial position or
  - $y(t) = \frac{a}{2}(t - t_v)^2 + y_v$ where t_v and y_v are the coordinates of the vertex of the parabola. Use an additional coordinate and solve for a

$y(t) = 5t + \frac{-1.\overline{3}t^2}{2} = \boxed{5t - \frac{2}{3}t^2}$

By analyzing the function's attributes, students identified the object's constant downward acceleration and formulated the corresponding vertical position function. They then constructed and sketched the associated velocity and acceleration functions.

Collectively, these activities enabled students to integrate algebraic structure, graphical interpretation, and physical meaning into a coherent mathematical representation of projectile motion.

Data Collection

Student learning was assessed using quantitative and qualitative measures. Quantitative data consisted of students' responses to conceptual kinematics tasks administered after the instructional intervention. Qualitative data were obtained from an open-ended survey question:

Which learning experience most helped you understand projectile motion as a combination of different algebraic functions?

Students selected one of three responses:

- Observation of classroom demonstrations
- The three-unit laboratory sequence
- Prior experiences outside the classroom

Students also provided written explanations to justify their selections.

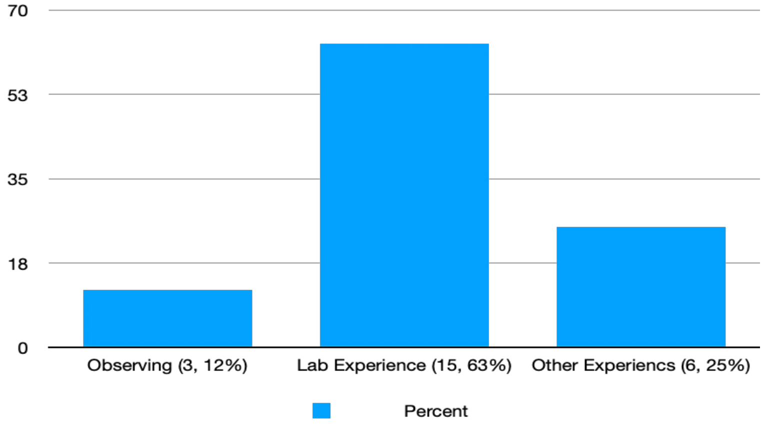
Data Analysis

Descriptive statistics summarized students' survey responses and performance on conceptual tasks. Written responses were analyzed thematically to identify evidence of students' understanding of functional decomposition, the independence of horizontal and vertical motion, and the interpretation of projectile motion using parametric equations. These analyses were used to evaluate the intervention's effectiveness in promoting model-based reasoning and conceptual understanding of two-dimensional motion.

Survey results indicated that the three-unit laboratory sequence had the greatest impact on students' conceptual understanding of projectile motion.

Figure 9

Distribution of Students' Survey Responses



Observation of Real Demonstrations (3, 12%)

Students in this group described observable features of projectile motion, such as the object slowing as it rises, momentarily stopping at its highest point, accelerating downward, and following a parabolic path. These responses suggest that demonstrations promoted descriptive observation rather than mathematical reasoning.

Three-Unit Laboratory Sequence (15 students, 63%)

Most students identified the laboratory activities as the most influential learning experience. Their explanations emphasized that the horizontal motion was represented by a linear function with constant velocity and zero acceleration, while the vertical motion was modeled by a separate function. Many specifically referred to parametric equations as helping them distinguish the independent horizontal and vertical components of projectile motion. Overall, these responses demonstrated strong integration of mathematical representations with physical interpretation. Students also cited the corresponding algebraic functions as evidence of the two-dimensional nature of projectile motion.

Experiences Outside the Classroom (6 students, 25%)

Students who selected this category referred to experiences such as sports, firing projectiles, or observing rolling objects. Although these experiences supported intuitive predictions about projectile motion, they rarely involved constructing or interpreting formal mathematical models.

Overall, the survey findings indicate that the laboratory sequence most effectively promoted students' ability to connect graphical and algebraic representations with the physical interpretation of projectile motion through parametric modeling.

Discussion

Taken together, the classroom observations, students' written work, and survey responses indicate that the instructional intervention strengthened both students' mathematical modeling skills and their conceptual understanding of projectile motion.

The inquiry-based learning environment encouraged students to investigate motion by analyzing simulation-generated data rather than applying memorized equations. Constructing separate horizontal and vertical component functions helped students recognize projectile motion as the combination of two independent one-dimensional motions.

Students then translated these observations into parametric equations, treating them as mathematical models derived from empirical evidence rather than formulas to memorize. Constructing and graphing the corresponding position, velocity, and acceleration functions further strengthened their understanding by linking graphical features with the physical behavior of the projectile. Students associated the linear horizontal function with constant velocity and the quadratic vertical function with uniformly accelerated motion under gravity.

The intervention also strengthened the connection between mathematics and physics by showing how familiar mathematical concepts, including linear and quadratic functions, parameters, and function composition, describe physical phenomena. As a result, students increasingly viewed parametric equations as a natural framework for representing two-dimensional motion.

Overall, the findings suggest that introducing parametric modeling early in kinematics promotes conceptual understanding by integrating experimental observations, graphical representations, algebraic models, and physical principles. Beyond supporting computation, mathematical modeling served as an effective vehicle for scientific reasoning and the analysis of two-dimensional motion.

Conclusions

This study investigated the use of parametric equations to strengthen students' understanding of projectile motion through mathematical modeling. The findings suggest that introducing parametric equations early in kinematics provides an effective instructional approach for helping students interpret projectile motion as two-dimensional vector motion with independent horizontal and vertical components. By constructing parametric models from empirical data, students developed stronger connections between physical principles and their mathematical representations.

The instructional intervention also demonstrated that mathematical modeling could extend students' reasoning beyond traditional demonstrations. For example, students verified algebraically that projectiles with different horizontal velocities share identical vertical motion and reach the ground simultaneously. This mathematical verification complemented experimental observations and reinforced the value of parametric equations as analytical tools for explaining physical phenomena.

More broadly, the study suggests that constructing mathematical models from empirical evidence encourages students to view functions as explanations of physical systems rather than formulas to memorize. As a result, mathematical reasoning became a means of interpreting motion, promoting conceptual understanding and supporting subsequent problem solving.

The study was exploratory and involved a single group of students; therefore, the findings should be interpreted cautiously. Future research should examine whether instruction based on parametric modeling leads to more effective and transferable problem-solving strategies than traditional approaches and whether these benefits extend to broader topics in two-dimensional kinematics and dynamics.

Study Limitations

Several limitations should be considered when interpreting these findings. First, the study involved a relatively small sample of 24 students, limiting the statistical power and generalizability of the results. Replication with larger and more diverse populations is needed to establish the robustness of the observed effects.

Second, the absence of a control group prevents attributing students' learning gains solely to the instructional intervention. Although the findings suggest that parametric modeling improved students' understanding of two-dimensional motion, other factors, including normal instructional progression and instructor effects, may also have contributed. Future studies employing experimental or quasi-experimental designs are needed to establish causal relationships.

Finally, the intervention was conducted over a relatively short period and did not examine the long-term retention or transfer of students' understanding to subsequent topics such as dynamics, work and energy, or rotational motion. Longitudinal studies would help determine the durability and transferability of these learning gains.

Despite these limitations, the findings provide preliminary evidence that integrating parametric equations into introductory kinematics is a promising instructional approach. Rather than offering definitive conclusions, this study serves as an initial step toward a more integrated understanding of these mathematical tools in physics.

Suggestions for Future Research

The findings of this study suggest several directions for future research. First, the instructional approach should be replicated with larger and more diverse student populations to evaluate its generalizability. Studies employing experimental or quasi-experimental designs with control groups would provide stronger evidence of the effects of integrating parametric equations into projectile motion instruction.

Future research should also examine the long-term impact of this approach by investigating whether students continue to use parametric equations when studying topics such as two-dimensional dynamics, circular motion, oscillatory motion, and other vector-based phenomena.

Such studies could determine whether early exposure to parametric modeling promotes a more integrated understanding of these mathematics tools in physics.

Additional studies should compare the effectiveness of different instructional settings, including traditional lectures, inquiry-based laboratories, simulations, and technology-supported modeling, while also examining how students with different levels of mathematical preparation respond to parametric modeling.

Finally, qualitative studies using classroom observations, interviews, and think-aloud protocols could provide deeper insight into how students develop connections between mathematical representations and physical concepts, informing future curriculum design and instructional materials.

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