Complex Mathematics Tools in Urban Studies

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Abstract
During centuries, the Theory of Architecture and Urbanism has been trying to describe and understand complex realities: the concept of beauty, how creative process are develop, or the meaning of a city, for example. In order to achieve explanations, sometimes we have built theories based in social issues; in other occasions we turn to Art with the aim of finding answers. And sometimes, we also use Science as a tool of understanding. About this last resource, it is possible to verify how the use of a rational deterministic point of view, has produced from buildings to urban models. Nevertheless, in the last decades it is possible to find approaches to Architecture and Urban studies based in a different kind of science, the one which we usually know in general as "science of complexity". In this paper we would like to reflect about how graphs theory allow us to have an instrument of analysis and understanding of complex systems such as old towns. We will see how in any urban tissues in which is difficult to find an order, traditional science tools seem to be incapable to describe the real complexity of them. Using graphs, then implementing a special algorithm, and finally with a proper graphic tool we are able to visualize the impact of different matters in these systems. To analyze the potential but also the limits of this kind of studies, is the aim of this paper

Keywords: *Complexity, Mathematics, Urban studies, architecture*

Introduction

History shows us that new scientific ideas are never developed *in vacuo*, but on the contrary they are in many senses the result of a *global understanding* about ourselves and our place on the universe: this is the kind of knowledge that traditionally has been named as *Philosophy* by the western culture [1]. For this reason, it shouldn't be too difficult for us to accept that there is a close relationship between the world of science, and other expressions of culture and creation, such it is art for example, or architecture [2].

But despite this reality, the fact is in many senses we live in a kind of a dual world, called by Fuller for example, the "two cultures world" [3], an expression also used by other authors in different fields [4]. The fact is that it seems to persist the idea of a world of "Scientifics", and perhaps in opposition to that world, or at least separated from that world, the universe of what we could called *humanistics*. Latour, among others, finds this situation senseless [5], but if we take a look at many official curricula in many academicals institutions, like my University for example, we will hardly find spaces for multidisciplinary experiences, such IAFOR promotes. Cecil Balmond, summarized this situation with this words: "*A classical understanding of engineering conspires in order to keep science away from art*" [6].

Architecture supposes as a discipline an ideal area in this sense. In its own nature, it exists a hybrid condition which allows to place the role of an architect in a blur territory between different kinds of knowledges, between science and art. Perhaps thanks to that situation, it has been possible for someone belonging to a humanistic field such it is Theory of Architecture, will develop different experiences of research with colleagues belonging to the Department of Mathematics. This paper is the result of some of those experiences, which have in common the use of complex mathematics tools to understand some aspects of the territory and of the cities: in this case, those tools come mainly from the *network theory*.

Figure 1: City as a network. Murcia Old Town

Evolutionary algorithms

The so called "neural network algorithms" are a kind of evolutionary algorithms inspired in the way real neural networks work in living beings: they "learn" progressively [7]. Used mainly in Artificial Intelligence, they may constitute a heuristic method to find a simpler system from a complex one which is able to keep the ***same topology*** as the primal system. In Figure 2 we have an illustration of this idea. The original face on the left may be described by a cloud of thousands of points, which is a complex system, and for that reason something difficult to work with and even to understand. In order to do so, that would be very convenient to find a simpler system formed by, for example, only 20 % of the original points. But the question is, would it be the same face? That's precisely what this tool may achieve: it is able to propose a new system formed by a network of much less points than the original, but in order to keep the same topology as the original objects, those are not just any point: they are *special points* in which the core condition of the primal system remains.

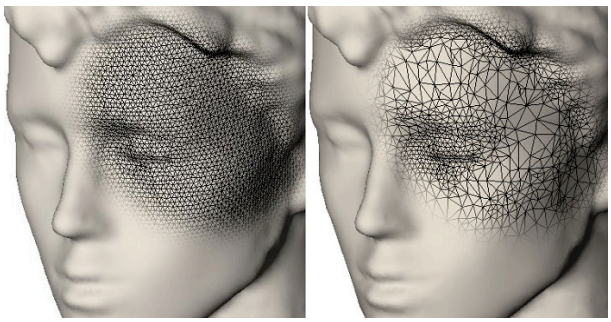

Figure 2: Mesh simplification

The objective of this research project was to investigate the possibility of applying this idea to complex situations that we could find in a larger scale: cities or territories. With *complex* we mean urban concentrations without precise limits or without a recognizable geometry patterns. These kind of systems are not easy to be understood by the use of "classic" architectonical geometrical tools, because their level of complexity and the lack of hierarchy make very difficult to identify the critical points, and consequently to decide where to act. In those context, this model could be useful to find a new simpler representation of that reality, formed by key points as we have seen with the face. As long as they are relevant points, we may think they play a strategic role for the network, and therefore, we may understand that they may indicate us where to apply some actions that will affect the whole system.

One field in which this research line has been applied is based in the idea of **density**, and its relationship with the public space. With the use of a neural network model we try to redefine from that point of view some examples of *urban sprawl* developed in the Mediterranean coast of Spain in the last decade. It is not necessary here to insist once again on the questionable results of this urban model: in terms of environmental impact, efficiency in energy or transport, or social sustainability we may agree it is a quite deficient one. Therefore there is here a real opportunity to debate about those settlements and to suggest some proposals.

What we have in this case is a complex system formed by hundreds of isolated houses in which it is difficult to find a global geometrical order, and in consequence to know where to act. In order to understand it on the first place, and then to handle with this system, we model it as a two dimensional triangle mesh in which each node is a

house. Applying a network optimization based on the Growing Neural Gal (GNG) model, we obtain a simpler system which - as we said - keeps the same topology as the original one, because it is formed only by critical points. Those are precisely the nodes in which we concentrate the density of a cloud of houses from the original system. This is not the place to explain the detail of the mathematics methodology, especially because it can be checked in other works [8]. But we can analyze the process from a critical point, and then we can conclude that this mathematical tool, offers to the urban planners a promising way to work with the geometry of these kind of systems. It constitutes a way to achieve a handy simplification from reality, but at the same time keeping the level of conceptual complexity that a self-organized model like this one implies.

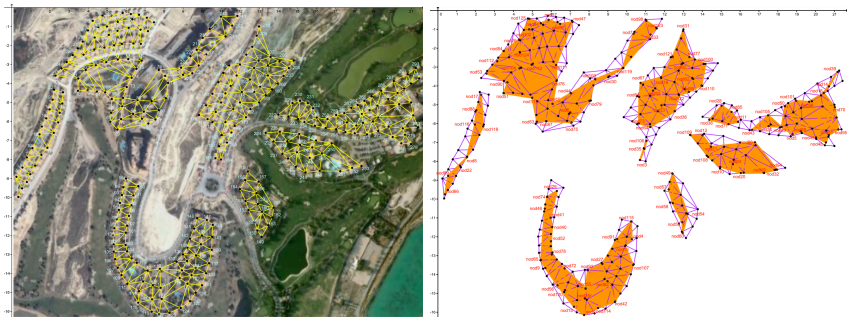

Figure 3: Original and Final mesh

In this case, we have read those points in terms of density as we said, but those critical points can also be understood in other meanings. Of fact, from a conceptual point of view, they are just core points in which some system can be simplified. And therefore, it is possible to use this methodology to design urban transport networks based in this kind of self-organized model [9], where the points will represent nodes of transportation; or for example, in a more wide approach, we can use this methodology to identify the key nodes – the *pressure points* using the terminology – where we may propose actions to activate the whole system, under the philosophy of what we call *urban acupuncture.* [10]

Network theory and the city

These previous experiences show that it is possible to describe the city – or any urban reality – as a system formed by nodes interacting and connected by edges, something which on mathematic language is called a *graph*. We have also seen that it may be quite interesting to distinguish on those systems the relative importance that some points have for the whole system.

Our proposal is then to consider an urban network as a specific type of complex network, or in other words, a *primal graph.* We are interested in using the primal graph as a data structure because is a very handy model to describe the street pattern of a given city. And once we have that structure, it is possible to create mathematics tools which may identify which are the key nodes for the system, and even more important, how some transformations in some nodes may affect to the whole system. (Figure 4.)

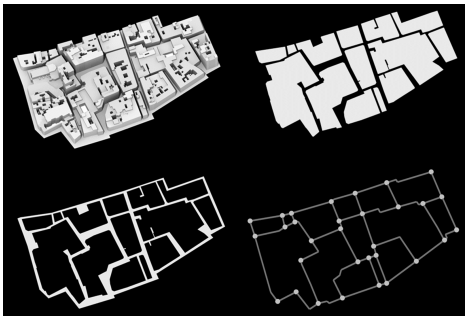

Figure 4: Primal graph configuration

In our research we wanted to use this idea to test the commercial activity of a real city, in this case the city of Murcia, on the South East of Spain (350.000 inhabitants). As we have anticipate, our approach begins with a process to convert the real urban streets into a primal graph representation, and then assigning to the nodes the kind of information we want to take in count in order to evaluate their importance in the system, which in our research is the commercial activity.
In order to stablish a ranking of nodes from this point of view, we propose an adaptation of the PageRank model that we call *APA (*Adapted PageRank Algorithm). The resulting data matrix, and the numeric details can be checked at [11], but here we can say that this process allows us to have a very clear and expressive representation of the *hot* and *cold* areas on the city from a commercial point of view. (Figure 5.)

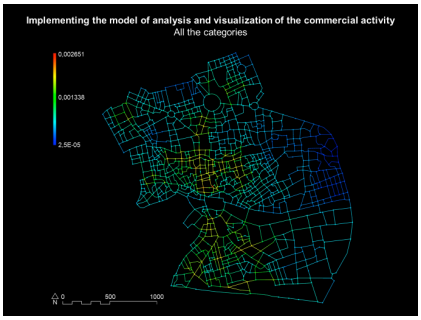

Figure 5. Visualization of the commercial activity

One of the most interesting aspects of this kind of approach, is its dynamic condition. As we know, the commercial activity in a town is constantly changing, and this methodology allows us to modify the information in a very efficient way. Once we have built the primal graph, the allocation of the information to the nodes is quite agile, and for this reasons it seems an interesting application to monitor the commercial activity with dynamic data bases. But not only. The APA also may allow us to propose simulations of the impact of new activities in some area, showing the influence in the area on the new activity. (Figure 6)

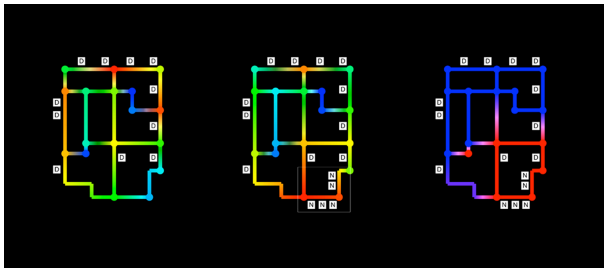

Figure 6. Study of the impact of new commercial activities

And similar simulations may be carried out in order to evaluate for instance an inner urban reform plan, allowing to visualize the impact that has on the network the development of new actions, something which could be checked in its mathematics details in [12].

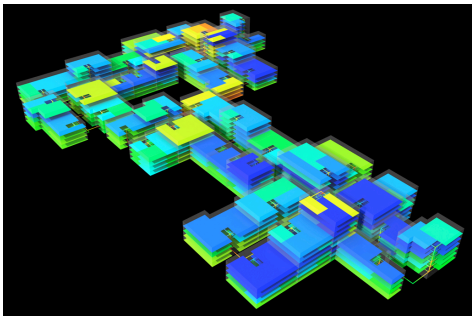

Figure 7. Study of new inner urban reform plan

Conclusion

Understanding a city as a complex network, allow us to use complex mathematics tools from the network theory in order to handle with some aspect of it. Obviously, these are not the only possible points of view when you work with a city, but the truth is they may complete other ones. And as we have seen, the results may be considered promising in many aspects.

But the kind of evolutionary algorithms we use, promote results which are non-deterministic, and this may make us question what is the role of the urban planner in this process. About that, we have to conclude that is as relevant as always has been in this matter. On the first place, at the beginning of the process, taking the decision about what to analyze and for what purpose, decision which of course is the result of a previous conceptual debate. Then, during the process, is the planner who evaluate and measure the evolution of the system. And finally, it is also us who at the end interpret the resulting data from a quality point of view.

In conclusion, we think that the use of some complex mathematics tools in urban studies, in particular the ones we have explained, allow to open a vast opportunity for understanding many aspects of the city, with the aim of giving us certain arguments to propose creative actions.

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