Diagnosing Misconceptions in First-Year Calculus

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Abstract
Learning tertiary mathematics is challenging and has been an obstacle for many students. Calculus students are unaware of their misconceptions. This makes understanding and progress difficult and prevents students from being successful in mathematics, especially in first-year calculus modules. Focusing on misconceptions in calculus is essential for enhancing student learning and is needed in developing conceptual understanding. Action research and reflective practice have both change and enhancement as aims. This paper reports on the first phase of the action research cycle – the diagnosing phase. Since the lecturer-as-researcher endeavors to be a reflective practitioner and given that reflective practice can lead to improved learning, the reflective practice method was used to diagnose misconceptions in first-year calculus and to develop an understanding of what students know about important concepts in first-year calculus. Two misconceptions were identified, namely limits of a function and the notation of inverse functions. Provided that the limit of a function is a fundamental part of learning calculus, and the inverse function is one of the concepts which is compulsory to be learned in calculus and appears to be a concept indispensable for some students in different study programs, these two misconceptions will be the focus of this paper. Intending to address the existence of misconceptions in calculus, the paper concludes with proposed strategies for the second action research cycle, the action-planning phase.

Keywords: Calculus, Misconceptions, Limits, Inverse Functions, Action Research, Reflective Practice

Introduction

Tertiary mathematics is difficult to learn and has long been a hurdle for many students (Rabadi, 2015, p. 3). According to Rabadi (2015, p. 10), students encounter challenges in learning calculus which include misconceptions in their understanding of certain key concepts. These misconceptions may prevent students from succeeding in mathematics, particularly in first-year calculus modules (Ay, 2017, p. 21).

Given the misconceptions, a transformation in learning is required (Eschenbacher & Fleming, 2020, p. 660). Mezirow (2000, p. 93) describes transformative learning as "incorporating the examination of assumptions, to share ideas for insight, and to take action on individual and collective reflection." Transformative learning begins, in accordance with Nerstrom (2014, p. 327) when people contemplate their views of what they consider to be true, accurate, or real. Critical reflection is the on-going process of purposefully or accidentally modifying and evaluating ideas to clarify the significance of experiences both individually and collectively.

The pedagogy of reflective practice incorporates critical reflection processes into action research. The extension of students' reflection on their behavior and learning, according to Leijen *et al*. (2012, p. 203), is one of the most crucial learning objectives in tertiary education. Reflective practice is defined by Moon (2000, p. 157) as a "cognitive process carried out in order to learn from experiences" through "individual inquiry and collaboration with others" (Dewey, 1933, p. 271). The teaching and learning process in a tertiary education framework can be understood from various angles by using reflective practice.

A calculus lecturer can utilize efficient approaches to help students recognize or correct their conceptual misconceptions by having a thorough understanding of the nature of these misconceptions and the factors that lead to them (McDowell, 2021, p. 3). According to Denbel (2014, p. 24), teaching these days prioritizes conceptual understanding above the mastery of abilities. The study by Jensen (2009, p. xiv) stresses that calculus students frequently accomplish tasks of a procedural character quite well, even though they have an insufficient grasp of a particular mathematical concept.

Thus, this study will be focusing on the diagnosed misconceptions in first-year calculus in a South African context.

Literature Background

With reference to the Oxford dictionary, Voon *et al*. (2017, p. 18) define a misconception as a "view or belief that is incorrect because of *faulty thinking* and *understanding*." Leinhardt *et al.* (1990, p. 5) explain that misconceptions are "features of a student's knowledge about a specific piece of mathematics knowledge that may or may not have been instructed." Furthermore, they suggest that a misconception may result from an important right concept that developed from an overly simplistic result. Instead of randomly and unconsciously, misconceptions typically arise recursively and/or explicitly. In addition, Voon *et al*. (2017, p. 18) note that an error denotes the improper application of the methods, conceptions, or techniques, whereas a misconception denotes the incorrect interpretation of a concept or specific notion. It leads to misconceptions by repeatedly making identical mistakes. Likewise, unnoticed misconceptions would be detrimental to a student's future learning and are thus not beneficial to them (Weliwita *et al*., 2020, p. 1).

Research on calculus learning has revealed several concepts that are difficult for students to understand, as shown by the definitions and explanations of misconceptions provided above. These include the following:

- Rate of change (Bezuidenhout, 1998, p. 389): A rate of change is a measurement of how one quantity alters in proportion to another.
- Limits (Areaya & Sidelil, 2012, p. 1; Bezuidenhout, 2001, p. 1; Jordaan, 2005, p. 1; Sebsibe & Feza, 2019, p. em0573): According to the definition, a limit is an output (or value) that a function approaches given certain input values.
- Tangents and functions (Bailey *et al.*, 2019, p. 18; Gunawan *et al.*, 2021, p. 99; Rabadi, 2015, p. 1): The curve's slope at a specific location is represented by a tangent. It is the line that contacts the curve at any given place and travels in the same direction as the curve at that location. The unique kind of relations is called functions. In mathematics, a function is represented as a rule that produces a distinct output y for each input x.
- Inverse functions (Delastri & Muksar, 2019, p. 1; Ikram *et al.*, 2020, p. 592; Nolasco, 2018, p. 15; Quaily & Agrawal, 2021, p. 123): An anti-function, also known as an inverse function, is a function that can transform into another function when reversed. A function that reverses the action of a function is called the inverse function, or f^{-1}.

The difficulties listed above have been specifically identified as ones that students face. Booth *et al*. (2017, p. 63) argue that calculus lecturers should encourage students to build conceptual understanding since students rather focus on mastering only the procedural techniques when solving calculus problems, as supported by this research.

There may be more misconceptions as a result of the fact that COVID-19 was a factor in many students' mediocre mathematics achievement. Ludwig (2021, p. 31) investigated how undergraduate mathematics and finance students did during COVID-19 and discovered that the course had a negative impact on the student's aptitude for learning mathematics, which led to their low performance. There is also a great deal of concern regarding the vast number of students enrolled in calculus and the rote, manipulative learning that occurs (Denbel, 2014, p. 24). As stated by Engelbrecht *et al.* (2005, p. 701), the experience in South Africa is that secondary school mathematics instruction tends to be quite procedural, and students entering tertiary institutions are well-prepared to handle procedural problems rather than conceptual understanding. According to Odafe (2012, p. 214), lecturers are frequently under pressure to help students understand concepts before applying them to real-world mathematical situations.

In order to define the concepts of derivative and integral calculus, the concept of a limit is crucial (Denbel, 2014, p. 24; Wu, 2020, p. 2832). Therefore, it would be difficult to understand notions like continuity, derivative, and definite integral without first conceptualizing the essential parts of limits (Juter, 2006, p. 19). As mentioned by Sulastri *et al.* (2021, p. 1) and Beynon and Zollman (2015, p. 48), in basic calculus and real analysis modules, the concept of a limit of a function is commonly formed without being connected to the formal definition of limits. Consequently, the learning and use of a limit of a function for learning in advanced calculus modules may not be deeply conceptually defined by many advanced mathematics students. Earlier studies done by Liang (2016, p. 37) indicate that students' difficulties with the limit of a function are caused by their misconceptions. Furthermore, students bring their everyday experiences along with their knowledge of limits into the calculus class, which, however essential, might result in misconceptions and hence create learning barriers.

Misconceptions concerning the concept of infinity have an impact on misconceptions that arise regarding the concept of a limit. The value of a function, the length of a sequence, or an approximate limit can all be confused with the concept of limits by students who are not familiar with the infinite process (Cottrill *et al.*, 1996, p. 4). Such misconceptions, according to Williams (1991, p. 419), are difficult to rectify and are inevitable when comprehending. Thus, not only does a lack of knowledge of limits impact that understanding, but it also makes it difficult to understand subsequent concepts like continuity, derivatives, and integrals (Sulastri *et al.*, 2021, p. 1).

As discussed by Sebsibe and Feza (2019, p. 5) and others, concepts of limits are frequently confounded by whether or not a limit is:

- **Unreachable**: A limit is a number or point the function gets closer to but never reaches (Güçler, 2013, pp. 445-447; Odafe, 2012, p. 218).
- **A boundary**: A limit is a number or point past which the function cannot go (Odafe, 2012, p. 218).
- **A dynamic process (motion) or static object (closeness)**: Limits are dynamic processes (motion) or static objects (closeness) (Cottrill *et al.*, 1996, p. 5; Williams, 1991, p. 219). Thus, limits are inherently tied to motion concepts (Bezuidenhout, 2001, p. 491; Tall & Vinner, 1981, p. 160).
- **An approximation**: A limit is an approximation that can be made as accurately as you wish (Güçler, 2013, p. 446; Parameswaran, 2007, p. 194; Sebsibe & Feza, 2019, p. 6).
- **Substitution**: Students think that limits simply entail substituting the value at which the limit is to be found, into the expression (Thabane, 1998, p. 65).
- **A function value that is the same as a limit value**: The limit of a function is the value of the function at the limit point (Sebsibe & Feza, 2019, p. 6).

One of the concepts that are also fundamental to calculus learning is the inverse function. A function called the inverse function turns the original function on its head. An inverse function, denoted by the notation f^{-1} is a function that maps B to A, for instance, when f translates domain A to range B. In order to master the idea of an inverse function, students must first grasp the idea of a function. When addressing an inverse function question, conceptual knowledge is required (Delastri & Muksar, 2019, p. 1). Even though the processes for calculating reciprocal and inverse functions differ, both functions are applications of the concept of an inverse (Kontorovich, 2017, pp. 278-279).

Two apparently unconnected mathematical concepts – reciprocal and inverse functions are both represented by the same superscript (-1) symbol. This superscript (-1) symbol confuses many students as observed by Chin and Pierce (2019, p. 6). Students had seen x^{-1} before encountering $\sin^{-1} x$, therefore it looked like they were able to apply the significance of previously encountered negative exponents in the context of real numbers to trigonometric functions. They mindlessly applied what they had learned from prior context to the current context while concentrating on the physical characteristics of the superscript (-1), so they conceptualized $\sin^{-1} x$ as if the statement was $[\sin x]^{-1}$. These participants seem to have concentrated on the well-known features without considering the significance of the context alterations and the physicalness of the composite symbols (Chin & Pierce, 2019, p. 6). If the students notice the changes in syntax and thereby conceptualize x^{-1} and $\sin^{-1} x$ as two composite symbols with various bases, then the physicalness of these two symbols might be regarded as different. Therefore, "-1" as a superscript has a different meaning in each scenario.

Methodology

The research methodology used in this study was action research, which is integrated with reflective practice. Lincoln and Guba (1986, p. 75) state that it offers a framework for the researcher to consider how her own approach may be improved. According to Mathew *et al.* (2017, p. 130), action research is also a type of reflective practice. Action research requires a commitment to reflective thought, which may include "becoming aware of what you need to do to improve your practice in your workplace, doing it, and then describing and explaining what you have done, how you have done it, and why you have done it" (McNiff, 2016, p. 51).

Five iterative phases can be found in an action research cycle (Susman & Evered, 1978, p. 588) :

- *Diagnosis*: Identify or diagnose a problem.
- *Action planning*: Consider alternative courses of action for solving a problem.
- *Action taking*: Select a course of action.
- *Evaluating*: Studying the consequences of an action.
- *Specifying learning*: Identify general findings.

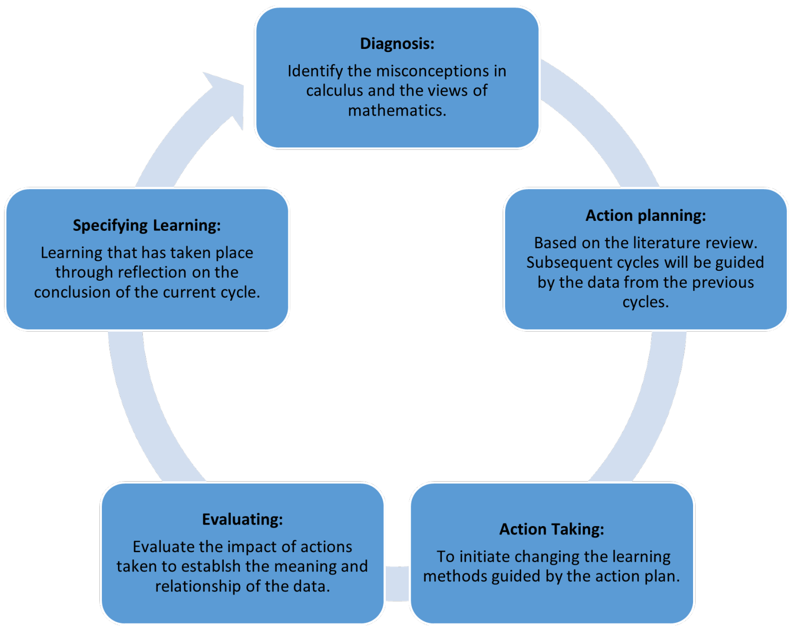

Figure 1: Action research phases

To diagnose misconceptions that exist in first-year calculus, this paper covers the first phase of the action research cycle. Prior to selecting appropriate tactics and theories to guide the action-taking phase, a diagnosis of the quandary was conducted to get some insight into the students' misconceptions of certain concepts in calculus. Reflective practice was the method utilized during the diagnosis phase.

Reflective practice is projected to be the way for lecturers and students to enhance learning in the constructivist means of inquiry known as critical reflection (Davis, 2003, p. 243). To gain a comprehensive understanding of the teaching and learning process in a tertiary education context, reflective practice can be used (Makura & Toni, 2015, p. 43). Ngololo and Kanandjebo (2021), Pavlovich (2007), Carey *et al.* (2017, p. 99), Lee (2010, p. 42), and

Carey *et al*. (2017, p. 99) all looked at reflective journals as a way to assist students to develop a deeper understanding of the crucial mathematical processes.

This study's data collection process involved multiple phases. Misconceptions regarding calculus were identified during the diagnosis phase. Literature reviews, reflections on the researcher's individual experiences, and previous assessments were used to identify misconceptions.

Results and Discussion

The inability of students to comprehend the concept of a limit is a major contributor to their struggles with other calculus concepts. There are explicit and implicit ways to explain the concept of a limit. An informal (implicit) definition of a limit, according to Stewart *et al*. (2021, p. 57), is:

> Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write $\lim_{x \to a} f(x) = L$ and say "the limit of $f(x)$, as x approaches a, equals L" if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.

Stewart *et al*. (2021, p. 74) also outline the formal (explicit) definition of a limit as follows:

> Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the limit of $f(x)$ as x approaches a is L, and we write $\lim_{x \to a} f(x) = L$ if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$.

By considering the two definitions of limits given above, the following misconceptions from past assessments were identified.

Misconception 1: Substitution of Infinity

The first misconception students saw, is that infinity is an exact number and substituted it into *x* like a value, as seen in Figure 2.

$$\lim_{x \to -\infty} \frac{5x}{\sqrt{3x^2 - 2x}}$$

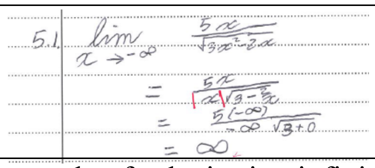

Figure 2: An example of substituting infinity

The understanding that *x* approaches negative infinity is not grasped by this student. This perspective is consistent with the assumption that infinity is an actual object because it is thought of as an existing entity, as emphasized by Oehrtman (2009, p. 417).

Misconception 2: Substitution of Numbers

Substituting an exact number into a variable is the second misconception identified by the researcher.

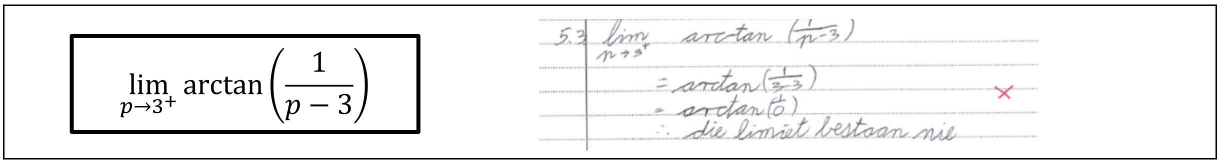

Figure 3: An example of substituting a certain value

As illustrated in Figure 3, the variable p was substituted by the exact value 3. This student did not acknowledge the fact that p approaches 3 only from the right-hand side.

Misconception 3: Function Values Are the Same as Limit Values

Many students think that the function value at a specific point is the same as the limit value at that point. Thus, this is the third misconception identified from past assessments.

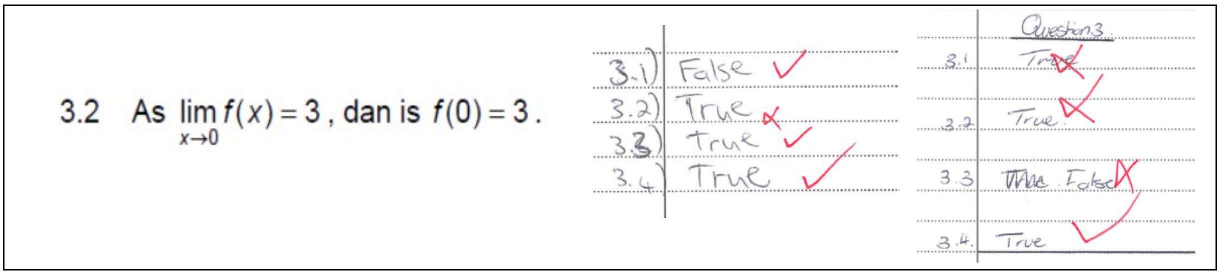

Figure 4: Examples illustrating the misconception that function values are the same as limit values

The function value of 3 at the point $x = 0$, is according to these students, the same as the value of the limit when x approaches 0, illustrated in Figure 4. According to Bezuidenhout (2001:495), this misconception may primarily be the result of using the method of substitution to discover limits algebraically without grasping the underlying conceptual understanding of the procedure. Such an answer might also be influenced by earlier classroom experiences in secondary school where students were only exposed to evaluating the limits of continuous functions. Because $\lim_{x\to a} f(x) = f(a)$ is true for continuous functions, they may have drawn the faulty conclusion that the results they received were the product of direct substitution. The only circumstance in which $\lim_{x\to a} f(x) = f(a)$, is when f is a continuous function (Bansilal & Mkhwanazi, 2022, p. 2093).

Misconception 4: Inverse and Reciprocal Trigonometric Functions

The fourth misconception identified by the researcher was the misunderstanding the students appear to have between inverse and reciprocal trigonometric functions together with the confusion about the use of the superscript (-1).

Stewart *et al*. (2021, p. 401) define an inverse function as follows:

> Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \Leftrightarrow f(x) = y$ for any y in B.

The notation of superscript (-1) to indicate inverse functions and reciprocal functions, and in this case, applied specifically to inverse and reciprocal trigonometric functions, seems to be challenging for many students, which will be illustrated in the examples that follow.

For clarification of the use of the superscript (-1) at inverse and reciprocal trigonometric functions, Table 1 gives the notation of inverse and reciprocal trigonometric functions alongside the use of the superscript (-1) of three of the six trigonometric functions.

INVERSE TRIGONOMETRIC FUNCTIONS	RECIPROCAL TRIGONOMETRIC FUNCTIONS
$\sin^{-1} x = \arcsin x$	$\frac{1}{\sin x} = (\sin x)^{-1} = \csc x$
$\cos^{-1} x = \arccos x$	$\frac{1}{\cos x} = (\cos x)^{-1} = \sec x$
$\tan^{-1} x = \arctan x$	$\frac{1}{\tan x} = (\tan x)^{-1} = \cot x$

Table 1: Notation of inverse and reciprocal trigonometric functions

Definition of Inverse Trigonometric Functions

Figure 5 illustrates the misconception this student, and many other students, appear to have about inverse and reciprocal trigonometric functions, specifically the use of the superscript (-1) when defining the function $\arctan x$.

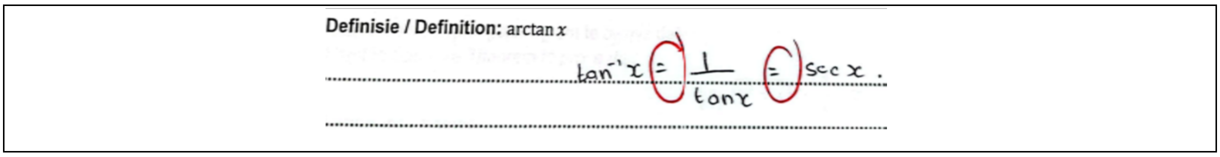

Figure 5: An example of an interpretation of superscript (-1)

Given the example seen in Figure 5, this student interpreted the superscript (-1) as the reciprocal of the trigonometric function. This confusion was also recognized by the research done by Chin and Pierce (2019, p. 6). Furthermore, this student says that $\frac{1}{\tan x} = \sec x$, which is correct, but this implies then that $\tan^{-1} x = \sec x$, which is incorrect.

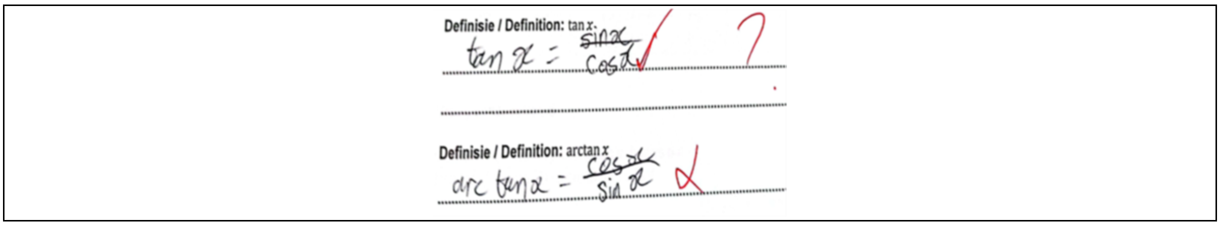

Figure 6: An example of an interpretation of the definition of $\tan x$ and $\arctan x$

Another example that points out the abovementioned misconception, is illustrated in Figure 6. This student defines the tan-function correctly but then states the inverse tan-function incorrectly. Hence, this student confused the inverse trigonometric function with the reciprocal trigonometric function even though the superscript (-1) was not applicable here.

Combination of Limits and Inverse and Reciprocal Trigonometric Functions

$$\lim_{t\to 3^-} \arctan\left(\frac{1}{t-3}\right)$$

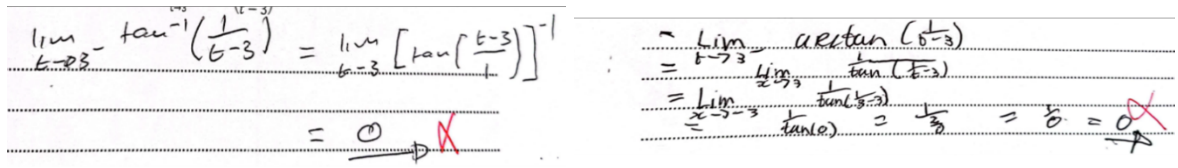

Figure 7: An example of two interpretations of inverse and reciprocal trigonometric functions at limit problems

In the last example (Figure 7) of the identified misconceptions, we have a combination of the misconception of a limit of a function and inverse and reciprocal trigonometric functions. The misconception about the limit of a function illustrated in both interpretations is the substitution of an exact number instead of *t* approaching 3 from the left-hand side.

Conclusion and Recommendations

From past assessments done by first-year calculus students at a certain South African university, the concept of a limit, and inverse and reciprocal trigonometric functions were identified as misconceptions. This paper adds to those voices who propose that if students do not have a conceptual understanding of the limit of a function as well as inverse and reciprocal trigonometric functions, students will have misconceptions about these very important notions in calculus. Thus, these misconceptions will influence their learning of concepts building from limits, like derivatives, integrals, and sequences (Ay, 2017, p. 21). Misconceptions regarding inverse and reciprocal trigonometric functions will also affect the calculations done with these types of functions.

For lecturers to teach for understanding is essential for first-year calculus students to succeed in first-year calculus in addition to any further studies in mathematics. To continuously be aware of and address misconceptions, lecturers make it possible for students to recognize their own misconceptions, address the misconceptions and try to eliminate those misconceptions for better understanding and to be more successful in calculus. The findings of this study show that the skills regarding limits are purely mechanical for those first-year students who are part of this study. The misconceptions found are typically comparable to those found by previous researchers. According to this study's results and emphasized by Jensen (2009, p. xiv), a lot of students' knowledge and comprehension are based mostly on discrete facts and procedures, and they have a poor conceptual comprehension of concepts like limits, inverse and reciprocal trigonometric functions, and infinity.

I conclude that students will develop further misconceptions if they have inadequate conceptual knowledge of these important concepts. In pursuit of an action plan to lessen or eliminate additional misconceptions, lecturers should be aware of their students' misconceptions and should carefully plan their teaching sequence.

The way forward will be the implementation of the second phase of the action research cycle, namely the action-planning phase (Susman & Evered, 1978, p. 588). During the action-planning phase, the researcher will create new techniques and strategies to address the diagnosed misconceptions. Given that the research topic was inspired by the researcher's desire for clearing up misconceptions and her own teaching experiences in the classroom when teaching calculus, the researcher’s reflections will be a valuable part of the action-planning phase. Therefore, the researcher will address the misconceptions by developing worksheets concerning the limits of a function and inverse and reciprocal trigonometric functions based on evidence through consulting the current scholarly literature and the researcher’s reflections.

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