Designing Math Problems for Primary Education: A Comparation of Outdoor Mathematics and National Testing

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Abstract
In the contribution, we focus on the process of creating math problems for pupils of all grades of primary schools. We focus on comparison math problems created with different trend, namely math problems designed for outdoor mathematics and math problems designed for national wide testing. Outdoor mathematics is implemented by a math trail, during which pupils can discover and solve math problems. These can be standard or non-standard, directly connected to real objects, which allows to see the use of mathematics in the real world. Math problems are created and solved in the application Math City Map. The national mathematics tests are intended for pupils in the 3rd, 5th and 8th grades of primary schools. Math problems are created on the base of the theoretical framework and Bloom's taxonomy. They are part of the compiled criteria tests. Math problems determine the level of mathematical literacy and are in paper form. We state necessary conditions for creating math problem in both cases. We created separated math tasks in cooperation with students - future mathematics teachers and real mathematics teachers. In the contribution, we also focus on the evaluation of separated tasks from math trails as well as criteria tests.

Keywords: Math Problems, Outdoor Mathematics, National Testing

Introduction

The goal of teaching mathematics is to prepare pupils learners for real life, among other things, by solving mathematical problems. The correct understanding and solving of mathematical tasks and problems can also be helped by the creation of mathematical tasks according to various criteria, e.g. creating tasks for leisure activities, outdoor activities, or national testing.

Mathematics Trails

Motivation is an important factor that has an impact on the teaching of mathematics. Much research (See: Wilkie & Sullivan, 2017; Buchholtz, 2020) has shown that various unconventional, creative and innovative learning activities can stimulate pupils' interest in maths and foster positive dispositions. An interesting educational activity that combines outdoor learning, group collaboration and mobile technologies is the math trail.

A math trail is an engaging activity for students and the public. Participants explore their surroundings, looking for and solving math problems on objects from the real world (Cahyono & Ludwig, 2017). The tasks are placed in a walk-in area. They often require the use of different measuring tools to solve them (Ludwig & Jablonski, 2019). Math trails promote teamwork as groups collaborate, communicate, and develop effective strategies for solving problems. By linking maths to topics such as history, art, or architecture, maths trails aim to stimulate enthusiasm and deeper engagement with the subject. This approach supports the integration of interdisciplinary content into the curriculum and adapts it to diverse learning needs.

Although the concept of math trails has a long tradition, the Erasmus+ project "Mobile Math Trails in Europe" introduces a modern twist by integrating them with mobile technology. The technical framework of the project includes the Math City Map (MCM) portal and the MCM application (Čeretková & Bulková, 2020; Jesberg & Ludwig, 2012). The free MCM application allows participants to access mathematical trails on their mobile devices, which include maps, images of objects associated with mathematical tasks, and assigned challenges. Math trails are created using the MCM web portal. After a short registration, users can create their own tasks and combine them into a complete maths trail. Each math trail must consist of at least four maths problems that have successfully undergone a validation process.

Geometry problems are one of the most common tasks of math trails, and students who participate in math trails solve a variety of application problems related to real-world geometric objects. Geometry-focused math trail lead to better retention of material and subsequent development of geometric thinking, because in solving these problems, students see the direct application of what they have learned and learn to think in context. Geometry problems on the Maths Trail are always adapted to the pupils' level of geometric thinking. Suitable tasks for developing geometric thinking are discussed for example by (Bočková, Pavlovičová, & Rumanová, 2024). Math Trail tasks can be formulated at the level of visualisation, analysis, informal deduction or formal deduction.

National Testing

In the PISA monitoring of mathematical literacy, Slovak students consistently perform below the OECD average (PISA, 2018). Some countries, like Slovakia, Denmark, responded to the

PISA results by introducing national mathematics testing (Andersen & Nielsen, 2016). Research was conducted in Sweden to examine the relationship between the TIMMS test and Swedish national measures. The result is a very strong correlation between the two tests, meaning that if a student was successful in the national test, he was also successful in the TIMMS measure (Wiberg, 2019). The aims of national wide testing are: to obtain objective information about the performance of pupils when entering the 2nd grade of elementary school, to verify the level of their knowledge and skills, the ability to apply knowledge in practical tasks and the ability to think logically, to provide schools and the general professional public with feedback and a more comprehensive picture of the students' knowledge and skills from the tested subjects, which will help in improving the quality of teaching. Mathematical tasks for national wide testing are created in the National Institute of Education and Youth in cooperation with elementary school teachers.

Problem posing is one of the important methods in mathematics education. It contains a whole group of new problems that are related to creating a problem or reshaping an already formulated problem. Problem posing can also be used as a diagnostic or assessment tool that helps teachers identify deficiencies and barriers in students' knowledge. Problem posing can also be used as a diagnostic and assessment tool (Papadopoulos et al., 2021). This method can improve the educational process in primary school. Its effective use depends on teacher preparation and appropriate integration into the curriculum. Teachers should be able to formulate and set meaningful mathematical problems for their students. The problems that the teacher poses can affect mathematics learning in the classroom and "their further mathematical goals for the class" (Ferrini-Mundi, 2000, p. 53). Teachers can use problem-based tasks to learn how their students understand and apply mathematics in real life (Cai & Hwang, 2002; Cai & Hwang, 2020; Kotsopoulos & Cordy, 2009). Research by Hamidi, Soleymani, Dazy, and Meshkat (2024) also shows a connection between problem posing and problem solving on the TIMMS and the PISA measure. Many experts, as well as ministries of education in several countries, are trying to incorporate problem posing into teaching as part of curricular changes to increase mathematical literacy (e.g., Brown & Walter, 1983; Healy, 1993; Ministry of Education, Slovak Republic, 2023).

Methodology

Our research focuses on the creation of tasks by future mathematics teachers and in-service mathematics teachers. Participants created the math trail task and tasks for the national testing.

As a research sample of future teachers, we used students at the University of Constantine the Philosopher who had already taken a course in the Didactics of Mathematics. As a research sample of in-service teachers, we used teachers who participated in the KEGA project (Innovation of Education in the Training of Mathematics Teachers in the Field of Task Creation with Emphasis on Criteria Testing) and created tasks for national testing.

The following research goals have been state:

- to create math tasks in cooperation with students – future mathematics teachers and in-service mathematics teachers,
- to evaluate the task for outdoor mathematics and national testing,
- to identify the most common mistakes in the creation task for outdoor mathematics and national testing

Criteria for Creation MathCityMap Task

Each task created in the MCM portal must meet certain criteria (Figure 1) (MathCityMap, 2024):

- Photo of the task - an image of the location associated with the task, but the task cannot be solved immediately from this image.
- Title of the task - corresponds with the problem.
- Definition of the task - clear and unambiguous, it should be obvious to the solver what to do with the task.
- GPS position of the task - every math trail task requires a GPS location of the task.
- Augmented Reality Scene - provides an option to insert a link to AR scene.
- Task type – can choose between different types of answers: interval, exact value, multiple choice, fill in the blanks, vector and fraction.
- Sample solution – written in the form of text or in the form of an image. The sample solution must be clear and understandable for each solver.
- Hint 1- 3 – each task must contain at least two hints in the form of text, image or video.
- Information about object – an optional part, to be used if you want to write something interesting about the object
- Grade – the grade for which the task is appropriate according to the educational standards.
- Tools – you can choose tool from the list (for example set square, chalk, measuring tape, etc.)
- Tags – keywords related to the task.
- Name – authors name.
- Email – authors email.

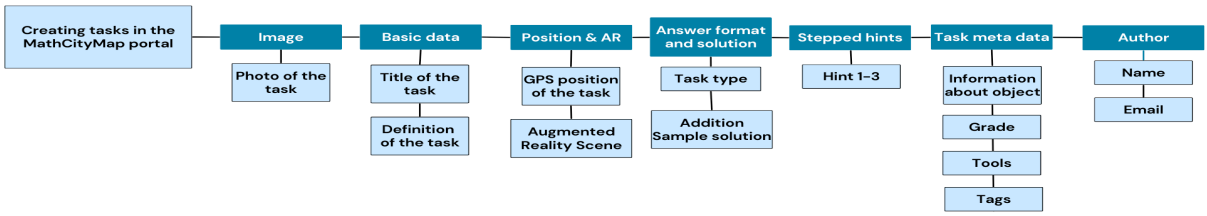


Figure 1: Criteria for the Creation of Tasks in the MathCityMap Environment

Criteria for the Creation of National Testing's Tasks

Mathematical tasks in national measurements must meet certain criteria too (Figure 2):

- Tasks must be clearly specified.
- Tasks are based on real situations.
- Tasks are open or closed.
- The answer to open tasks must be a number.
- The tasks are based on a theoretical framework and are focused on a specific area of education.

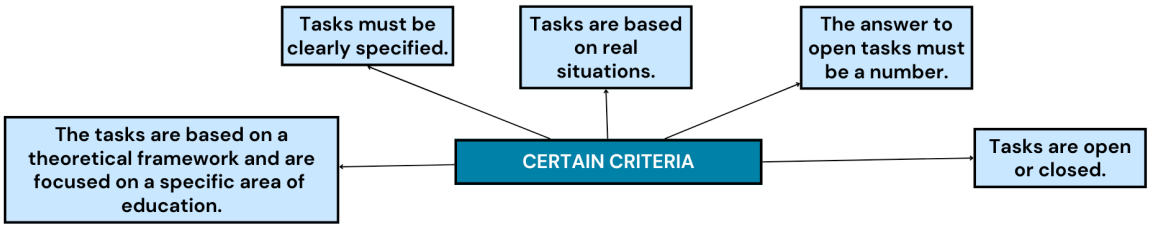


Figure 2: Criteria for the Creation of Tasks for National Testing

Results

Throughout the semester, we had the opportunity to view, evaluate and edit a wide variety of designed tasks created by future teaches and in-service teachers on the MathCityMap portal and for national testing. Different types of mistakes made by future teachers and in-service teachers in the formulation task were recorded.

Students' Suggestions – Incorrect Outdoor Geometry Problems

In the tasks listed in this sub-section, we give concrete examples of tasks designed by future mathematics teachers. The tasks were subsequently modified. After modification, they were suitable for school practice.

Title of the task: New paint
The task required a revision of the task – definition of the task, the type of answer, the simple solution and the hints.
Photo of the task: Figure 3

Figure 3: Photo of the Task – New paint

Original task
Definition of the task: How many tubes of paint do you need if you want to paint a circle of equality using just one color?
The circle with the lettering will remain on the outside. We need 17 ml of paint in a tube for 1 dm².
Task type: multiple choice
a. 17
b. 5
c. 8
d. 4

Modified task
Definition of the task: How many tubes of paint are needed to fill the inside of the circle? Leave the outer circle with the lettering. To cover 10 cm², one tube is enough.

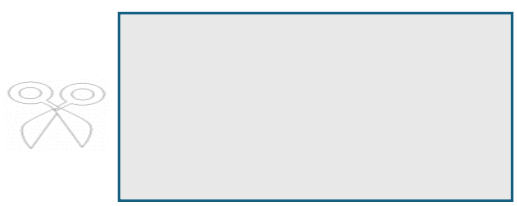
Task type: interval

Sample solution:
$S=\pi r^2$
$S=\pi\ (50,5)^2$
$S=8007,785\ cm^2 = 80,08\ dm^2$
80,08:17=4,71
4,71≈5
Hints:
Hint 1: The circle is bounded by a black frame around it.
Hint 2: Round off the area of the circle to the nearest whole number.
Grade: 8
Tools: measuring tape, calculator

Sample solution:
$S = \pi r^2$
$S = \pi\ (50,5)^2$
$S = 8007,785\ cm^2 = 80,08\ dm^2$
80 ÷ 10 = 8
8 tubes of paint are enough for painting.
Hints:
Hint 1: Use the formula $S = \pi r^2$ to calculate the volume of a circle.
Hint 2: You will need to buy an extra tube if the area to be painted is larger than one tube will cover.
Grade: unchanged
Tools: unchanged

Teachers' Suggestions – Incorrect Geometry Problems

Original task
Definition of the task: In art, Lenka cut paper. She cut three times from right to left and five times from top to bottom. She gave a third of the cut paper to Katka. Zuzka took two pieces of paper less than Katka. Lenka took 4 pieces of paper more than Zuzka.



How many parts did Lenka cut the paper into? Lenka cut the paper into ____ parts.
Katka received ____ pieces of paper.
Zuzka took _____ pieces of paper.
Lenka took _____ pieces of paper.
According to how many pieces of paper the girls got, sort the numbers from largest to smallest.
How many parts of the paper were left after Katka, Zuzka and Lenka took them?

Modified task
Definition of the task: In art education, Lenka cut paper. She cut three times from right to left and five times from top to bottom.

How many parts did Lenka cut the paper into?

Lenka cut the paper into ____ parts.

The Most Common Mistakes Made by the Creators of MathCityMap Tasks

Mistakes made in designing the task could be categorized as follows:

- The problem can be solved without the need to go to the place where it is located. All the data needed to solve the problem is captured in the task photo. The error is removed by cropping the photo or overlaying the search data.

Example of the task: *Determine all planar geometric shapes on the railing.* See Figure 3 for a photo of the task.

Figure 4: A Task That Can Be Solved from a Photo

- Missing the units in which the dimensions of the object should be measured. The error is removed by adding the units in which the correct result should be given.

  Example of the incorrect task: *Determine the diameter of the column.*

- The correct answer is given in the wrong type of answer. For tasks requiring measurement, the response format cannot be an exact value but an interval. The error can be corrected by changing the type of answer.

  Example of the task: *How much does the cuboid stone of the bench weigh? 1m³ weighs 2600kg. Give the result in kg.*

  Example of incorrect type of answer: *1349 kg*

- Hints do not give relevant information. Some hints are simply a repetition of the task. The error can be corrected by the formulation of a new hint that is more appropriate.

  Example of the task: *If you doubled the length of the beam, how many metre-long prisms could be placed on it?*

  Example of incorrect hints: *Measure the length of the beam. Double the beam length.*

- The location of the problem is not correct. Students must specify the exact task location in the MathCityMap portal. The error is removed by determining the correct position of the task.

The Most Common Mistakes Made by the Creators of Mathematical Tasks for National Testing

Mistakes made in designing the task could be categorized as follows:

- Ambiguous assignment - the assignment of mathematical tasks is unclear, an inaccurate reproduction of previously seen material and what is to be reproduced is not clearly and not directly stated.
- High difficulty – the math tasks are too difficult for the given age group of pupils.
- A lot of redundant data that makes it difficult to solve the task correctly.

- Overly long questions or assignments often discourage pupils from solving given math problems.

Conclusion

In the article, we have approached the process of creating mathematical tasks from two perspectives: tasks for outdoor mathematics and tasks for national testing. Such tasks, as well as their creation, help develop creativity, mathematical and geometric thinking in students and teachers. We have focused on the criteria that must be followed when creating such tasks. We have also pointed out the most common mistakes of creators that occur in the first phase of creating tasks of both types. We have also presented the modified task assignments.

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