***Mathematical Modeling: Synthesis of Qualitative Research***

Andrzej Sokolowski, Texas A&M University, USA

The IAFOR International Conference on Education - Dubai 2015
Official Conference Proceedings

**Abstract**
The purpose of this study was to synthesize qualitative research findings about mathematical modeling at the high school and college levels focusing on the inquiry processes applied during modeling. A total of 19 primary studies published in peer-reviewed journals between January 1, 2000, and February 28, 2013, with a total of 1,290 subjects met the inclusion criteria.

The research findings revealed that mathematical modeling can enhance students' problem solving techniques and that it has a potential to be supported by scientific inquiry methods. As such, this paper can be of interest to mathematics curriculum designers and practitioners who seek ways of integrating the methods of mathematics with other disciplines.

**Keywords:** Mathematical modeling, scientific inquiry, inductive reasoning.

## Introduction

Mathematical modeling (MM) has been described diversely; for instance, Lesh and Harel (2003) defined MM as finding patterns and quantifying and generalization of a phenomenon, while Blum, Galbraith, Henn, and Niss (2007) defined MM as a process of "learning mathematics so as to *develop competency in applying mathematics and building mathematical models* for areas and purposes that are basically extra-mathematical" (p. 5). Modeling processes constitute central methods of science knowledge acquisition (Schwarz & White, 2005). Modeling provides a means for analyzing data, formulating theories—often expressed in symbolic mathematical forms—and testing those theories.

As such, learning via modeling plays a vital role in developing students' skills in both science classes (Wells, Hastens, & Swackhamer, 1995) and mathematics classes, especially during problem solving (Lesh & Harel, 2003). One of the many advantages of modeling, as compared to problem solving, is shifting the learning focus from finding unique solutions to enhancing solution process skills through transforming and interpreting information, constructing models, and verifying the models (Lim, Tso, & Lin, 2009). One of the obstacles to adopting such inquiry-based learning in mathematics is the gap between problem solving in mathematics and inquiry in science (Schoenfeld & Kilpatrick, 2013).

The purpose of this research was to synthesize findings of studies that immersed students in modeling activities defined as finding patterns, generalizing the patterns, and expressing the patterns using mathematical apparatus and search for ways of merging the process of mathematical modeling with scientific inquiry methods.

## Theoretical Background

Pinar (2004) contended that interdisciplinary curriculum fosters intellectual development and students' capacities for critical thinking, while Kelly (1989) maintained that to acquire knowledge is to experience, observe, and form hypotheses. Dewey (1997) suggested that learning results from action and practical consequences of real effects that are vital components of meaning and truth in education, and defined *learning* as a process in which experience is the motor for new knowledge guided by inquiry.

Ernest (1998) claimed that many concepts of mathematics are derived by direct experience of the physical world, by generalization and reflective abstraction of previously constructed concepts, by negotiating meanings with others during discourse, or by some combination of these means. Piaget (1964) contended that physical knowledge and logico-mathematical knowledge are learned simultaneously; when physical characteristics are learned, mathematical knowledge is used to quantify the characteristics and vice versa.

What makes science and mathematics conjoint disciplines? Multiple opportunities of integrating the methods of mathematics and other disciplines set the foundation for a curriculum that reflects a complex field of scholarly inquiry that endeavors to understand concepts across school subjects and academic disciplines (Kelly, 1989). As science guides the search for patterns and their qualitative explanations, mathematics guides the search for describing and explaining the patterns using quantity, size, and shape, which illustrates why these two disciplines are conjoined (Yore, Pimm, & Tuan, 2007).

Natural phenomena cannot be completely apprehended without the tools of mathematics, and the purpose of studying mathematics intensifies when mathematics is used to analyze rich scientific contexts. Mathematics helps model empirical data and formulate them as timeless and universal representations (Sokolowski, Yalvac, & Loving, 2011). However, in order to have mathematics students appreciate all aspects of such inquiry, they need to be immersed in meaningful learning environments that provide opportunities not only for evaluating abstract mathematical representations but also for formulating hypotheses and validating derived models in new modified situations.

The process of  MM was first introduced into mathematics classrooms about four decades ago (Pollak, 1978), and its effects on mathematics education research have been increasing (Blum & Leiss, 2007). By promoting transfer of knowledge, problem solving, and scientific thinking (Kuhn, 2007), the original ultimate goal of mathematical modeling was to bridge the gap between reasoning in a mathematics class and reasoning about a situation in the real world (Blum, Galbraith, Henn, & Niss, 2007). Over the years, the descriptions of mathematical modeling have undergone many modifications, ranging from deductively situated authentic problem modeling activities seeking unique solutions to given problems (English & Sriraman, 2010) to inductively organized inquiries leading students to find general patterns and express the patterns using the tools of mathematics (Lesh & Zawojewski, 2007).

Modeling offers a systematic way of understanding and working with the relationship between mathematics and problem situations or phenomena in other disciplines (Artigue & Blomhøj, 2013). The products of MM; elicited models, can take various forms, ranging from physical objects (e.g., solids or plane figures) to mathematized statistical models, functions, or differential equations.  Mathematical modeling that utilizes real scenarios and leads learners to a pattern formulation is often classified as an exploratory type of learning (Thomas & Young, 2011). As such, multifaceted cognitive goals are achieved by learners undertaking modeling activities. Bleich, Ledford, Hawley, Polly, and Orrill (2006) claimed that such activities (a) expand students' views of mathematics by integrating math with other disciplines, especially science; and (b) engage students in the process of mathematization of real phenomena.

Problem solving and MM and are interwoven. While *problem* is defined as a situation carrying open questions (Blum & Niss, 1991), mathematical modeling is a process of moving from a real-world situation to a model, and then using the model to further understand and develop new knowledge or solve real-world problems (Crouch & Haines, 2004). Thus, the contexts of problem solving and modeling are tightly integrated. A major contribution to problem solving was made by Polya (1957), who formulated four stages of the process: understanding the problem, devising a plan, carrying out the plan, and looking back.

This sequence was further extended and redefined by Bransford and Stein (1984), whose model included the stages of identifying the problem, defining goals, exploring possible stages, anticipating outcomes, and looking back and learning. According to Arthur and Nance (2007), the stage of exploration, which leads the solver to model formulation and validation, is one of the most important in the process of problem solving.

Modeling is a core practice in science and a central part of scientific literacy (Schwarz et al., 2009). Situation, model, and analysis of the model are thereby also essential elements of scientific modeling. An important unanswered question is the following: Should scientific elements remain silent in math modeling activities, as they do in the current literature, or should they be incorporated to produce coherent methods that students should be able to transfer and apply in their mathematics classes, as is suggested by curriculum theorists?

This section provides insight into how scientific inquiry is organized. The Oxford English Dictionary defines *scientific method* as “a method or procedure that consists of systematic observation, measurement, experiment, the formulation, testing, and modification of hypotheses” (Scientific Method, n.d.), whereas the National Science Education Standards (NSES; National Research Council, 2000) define *scientific inquiry* as a way in which scientists study the natural world and propose explanations based on the evidence derived from their work. Scientific inquiry not only reflects on how scientists explain the phenomena occurring in the natural world but also gets at the heart of how students learn (Sandoval, 2005).

While scientific inquiry defines ways of investigating natural phenomena, scientific methods encompass techniques of such analysis (Hestens, 2013). The NSES identify five elements of inquiry methods for teaching and learning; learners must (a) be engaged in two types of questions: existence (*why*?) or casual (*how*?) scientifically oriented questions; (b) give priority to evidence by observing and measuring; (c) formulate explanations from evidence to address hypotheses; (d) evaluate their explanations; and (e) communicate and justify their proposed explanations. In sum, in order to be termed scientific, a method must be based on empirical and measurable evidence followed by hypothesis stating, evidence gathering, model formulating, and testing.

## Findings of Prior Research

Although mathematical modeling was implemented in mathematics education several decades ago, its contribution to mathematics education research has gained momentum recently. This section synthesizes major findings from prior studies. In supporting the objectives of this study, we searched for research using ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, ProQuest Educational Journals, Science Direct, and Google Scholar. Although several research studies aimed at various aspects of conceptualization of mathematics ideas were located, a synthesis of qualitative research pertaining to mathematical modeling was not found. The lack of such studies further supports the need for undertaking a qualitative study on mathematical modeling.

Research has shown a positive connection between mathematical modeling and learning outcomes. For example, research conducted by McBride and Silverman (1991) revealed that mathematical modeling used during integrated lessons increased student achievement in all involved subjects. Research has also identified several pitfalls connected to mathematical modeling that educators must consider. Zbiek and Conner (2006) questioned whether the skills of mathematical modeling should be considered as separate assessment items. The research reported positive effects on student learning when contexts were used to enhance math learning objectives. As cognitive and affective effects on students' math knowledge and aptitude are well exploited and researched, the literature revealed several unanswered questions and unresolved issues regarding instrumental implementation of this learning method in school mathematics along with its relation to science. This study, described in detail next, attempted to answer some of these questions.

## Method

The analysis of the pool was guided by a systematic classification process of coding and identifying themes and patterns as described by Hsieh & Shannon (2005) focusing on the development and evaluation of the elicited models formulated by learners.

As an emerging method of mathematical knowledge acquisition, MM that uses exploratory environments still faces unresolved issues that prevent the methodology of framework design from solidifying. Guided by the conceptual framework, the following research questions emerged:

How do currently used mathematical modeling activities reflect on scientific inquiry methods when used in mathematics classes?

What is the role of mathematical modeling in problem solving?

While the answer to the first research question might provide prompts for a possible modification of the currently used modeling cycles, the answer to the second one will generate means for identifying areas where both can merge. In the attempt to more comprehensively answer these questions, other auxiliary themes were considered: Should mathematical modeling be limited to just formulating mathematical representations, or should mathematical modeling be perceived as a bridge linking mathematics with other academia and provide more opportunities for enhancing students' scientific inquiry skills? What phases of mathematical modeling help students improve their problem-solving skills and enrich their techniques of formulating solution designs?

Several key terms were formulated to guide the systematic literature review. The terms are summarized in Table 1.

Table 1: Summary of Key Terms Used to Locate and Scrutinize the Research Pool

| Key Term | Description |
|---|---|
| Math modeling | Process of moving from a real-world situation to a mathematical model (Crouch & Haines, 2004). |
| Medium applied | A form of information presented that can be categorized as data tables, a written text problem, computerized interactive scenario, or real experiment. |
| Research type | Quantitative |
| Grade levels | High school and college levels. Initially, we intended to include the entire schooling spectrum, yet due to a lack of literature pertaining to lower grade levels (English & Sriraman, 2010), the grade levels were limited to high school and college. |
| Time span | From January 1, 2000, to February 28, 2013. This time span marks an increase in research on math modeling (Lesh & Harel, 2003). |
| Descriptive parameters | Locale where the studies were conducted, sample size representing the number of subjects under investigation groups, date of the study publication, duration of the study, and total time interval that the subjects were under treatment. The total treatment time was introduced due to a high diversity of treatment frequency; thus, for instance, if the study lasted 2 months and the treatment was applied twice a week for 3 hours each session, the reporting is depicted as 2 months/48 hours. |
| Publication bias | Only peer-reviewed studies published as journal articles were extracted. By enacting these criteria, a publication bias was reduced. |
| Scientific inquiry | *Scientific inquiry* as a method or procedure that consists of experiment, hypothesis, and systematic observation and measurement, along with formulation, testing, and modification of model and hypotheses. |
| Activity organization | Established to reflect on the inquiry method during modeling activities and learn if scientific methods such as hypothesis stating, measurements, observations, data analysis, model formulation, and its validation were present. Within this domain, MEAs constituted a separate category. |

In the process of collecting the research literature, ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest educational journals, as well as Science Direct, Google Scholar, and other resources available through the university library, were used. The following terms and their combinations were utilized to locate the relevant literature: *mathematical modeling*, *model-eliciting activities, simulations*, *computers and mathematics modeling, inquiry in mathematics learning, student achievement*, *high school,* and *college*. These search criteria returned 145 articles. A review revealed that eight of these research studies satisfied the criteria.

The majority of the rejected studies focused on examining formulated models in the professional fields of engineering or medicine, and some represented position papers on modeling. In order to increase the pool, a further search was undertaken with broader conceptual definitions.

This search included auxiliary terms that were found in descriptions of mathematical modeling activities, such as *investigations in mathematics, exploratory learning in mathematics,* and *computerized animations and learning*. These modifications returned 37 research papers. After additional scrutiny, 11 more studies were coded as satisfying the research criteria. In total, the meta- analysis included 19 primary studies, out of which four were conducted using mixed research design.

## Descriptive Analysis of the Pool of Studies

This analysis encompasses findings conducted with 1,256 students at the high school and college levels. Several evaluation instruments were used in the qualitative research analyzed, including interviews with participants, surveys, observations, and questionnaires.

The sample sizes of the research pool ranged from three subjects (Cory & Garofalo, 2011) to 228 subjects (Leutner, 2002); the average sample size was 60 subjects. When categorized by school level, 14 of the studies (or 74%) were conducted on the college level and involved mainly calculus students, and four (or 21%) were conducted at the high school level. Four studies (or 21%) were conducted at the college level and involved students from teacher preparatory programs (e.g., Carrejo & Marshall, 2007; Türker et al., 2010).

This trend indicates that preparing teachers to teach students modeling techniques has gained popularity in mathematics education. Considering the ratios of the populations, it is evident that the interest in examining the effects of applying MM gravitates toward college-level education. There was a noticeable diversity in the study durations, ranging from 1 hour (e.g., Cory & Garofalo, 2011) to 1 semester (e.g., Klymchuk et al., 2008; Yildirim et al., 2010).

## Inferential Analysis and Theme Formulation

While qualitative research unfolds as data are gathered, each study was considered as an individual source of information. With the goal of searching for keys that reflected scientific inquiry in MM along with descriptions that highlighted students' progression through the modeling cycles, a tabularization was generated; see
Table 2.

Table 2: Summary of Treatment Descriptions and Research Findings

| Leading Author | Treatment Description and General Findings | Medium Applied, Activity Organization |
|---|---|---|
| Yoon (2010) | Used MEAs after an instructional unit to teach the accumulation. Investigated change of student perception and interpretation of calculus tools. | PP, MEA |
| Lim (2009) | Used MATLAB and simulated volcanic ash fall. Investigated change in students' attitude toward mathematics. Mathematics appeared to generate a friendlier environment to students. | COMP, Analysis |
| Liang (2010) | Used interactive 3D objects to formulate patterns for volume and surface area. Investigated students' change of interpretations of geometry terms. | COMP, Analysis |
| Leutner (2002) | Used simulation called SimCity to enhance problem solving through modeling skills. Measured participants' comprehension skills. | COMP, Hypothesis Analysis |
| Chinnappan (2010) | Observed discussion of modeling techniques and approaches. Students' descriptions of math terms were more detailed and focused. | PP, Analysis |
| Crouch (2004) | Analyzed reflective questionnaire to distinguish between novice and expert modelers. Expert modelers used math tools with greater flexibility. | PP, Hypothesis Analysis |
| Yu (2011) | Teachers solved modeling problems and designed some. They perceived modeling as a bridge to problem solving. | PP, MEA |
| Diefes-Dux (2012) | Used web-based MEA resources to support modeling activities. The unit was applied after certain math concepts were introduced. | PP, MEA |
| Faraco (2012) | Students used Lab VIEW to develop simulated physical phenomena. Principle understanding is needed for successful modeling techniques. | COMP, Hypothesis Analysis |
| Iversen (2006) | Derived and evaluated algebraic functions. Assessed strengths and weaknesses of the modeling processes. | PP, MEA |
| Klymchuk (2008) | Used interactive life contexts. Learners priced correlations of the tasks to reality. Math modeling improved their problem-solving skills. | COMP, Analysis |
| Schorr (2003) | Pre-service teachers provided feedback about teaching the processes of modeling. Positive changes in students' attitudes emerged. | NP, Analysis |
| Türker (2010) | Participants worked on modeling activities. They reported that mathematics concepts became more tangible to them. | PP, Analysis |
| Soon (2011) | Students worked on modeling activities involving DEs and linear algebra. Auxiliary steps were provided. | PP, Analysis |
| Yildirim (2010) | Investigated students' progression through modeling cycle. Students had difficulties with hypothesis stating. | PP, Hypothesis and MEA |
| Cory (2011) | Used sketchpad to visualize the concept of limits. | COMP, Analysis |
| Schukajlow (2012) | Modeled problems in two different learning settings such as student and teacher centered. Student-centered modeling benefited students the most. | PP, Analysis |
| Flegg (2013) | Students learned how to construct and deconstruct mathematical models. | PP, Model Analysis |
| Carrejo (2007) | Teachers were involved in modeling activities. Need for implementing mathematical modeling in teacher preparatory programs arose. | Real Lab, Analysis |

*Note.* PP = pen and paper, COMP = computer, NA = not applicable, MEA = model eliciting activity

When categorized by medium-supporting modeling activities, traditional pen-and-paper activities—used in 11 studies—dominated the pool. Computers were used in six of the studies, and a real lab was applied in only one study. When categorized by elements on inquiry, in the majority of the studies (12, or 63%), the processes were initiated by having students formulate a problem, analyze given contexts, and construct mathematical models. In four of the studies, students were required to state a hypothesis.

According to the procedures of analyzing a qualitative research study (Hsieh & Shannon, 2005), its inference concludes with formulations of themes that will be used to answer research questions. In this study, constant comparisons and debriefing of the accumulated research findings helped formulate the three themes that mirrored the study objectives. The first theme analyzed the stages of the modeling cycles and reflected on how they integrated scientific inquiry methods. The second theme reflected on the modeling–problem-solving interface and sequencing of modeling activities within math curriculum.

**Research Findings**

How do currently used MM activities reflect on scientific inquiry methods? The answer to this research question is clustered into three subthemes that surfaced in the research pool: formulating a hypothesis and converting reality into mathematical symbolism. A separate theme deals with a relation of MM activities and problem solving.

Learners were expected to state a hypothesis in four of the studies, yet several researchers (e.g., Crouch & Haines, 2004; Faraco et al., 2007) pointed out concerns about students' weak skills in formulating, proving, and disproving the hypotheses. Hypotheses build on the problem stated in the activity, and hypothesis formulation is a one of the most important elements of scientific inquiry that drives the process (Kelly, 1989). Once formulated, a hypothesis focuses the investigator's attention on a narrower area of investigation.

The hypothesis can be perceived as the investigator's proposed theory explaining why something happens based on the learner's prior knowledge (Felder & Brent, 2004). The role of a hypothesis is to confirm or correct an investigator's understanding of what the content of the modeling activity presents. As hypotheses in modeling activities will likely be verbalized such that they are aimed at testing mathematical concepts rather than scientific ones, the contextual balance between these two disciplines needs to be established. Reducing the problem statement in a way that students formulate only mathematical representation will not nurture the connection between the real world and mathematical world as defined by Blum and Leiss (2007).

As the research shows, the term *hypothesis* is rarely used during math modeling activities; therefore students' difficulties in hypothesis formulation signalize that more work is to be done to help students overcome the barriers. More elaboration in the differentiation between hypothesis and prediction is also needed, as both can be used in mathematical modeling. A hypothesis proposes an explanation for some puzzling observation, while a prediction is an expected outcome of a test of some element of the hypothesis (Lawson, Oehrtman, & Jensen, 2008). Thus, predictions will be associated with deductively organized modeling activities seeking a unique solution, as defined by Gravemeijer (1997).

In inductively organized modeling activities, a hypothesis will reflect on general mathematical structures and scientific context, whereas a prediction will constitute an extension of the activity, most likely supporting its further validation (see, e.g., Leutner, 2002). The use of these essential terms of scientific inquiry during math modeling activities is not visible in the current research.

A major concern voiced frequently in the research pool involved students' inability to transfer a text scenario description into its mathematical embodiment (e.g., Soon et al., 2011; Yoon et al., 2010), which usually constitutes the pivotal element of the modeling process. During the process of analyzing the accumulated research with the intention of finding suggestions for improving this stage, two further questions emerged: Is the deficiency due to a weak student understanding of mathematical structures (e.g., the properties of periodic functions, the differences between rate of change and a percent change, the techniques of solving differential equations, and so forth), or is the deficiency due to difficulties in identifying conceptual patterns in given problems (scientific principle) and mapping the patterns on corresponding mathematical embodiments? Thus, the interface of integrating the two different worlds—real situations and their corresponding mathematical representation, as defined by Blum and Leiss (2007)— needs further investigation to help students see the link. Flegg et al. (2013) concluded that in the process of understanding mathematical model formulation, students look for relationships between the math elements of the model and the context and try to relate the model to what they already know, which is not sufficient.

Another theme that evolved from the qualitative analysis involved the investigation of how modeling activities support students' problem solving skills, what their strongholds are, and which elements appear to be still unsolved. In their study, Yu and Chang (2011) concluded that "developing the modeling ability promotes students' problem solving ability" (p. 152). However, they also noticed a lack of theoretical background on how to transition the process of mathematical modeling to problem solving. There is though strong research supporting the thesis that carefully designed modeling environments can foster and solidify students' problem-solving skills (e.g., see Chinnappan, 2010).

In this venue, the issue of sequencing modeling activities within math curriculum can also be discussed. One of the themes that emerged from the high school modeling research findings was the sequencing of modeling activities within a chapter domain. There are two distinct voices regarding this matter: one, advocated by Blum et al. (2007) and Lesh and Zawojewski (2007), suggests that modeling activities be implemented prior to new content being taught, while the other view, as presented, for example, by Leutner (2002) and Chinnappan (2010), proposes that modeling activities be implemented after new content is delivered. Both strategies seem to benefit learners, but caution needs to be taken with the inquiry design of the activities.

Lesh and Zawojewski (2007) supported their claim by pointing out that modeling implemented as a concluding activity of the instructional unit guides students along necessary trajectories but turns the activity into mathematical applications, which is not intended. A legitimate question in this context arises: Does associating mathematical modeling with the exercising of mathematical applications diminish the virtue of modeling activities?

Considering the content of the simplicity principle (see Lesh & Kelly, 2000), it seems that implementing inductively organized activities after content delivery would produce higher learning effects as compared to the reverse order. There is further support for such sequencing; Leutner (2002) advocated that students' pre-domain knowledge correlates with their achievement in modeling and problem-solving activities.

A similar conclusion was reached by Chinnappan (2010), who stated that if the goal of teaching math is developing students' structural understanding of concepts and embedding the concepts in realistic contexts, students need to learn the structures *before* exercising their applications, and this arrangement is a precursor of students' success.

Since modeling activities are often depicted by diagrams (e.g., see Blum & Leiss, 2007; Pollack, 1978), we determined that suggesting some modifications to the existing cycles to reflect the current research findings might serve as a way of expressing the research conclusions (see Figure 1). The modifications of these cycles reflect the research findings and are guided by an attempt to increase the presence of scientific inquiry methods in the math modeling activities, especially in the initial and concluding stages of the process.

Although the general structure is not new, we suggest emphasizing certain stages that emerged through debriefing of the primary research. The concluding section of the research has a bifocal purpose: to elaborate on the general structure of the proposed modified modeling cycle by pinpointing particular research findings that led to its emergence and provide suggestions for modeling activities rooted in this design.

The stem of the process is organized in a manner that will simultaneously provide the learner with foundations for problem solving and offer the opportunity for intertwining mathematics and science (or other) concepts. Constant revision of the process stages is also strongly suggested.

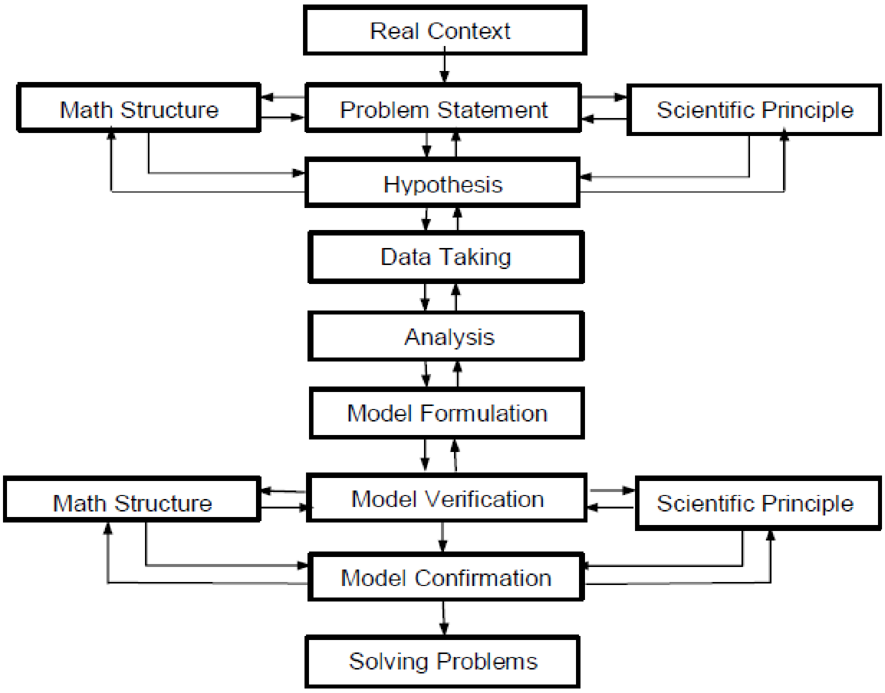


Figure 1: Integrated Modeling Cycle

The process illustrated in Figure 1 is initiated by providing students with a real context and a problem/question to solve. It is important for the context to provide opportunities for taking measurements and collecting data that will represent not only scientific evidences for the activity but also create more prompts for hypothesis formulation. Thus, based on their prior knowledge, students formulate a hypothesis for the problem solution, not only reflecting on the mathematical structure but also supporting it by scientific or other subject knowledge. Data taking, analysis, model formulation, its verification, and its confirmation constitute the central stage of the modeling activity.

The mathematical structure enacted will appear as the optimal solution to the problem posed. Through interactions with an appropriate medium, students progressively build, revise, or adapt their initial strategies if necessary. Students then validate the model's mathematical structure (e.g., type of algebraic function) and the coherence of the structure with the embedded scientific principle. Once validated, the model is ready to be used in other contexts or applied to solve word problems of a similar content domain. The details of how the stages were assembled follow.

**Implications**

This study carries certain limitations, some of which can be attributed to the limited number of studies available for analysis, and others that can be attributed to the diversity of mathematics curricula and competencies across the countries where the research on modeling was conducted. Accounting for such diversity was not possible in the present study. Moreover, though the view of this research was that mathematical modeling is intended to support problem solving, a moderator link testing the modeling impact on students' problem-solving techniques could not be established. In some of the primary studies, student achievement resulting from mathematical modeling activities was evaluated on a broader scope, seen through general students' math concept understanding but not focused on modeling skills, an issue that was raised by Inversen and Larson (2006).

This conclusion prompts more sophisticated research focused on investigating student perceptions of transitioning from mathematical modeling to problem solving and their success with the latter. The current research also shows the need for the establishment of a stronger link between mathematical modeling and problem solving in high school practice. It has been widely proven that mathematical modeling, even when taught in isolation to problem solving, helps accomplish multiple math learning objectives (e.g., see Chinnappan, 2010; Crouch & Haines, 2004). Yet, if set as a leading method of problem-solving techniques, its impact on students' mathematical knowledge acquisition is projected to be much higher.

One major question that arose and warrants further investigation is the following: How can educators organize experimental activities in a math classroom that is not typically designed for such activities? Advances in modern technology can be helpful. Several studies (e.g., see Finkelstein et al., 2005) have shown that computerized experiments not only effectively replace real experiments, but students who used simulations learned even more than students who used real equipment.

Thus, the potential to include virtual simulations that allow for manipulating variables and collecting data exist, and it seems that this potential is not fully explored in mathematics classes yet. Another question, and one of a rather philosophical nature, that emerged involves the possible augmentation of the math description: Are the methods of mathematics, defined widely as pattern seeking and conjecture formulation (Devlin, 1996), sufficient to reflect on their new role to lead learners through the MM processes?

Finally, MM has the potential to bridge math with other academia; thus, addressing the issue of a closer integration of science inquiry with math emerged as a venue for future investigation.

As one of the obstacles in adopting inquiry-based learning projects in mathematics is "a tremendous gulf between the language and traditions of problem solving in mathematics and inquiry in science" (Schoenfeld & Kilpatrick, 2013, p. 901), it seems that mathematical modeling supported by scientific inquiry can bridge these two learning methods. The author strongly advocates conducting further research in these regards.

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**Contact email:** asokolowski@tamu.edu