The Taboo of Negative Numbers in Primary Education

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The European Conference on Education 2023
Official Conference Proceedings

Abstract
Performing subtraction, as opposed to addition, is a rather daunting task for many primary school children; especially when using the so-called method of 'Subtraction with Regrouping' (SWR). We argue that the main reason for this is because the concept of negative numbers is not introduced at an early stage of primary education. Negative numbers have been considered as a taboo for too long and measures need to be taken to break this taboo to increase children's interest in mathematics. The SWR method is well-understood when a small number is subtracted from a large one; especially when there are no zeros in the large number and every of its digit is greater than that of the smaller number, like 758-231. Things can get rather complicated for children when the large number contains 0 as a digit or when they have to subtract a large number from a small one, for example 7045-2658. In the SWR method, a non-zero number is decremented by 1 when 10 is borrowed from it; for example, 5 becomes 4 when 10 is borrowed from it. However, the exception is that 0 becomes 9 when 10 is borrowed from it. This leads to an inconsistency in the procedure; hence, creating a confusion in children's mind. In this paper, we propose a direct method of subtraction, whereby the number 0 can be rightly replaced by -1 without disrupting the procedure. This can only be done when the children are taught the concept of negative numbers before tackling subtractions.

Keywords: Subtraction, Primary Education, Regrouping Method

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Introduction

For many primary school children, performing subtraction, as opposed to addition, is a rather daunting and difficult task. In effect, Baroody (1984) argue that some of the root causes for children's difficulties in subtraction stem from their informal subtraction strategies. However, in this paper, we argue that an alternative cause for children's difficulties with subtraction is due to the fact that the concept of negative numbers is not introduced at an early stage of primary education. In effect, negative numbers have been considered as a taboo by educators and developers of children's curriculum since the early 1900s. Consequently, there is a real need to take the required measures to break this taboo, so that young children can develop an interest in mathematics. The method of subtraction commonly and widely employed in primary schools is the so-called method of *subtraction with regrouping or borrowing* method as described in the following.

The Classical Subtraction With Regrouping Procedure

In the context of early primary education, the regrouping/borrowing method is generally employed when small numbers are subtracted from large ones so that the answer is always positive or zero. When there are no zeros in the large number and every of its digit is greater than that of the smaller number, then the subtraction is straightforward. For example, if we are asked to perform the following subtraction: 785-143. We start by writing the numbers column-wise as follows:

$$\begin{array}{cccc} & 7 & 8 & 5 \\ - & 1 & 4 & 3 \\ \hline & & & \\ \hline \end{array}$$

Then, we proceed by subtracting the rightmost digits and we gradually move to the left. That is, we subtract the numbers at the unit place, the numbers at the 10th place and finally the numbers at the 100th place as described below:

$$\begin{array}{cccc} & 7 & 8 & 5 \\ - & 1 & 4 & 3 \\ \hline & & & 2 \\ \hline \end{array} \quad \begin{array}{cccc} & 7 & 8 & 5 \\ - & 1 & 4 & 3 \\ \hline & & 4 & 2 \\ \hline \end{array} \quad \begin{array}{cccc} & 7 & 8 & 5 \\ - & 1 & 4 & 3 \\ \hline & 6 & 4 & 2 \\ \hline \end{array}$$

Step 1 ***Step 2*** ***Step 3***

Things can get rather complicated for children when some digits of the large number have a smaller value than that of the small number. For example, consider the following simple subtraction: 735-296. The subtraction with regrouping procedure for this example is done in three steps shown below:

$$\begin{array}{cccc} & 7 & \dot{3}^{2} & 5 \\ - & 2 & 9 & 6 \\ \hline & & & 9 \\ \hline \end{array} \quad \begin{array}{cccc} & \dot{7}^{6} & \dot{3}^{2} & 5 \\ - & 2 & 9 & 6 \\ \hline & & 3 & 9 \\ \hline \end{array} \quad \begin{array}{cccc} & \dot{7}^{6} & \dot{3}^{2} & 5 \\ - & 2 & 9 & 6 \\ \hline & 4 & 3 & 9 \\ \hline \end{array}$$

Step 1 ***Step 2*** ***Step 3***

As usual, we start by comparing the rightmost digits in the column (i.e. the unit column) and work to the left. Since 5 is less than 6 (and that 5-6 is a negative number), we borrow 10 from the number 3 at the tenth place. To symbolise this process, we put a dot on the top of the digit

3 and we write the number 2 (in small font) next to it to show that the number 3 has decremented by 1. Then, we add 10 to 5 which gives 15. After that, we calculate 15-6 which gives 9. Sometimes, we compute (10-6)+5 rather than (10+5)-6 which amounts to the same thing. Next, we repeat this procedure with the tenth column; that is, since 2-9 is negative, we borrow '10' from 7 and then compute (10+2)-9 (or (10-9)+2) to obtain 3. Since we borrowed 10 from 7, it decrements by 1 becoming 6. Finally, in the hundredth column, we subtract 2 from 6 to obtain 4.

Notice that, in the above procedure, we have avoided getting a negative result by all means by computing (10+5)-6=15-6 or (10-6)+5=4+5 in the first step. In other words, we have added 10 to 5 in order to get a number that is greater than 6 (i.e., 15) and then we have subtracted 6 from it. Similarly, for those proceeding as (10-6)+5, the idea is to subtract 10 from 6 which will always give a positive number (i.e. 4) and by adding a positive number to another positive number will yield a positive number.

The question is: *Why haven't we just computed (5-6)+10=-1+10 directly?* After all, we would have obtained the same answer! In other words, if we perform a direct subtraction 5-6, which gives -1, and then add the 10 which we have borrowed from 3, then we would get exactly the same answer as before. There is no need to follow a tortuous path to reach to the same answer! Similarly, instead of computing 10+2-9 (or 10-9+2) in Step 2, we could have just done a direct computation 2-9 which gives -7 and then we add 10 which we have borrowed from 7, giving again 3. This would yield the same answer.

Now, things can get even more complicated for children when they have to subtract numbers involving zeros. Consider, for example, 7045-2658. In the majority of textbooks, worksheets or online material, the solution to this subtraction, is expressed as follows:

$$\begin{array}{ccccc} & \dot{7}^{6} & \dot{0}^{9} & \dot{4}^{3} & 5 \\ - & 2 & 6 & 5 & 8 \\ \hline & 4 & 3 & 8 & 7 \\ \hline \end{array}$$

Here we proceed the same way as above except that we replace the ‘0’ digit with a ‘9’ when we borrow 10 from 7. To be more precise, we start by subtracting 8 from 5. Since 8 is larger than 5, we borrow 10 from 4. This gives 15-8 yielding 7 and we replace 4 by 3. Then, since 5 is larger than 3 we borrow 10 from 7 rather than from 0. In effect, in this regrouping method, it is said that we cannot borrow from 0. So, we have to borrow 10 from 7 which is then replaced by 6. Meanwhile, 0 is replaced by 9! Finally, we obtain the correct answer. Obviously, there must be some logical explanation of the above procedure, as it always yields the correct answer. We shall explain that subsequently.

However, at this stage, one might ask the following obvious question: *why can't we borrow 10 from 0 and why the '0' becomes '9' after borrowing 10 from the number 7? Why does it not become -1, as a matter of pure logical continuity? Why this lack of consistency in the subtraction procedure?*

Before answering to the above questions, we are going to propose a direct method of subtraction involving negative numbers.

Direct Subtraction Procedure

In fact, in the above example, there is nothing wrong in replacing 0 by -1 after borrowing 10 from it. In effect, we can perform the above subtraction in the following steps:

$$
\begin{array}{ccccc} & \dot{7}^{6} & 0 & \dot{4}^{3} & 5 \\ - & 2 & 6 & 5 & 8 \\ \hline & & & & 7 \\ \hline \end{array}
\qquad
\begin{array}{ccccc} & \dot{7}^{6} & \dot{0}^{-1} & \dot{4}^{3} & 5 \\ - & 2 & 6 & 5 & 8 \\ \hline & & & 8 & 7 \\ \hline \end{array}
\qquad
\begin{array}{ccccc} & \dot{7}^{6} & \dot{0}^{-1} & \dot{4}^{3} & 5 \\ - & 2 & 6 & 5 & 8 \\ \hline & & 3 & 8 & 7 \\ \hline \end{array}
\qquad
\begin{array}{ccccc} & \dot{7}^{6} & \dot{0}^{-1} & \dot{4}^{3} & 5 \\ - & 2 & 6 & 5 & 8 \\ \hline & 4 & 3 & 8 & 7 \\ \hline \end{array}
$$

Step 1 ***Step 2*** ***Step 3*** ***Step 4***

Starting from the right (Step 1), we compute 5-8=-3. Since -3 is negative, we add 10 to it, which we have borrowed from the digit 4 at the tenth place. This, therefore, gives 7 and the value of the digit 4 is decrement by 1 to become 3. Next, in Step 2, we compute 3-5=-2. Again, since -2 is negative, we borrow 10 from the digit 0 at the hundredth place and add it to -2, which gives 8. Now, we decrement the digit 0 by one to become -1, since we borrowed 10 from it. Then, in Step 3, we compute -1-6=-7. Since -7 is negative we add 10 to it, which we borrow from the digit 7 at the thousandth place, yielding 3. Again, the digit 7 then becomes 6 as we borrowed 10 from it. Finally, in Step 4, we compute 6-2=4, hence, yielding the required solution. We shall henceforth refer to this subtraction procedure as the "*direct subtraction procedure*".

The main question therefore is: *Why such a direct subtraction procedure is not used in early childhood education?*

Indeed, from several informal investigations carried out in several primary schools in the UK and abroad it was found that none of the schools used such direct subtraction procedure in their worksheets, reading materials and their reference books. In addition, the above subtraction procedure does not exist in any online learning resources and websites (like the Khan Academy, IXL, etc).

The main objective of this paper is to understand the underlying reasons as to why the direct subtraction procedure is not employed in primary schools. These are discussed in the next section. After that, we give the mathematical justification of the direct subtraction procedure. Finally, some recommendations and conclusions are given.

Discussion and Findings

In this section, we shall discuss and address the questions asked in the previous section. The two main questions asked with respect to the regrouping procedure were:

i) Why don't we employ the direct subtraction procedure albeit involving the use of negative numbers?

ii) *Why can't we borrow from 0 and why the digit 0 is replaced by 9 (and not -1) after borrowing 10 from a non-zero number in the classical regrouping method?*

At first sight, these questions might seem innocent. But when we ponder quite a while on it, one can notice that there is some kind of unconscious taboo on the introduction of negative numbers in early-stage education, as well as some misconceptions on the way subtraction is tackled and taught at primary education level.

Essentially, there seems to be a hesitancy from educators in introducing negative numbers at an early stage of children education; probably because there is a subconscious prejudice towards the ability of children in understanding some seemingly 'difficult or complex' concepts in mathematics. The original intention is probably not to 'confuse' the children with the concept of negative numbers. One can understand why that would be case in the past, where there would be very few real-life situations involving negative numbers. However, today we are surrounded with real-life examples and situations where negative numbers are omnipresent. For example, we have lifts and elevators going from the 5th floor to floor -2 in the basement, and we have thermometers with negative readings etc. Despite these, there is still some hesitancy to boldly introduce negative numbers as a natural concept in children education. One can certainly notice that, in the vast majority of schools, the rulers that children use do not contain negative numbers. In fact, rulers that contain both negative and positive numbers exist in the market. This can be a good starting point to teach negative numbers to children and relating them to directions; like 'up' for positive, 'down' for negative or 'going right' for positive and 'going left' for negative.

Furthermore, the classical regrouping method, which avoids the use of negative numbers by all means, has been repeated over and over again and is still being widely employed in schools' worksheets, reputable books, learning resources and websites (like the Khan Academy, IXL, etc). There is even a song in YouTube about subtraction with regrouping which goes like "zero minus one can't be done!" (Maths song with numberock, 2016).

In fact, if we look at how subtraction is taught at schools, they are very much the same as our grandparents learned during their school days in the early 1900; except that today we tend to gloss it with colour books and dancing videos and songs; like in BBC Bitesize series (see BBC Bitesize). One of the reasons why the methods of teaching subtraction has not changed in our primary curriculum is because we tend to think that the methods of subtraction are well established and that there is nothing else, we can do or see there. Finally, there is some reticence in accepting new ideas by primary school teachers and curriculum developers, which reflects some sort of rigidity in primary education.

On the other hand, children are very logical and consistent in their reasoning and thinking (see e.g. T. Nunes et al. (2007)). In fact, children like consistency, they do not like exceptions to a rule. This tends to confuse them. For this reason, it is important to develop a consistent methodology in performing subtraction in primary education.

By not introducing negative numbers at an early stage of children's education, educators have made matters worse by impeding on their creativity and insight. In effect, the UK is ranked 14th out of 79 in terms of performance in mathematics among 15-year-old students according to the Programme for International Student Assessment (PISA) results in 2018 (see PISA 2018 results).

Finally, for comparison purposes, we shall tackle a final example using the new direct subtraction method using the following example is borrowed from Math Meeting, (2016).

$$\begin{array}{rcccc} & \dot{9}^{8} & \dot{0}^{-1} & \dot{0}^{-1} & 4 \\ - & & 2 & 9 & 7 \\ \hline & 8 & 7 & 0 & 7 \\ \hline \end{array}$$

Starting from the unit place, we have 4-7=-3. We borrow 10 from 0 (at the tenth place) and add it to -3 to get 7 at the unit place. Then, we replace the 0 at the tenth place with -1. Next, at the tenth place we perform -1-9=-10. We borrow 10 from 0 (at the hundredth place) and add it to -10 to get 0 at the tenth place. After that, at the hundredth place we perform -1-2=-3. We borrow 10 from 9 (at the thousandth place) and add it to -3 to get 7 at the hundredth place. Finally, the digit 9 is replaced by 8. Subtracting 0 from the 8 at the thousandth place yields the correct answer. One can notice that the new method is more systematic and consistent.

Justification of the New Subtraction Procedure

Now, another obvious question one can ask is whether there is a rigorous mathematical justification of this 'new' method of doing subtraction compared to the standard classical way. Obviously, in the classical method, there is a rigorous mathematical reason as to why 0 is replaced by 9 when 10 is borrowed from another non-zero adjacent number. Otherwise, such a method would not have been taught for so long. In fact, for the above example, if we perform the subtraction by decomposing the numbers into tens, hundreds and thousands, we will clearly see as to why such is the case.

Classical Method

In effect, for the previous example, we have:

$$\begin{aligned} 7045 - 2658 &= (7000 + 40 + 5) - (2000 + 600 + 50 + 8) \\ &= 7000 + 40 + 5 - 2000 - 600 - 50 - 8 \end{aligned}$$

We can now perform this subtraction by an orderly fashion as above by subtraction unit numbers from unit numbers, then the tenth numbers from tenth numbers and so on. Hence, regrouping[1] the units, tenths, hundredths and thousandths number together, we get:

$$\begin{aligned} 7045 - 2658 &= (7000 - 2000) + (0 - 600) + (40 - 50) + (5 - 8). \\ &= (7000 - 2000) + (000 - 600) + (40 - 50) + (5 - 8) \end{aligned}$$

Here, we have written 0 = 000 simply to symbolise its location at the 100th place. By subtracting the unit numbers, we see that 5-8=-3, which is negative. Consequently, we borrow 10 from 40 as expressed below:

$$\begin{aligned} 7045 - 2658 &= (7000 - 2000) + (000 - 600) + (30 - 50) + (10 + 5 - 8) \\ &= (7000 - 2000) + (000 - 600) + (30 - 50) + 7 \end{aligned}$$

Next, by subtracting the numbers in the tenth place, i.e., 30-50, we get -20 which is negative. Since there is no strictly positive number at the hundredth place, we have to borrow 100 from 7000, i.e., from the number at the thousandth place instead[2]. These yields:

[1] This is where the term ‘regrouping’ comes from.
[2] That is also why we say that we cannot borrow from 0 in the classical regrouping method.

$$7045 - 2658 = (6900 - 2000) + (000 - 600) + (100 + 30 - 50) + 7$$
$$= (6900 - 2000) + (000 - 600) + 80 + 7$$

We then decompose the number 6900 again to get:

$$7045 - 2658 = (6000 + 900 - 2000) + (000 – 600) + 80 + 7$$

Then, we regroup the hundredth numbers again to obtain:

$$7045 - 2658 = (6000 - 2000) + (900 - 600) + 80 + 7$$

This is where the '9' appears in the classical subtraction by regrouping procedure.

Finally, by subtracting the numbers at the hundredth and thousandth place, we get the required answer:

$$7045 - 2658 = 4000 + 300 + 80 + 7 = 4387$$

New Method
Now, instead of borrowing 100 from 7000, we could have simply borrowed 100 from 0, as there is 0 at the hundredth place. More precisely, we have:

$$7045 - 2658 = (7000 - 2000) + (000 - 600) + (30 - 50) + 7$$
$$= (7000 - 2000) + (-100 - 600) + (100+30– 50) + 7$$

In this case, after subtracting the units and tens numbers, we would have:

$$7045 - 2658 = (7000 - 2000) + (-100 - 600) + 80 + 7$$

Note, that the '-1' superscript in the above new procedure appears in the -100 term. Therefore, at the hundredth place we have -700 which is again negative. Consequently, we borrow 1000 from 7000 to obtain:

$$7045 - 2658 = (6000 - 2000) + (1000 -700) + 80 + 7$$
$$= 4000 + 300 + 80 + 7 = 4387$$

This justifies the new method of subtracting numbers.

Remark
i) One might ask: "*why do we borrow only ten when we perform the calculation in a column-wise fashion rather than borrowing, 10, 100 and 1000 etc. when we perform the calculation using the decomposition method?*". The reason for this is simple: when we are subtracting in a column-wise fashion, we are doing the calculation in an orderly and systematic manner starting from the rightmost digit and moving to the left. Consequently, once we have subtracted the unit numbers, we can simply forget them and treat the numbers at the tenth place as the 'new unit' numbers, and similarly for the other numbers at the hundredth place and so on. It is for this reason we simply say that we are borrowing 10 rather than borrowing 100, 1000 etc. Consequently, ‘borrowing 10’ is just a misnomer.

ii) Note that the classical regrouping procedure is much more understandable when a decomposition method is used rather than using column subtraction.

Concluding Remarks and Recommendations

This paper has highlighted the need and necessity of introducing the concept of negative numbers at an early stage of primary education so that the direct method of subtraction can be employed in a consistent and systematic manner. This can be done by using day to day examples involving negative numbers such as elevators, lifts, financial transactions etc. and the use of rulers with both positive and negative numbers. Regardless of whether the classical regrouping or the new direct subtraction method is employed, it is imperative that both of these methods are explained through the decomposition method. This will give children the choice of the method with which they are comfortable with. Finally, training should also be provided to teachers and curriculum developers to embrace new ideas in primary education.

References

Arthur J. Baroody (1984). Children's Difficulties in Subtraction: Some Causes and Questions *Journal for Research in Mathematics Education*, Vol. 15, No. 3, pp. 203-213.

BBC bitesize. Column subtraction method. https://www.bbc.co.uk/bitesize/topics/zy2mn39/articles/zr3ytrd

IXL, Subtract across zeros. https://www.ixl.com/

Khan Academy, Subtraction with regrouping (borrowing). https://www.khanacademy.org/math/early-math/cc-early-math-add-sub-100/subtraction-within-100/v/introduction-to-regrouping-borrowing

Math meeting (2016). Subtracting large numbers. https://www.youtube.com/watch?v=3KZX41dU5ds

Maths song with numberock (2016). Subtraction with regrouping song. https://www.youtube.com/watch?v=nku3jVLbPBw

Pisa 2018 results. https://www.oecd.org/pisa/publications/pisa-2018-results.htm

Terezinha Nunes, Peter Bryant, Deborah Evans, Daniel Bell,Selina Gardner, Adelina Gardner and Julia Carrahe (20070. The contribution of logical reasoning to thelearning of mathematics in primary school, *The British Psychological Society, British Journal of Developmental Psychology* (2007), 25, 147-166.

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