Project-Based Learning in a Collaborative Environment: A Math Study

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Abstract
One of the concerns of higher education institutions is to provide students with a wide range of skills. Whether they are basic, cognitive, and professional skills, or social, problem-solving and teamwork skills. Project-Based Learning (PBL) is a teaching methodology that involves students designing, developing, and building practical solutions to a problem. And, if associated with a collaborative environment (CE), which promotes group work, the student becomes actively involved in an enriched learning process, allowing them to acquire hard and soft skills. This paper analyzes the interest in applying PBL-CE to first-year Electrical and Computer Engineering students to learn Mathematics. To motivate students' learning, acquire skills and verify the effectiveness of the PBL-CE method, an experiment was developed as a case study. The students, in groups, lived this experience, counting on the support of teachers, as advisors, helping them in the construction of knowledge. To assess the interest of the experience, two questionnaires were carried out, one before and one after the experience. 26 students indicated that their contributions were valuable and that they were a fundamental element for the group. That cooperation (80.0%), responsibility (69.2%) and commitment (61.5%) define their performance in the experience and that they are confident in what they learned (88.5%). All students enjoyed using PBL-CE and 92.5% stated that it contributed to the development of skills inherent to the engineering course and professionals. With this experience it is recognized that it is possible to develop a wide range of soft and hard skills in students.

Keywords: Project-Based Learning, Collaborative Environment, Case Study, GeoGebra, Trapezoidal and Simpson Rules

Introduction

Currently, one of the concerns of higher education institutions is to provide students with a wide range of skills. Both hard skills, such as cognitive knowledge skills and professional skills (Vogler et al., 2018), and soft skills, such as problem solving and teamwork (Casner-Lotto & Barington, 2006).

Project-Based Learning (PBL) is a learning model that uses projects as a first step in integrating new knowledge and skills based on real experiences (Loyens et al., 2023). Students engage in designing, developing, and building practical solutions to a problem. PBL makes use of groups of people and, therefore, requires constant communication, collaboration, and management of activities, with the collaborative environment considered as a support tool (Guo et al., 2020, Chistyakov et al., 2023, Uden et al., 2023).

The Collaborative Environment (CE) of learning promotes group work to enrich student learning (Caridade, 2021). A group of students work together to solve problems, complete assignments, or learn new concepts (Ng, Chan, & Lit, 2022). This approach actively involves students in processing and synthesizing information and concepts, rather than using rote memorization of facts and figures (Bjelobaba et al., 2022).

This paper intends to analyze the interest in applying PBL-CE to students of the first year of the Degree in Electrical and Computer Engineering for learning Mathematics. To motivate students' learning, the acquisition of skills and to verify the effectiveness of the PBL-CE method, an experience was developed in the classroom, as a case study, where the technologies allied to the PBL-CE were worked on. One of the syllabus contents of the discipline was chosen and the experience was carried out during two blocks of 1h30m in the last week of classes. The students, in groups, lived this experience, with the support of the teachers, as advisors, helping them in the construction of knowledge. To assess the interest of the experience, two questionnaires were carried out, one before and the other after the experience. The main results of these surveys are presented, analyzed, and discussed.

Methodology

The PBL-CE was implemented in Mathematical Analysis, one curricular unit of the first year in Electrical and Computer Engineering degree. With about 35 students spread over two theoretical-practical (1h30) and practical (1h30) classes during the last week of classes in the 1st semester of 2022/2023. It was elaborated and planned by the 2 teachers of the discipline on the application of the Trapezoidal rule and Simpson's rule for the calculation of numerical integration, one of the contents to be taught to these students, following the flowchart shown in Figure 1. The PBL-CE experience flowchart is divided into three moments "Before experience", "Classroom experience" and "After experience".

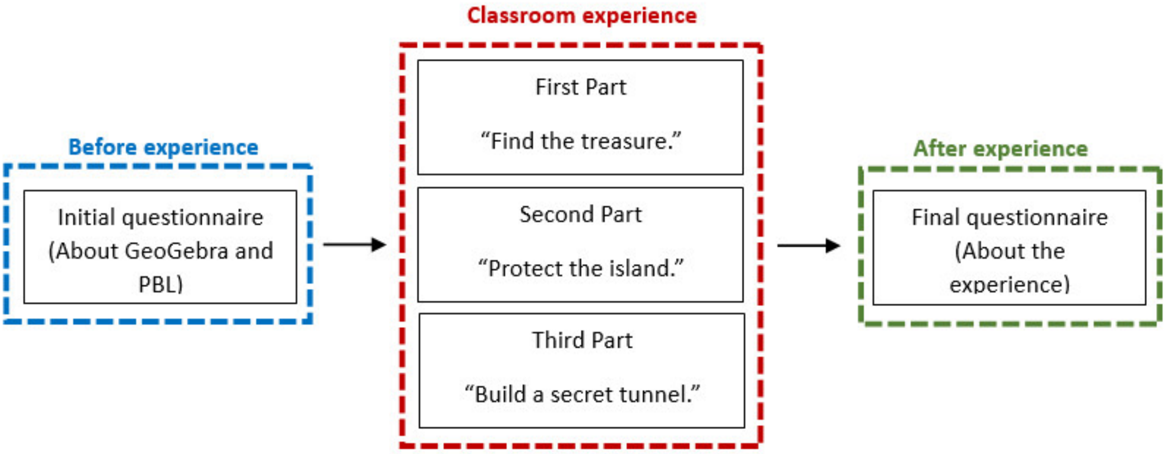

Figure 1: Implemented PBL-CE flowchart.

"Before the experience", students responded individually to a questionnaire to check their knowledge about GeoGebra and PBL-CE. In the "Classroom Experience", students, in groups of 2 or 3 elements, were integrated and led through a story where they had to solve problems about the applications of the defined integral. The story involved the students in three distinct parts: "Find the treasure", "Protect the Island" and "Build a secret tunnel", according to the support of a guide presented to them. The use of technologies and mathematical tools to solve problems was necessary and encouraged. In the final moment, "After the experience", students responded individually to a questionnaire, to evaluate their experience.

Experience in Class

Numerical integration is one of the important contents of Mathematical Analysis in engineering courses. Many engineering problems need to solve integrals that are impossible to solve analytically, or the analytical expression of the function is not known, only values of that function in a set of points. The PBL-CE proposed in this paper and entitled "A treasure island adventure" is a new experiment on numerical integration, programmed to be used in a classroom context. The experience consists of a story where each group of students is inserted and invited to participate according to a set of tasks described in a small guide composed of three parts: "Find a treasure", "Protect the island" and "Build a secret tunnel".

Each group of students is included in the story "A treasure island adventure".

First Part – "Find a treasure"

Story: *"Your team of pirates, traveling by boat, discovers the missing treasure island. To find the treasure more easily, the pirates decide to assign a region with the same area to each one. Therefore, it is necessary to determine the total area of the island (Figure 2) and divide it by the number of pirates in your team."*

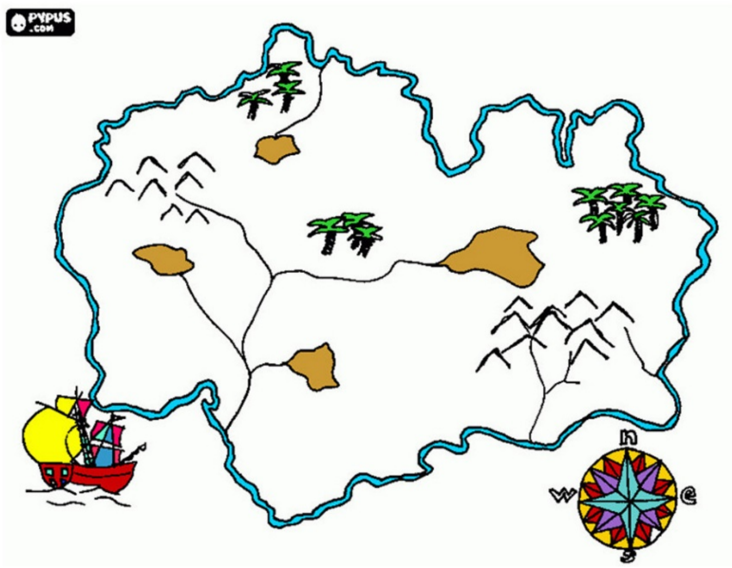

Figure 2: Treasure map
(https://www.colorirgratis.com/colorir-de-mapa-de-ilha-do-tesouro_115421.html).

To calculate the total area of the island, the group has a set of tasks to perform. It is necessary to load the image of the island in GeoGebra and adjust it to the coordinate axes, rotating it, if necessary, so that it is centered on the *X*-axis with a length of 12 units. Then they will have to define a set of 13 points equally spaced along the upper contour of the island, build the functions that interpolate these points and define the contour of the island, as can be seen in Figure 3. In this example, the northern part of the island is defined by two functions, one that interpolates points *A* to *G* (red) and the other the points from *G* to *M* (blue). The same process will be carried out to calculate the area of the southern part of the island, in the example also using two functions, one defined by points *M* to *U* (red) and another by points *U* to *A* (blue).

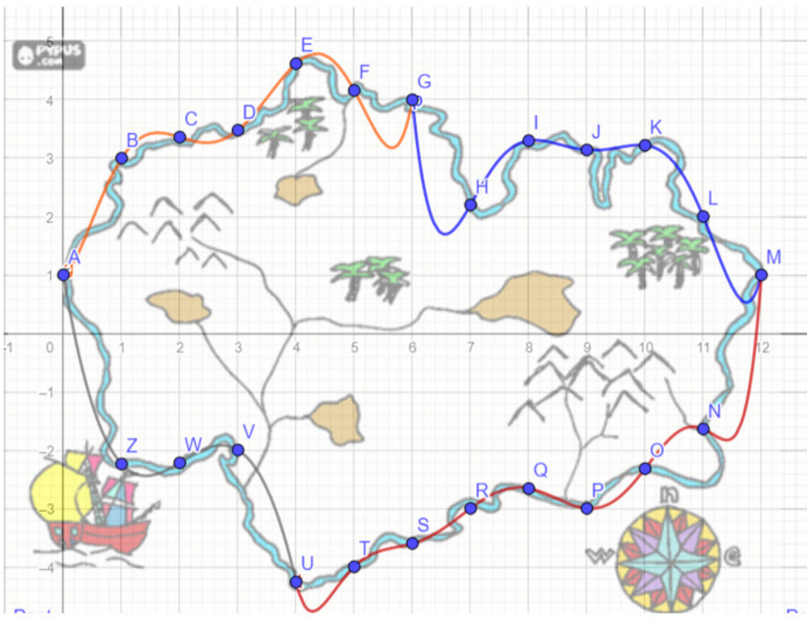

Figure 3: Definition of the points and functions that define the contour of the island (northern part and southern part).

Finally, using a scale of *1:100*, the area will be calculated using the Trapezoidal rule and Simpson's rule. Calculations are carried out using the technologies available in the classroom, such as the calculating machine and computers. Also here, the variety of options chosen by the students was immense, from the use of GeoGebra itself, to the use of applications available on the internet for mathematical calculations, such as Wolfram alpha, Symbolab, Math calculator, QuickMath, and so one. In Figure 4, it can be seeing an example of solving these calculations using Trapezoidal rule. The area of the northern part is 35.7 *units,* and the southern part is 29.87, so the total area is 65.57, which corresponds to 6557 *units* on the given scale.

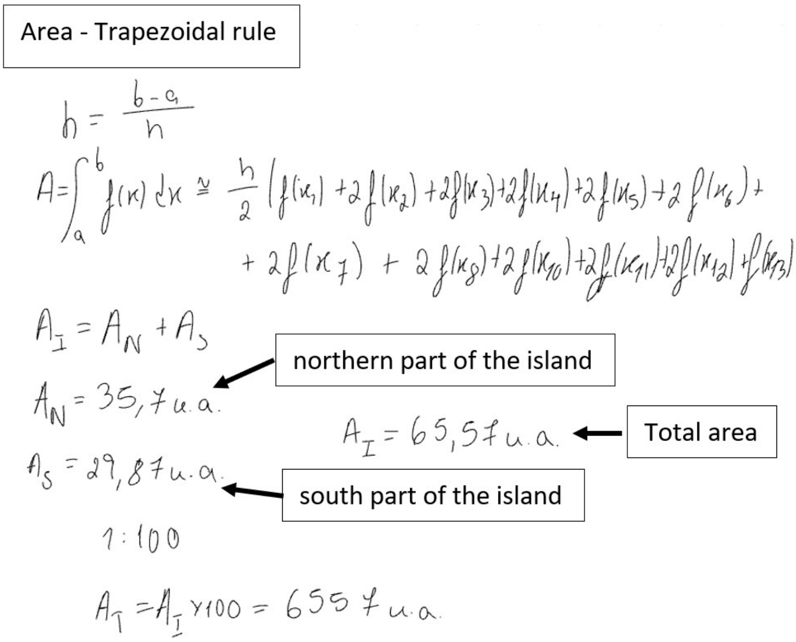

Figure 4: Calculations of island's area using the Trapezoidal rule.

After the area of the island has been calculated, it is necessary to divide it by the number of pirates (group members) and identify the area assigned to each pirate to find the treasure.

Second Part - "Protect the island"

Story: *"The northern part of Treasure Island is often attacked by pirates from other teams, so you need to build a wire fence along its entire border. If the fence is defined by the curves, you determined earlier, how much wire is needed?"*

After the treasure is found, it is necessary to defend the island from the attack of other pirates (other groups of students) by building a wire fence along the island's border. Here, the students had to calculate the perimeter of the northern part of the island, using the previously created functions and again applying the Trapezoid rule with 4 equally spaced intervals in each of the functions. In the Figure 5 you can see the calculations made by a group of students. In this case, the students only defined 3 interpolating functions (*f(x), g(x)* and *h(x)*) and determined the perimeter of the North part of the island, applying the perimeter formula in the Trapezoidal rule.

Perimeter - Trapezoid Rule

$$f'(x) = -0.13x^3 + 1.5x^2 - 4.7x + 4.3$$

$$g'(x) = -0.7x^2 + 6.7x + 16.9$$

$$h'(x) = 0.3x^4 - 11.5x^3 + 161x^2 - 995x + 2288.5$$

$$\frac{1}{2} * \left(\sqrt{1^2 + (f'(0))^2} + 2 * \sqrt{1^2 + (f'(1))^2} + 2 * \sqrt{1^2 + (f'(2))^2} + 2 * \sqrt{1^2 + (f'(3))^2} + 2 * \sqrt{1^2 + (f'(4))^2} + 2 * \sqrt{1^2 + (g'(4))^2} + 2 * \sqrt{1^2 + (g'(5))^2} + 2 * \sqrt{1^2 + (g'(6))^2} + 2 * \sqrt{1^2 + (g'(7))^2} + 2 * \sqrt{1^2 + (h'(7))^2} + 2 * \sqrt{1^2 + (h'(8))^2} + 2 * \sqrt{1^2 + (h'(9))^2} + 2 * \sqrt{1^2 + (h'(10))^2} + 2 * \sqrt{1^2 + (h'11))^2} + \sqrt{1^2 + (h'(12))^2}\right)$$

$$= 26.08$$

for the scale 1:100

26.08*100=2608

Figure 5: Calculations of the island's north perimeter using Trapezoid rule.

Finally, the perimeter of the northern part of the island found corresponds to the length of the wire fence needed to protect the island.

Third Part - "Build a secret tunnel"

Story: *"In order to transport the wire fence and other materials invisibly, it is necessary to build an underwater tunnel that connects the treasure island to the mainland. The tunnel is 1860 units long and 13 equally spaced circular sections were collected with the following areas S0=214, S1=250, S2=280, S3=300, S4=330, S5=365, S6=395, S7 =415, S8=435, S9=455, s10=475, S11=490 and S12=530 length units. What is the volume of the tunnel?"*

In the last part, it was necessary with the areas of the circular sections to determine their circumference radius (or diameters) so that, through the set of points, define the function that represents the tunnel. By defining the 13 circular sections (Figure 6) in GeoGebra, with the centres on the *X*-axis, it is possible to find the points that define the outer limits of the tunnel, so for example the point *P0* (represented in green) is calculated through the radius of the circumference *S0*, that is, $r0 = \sqrt{\frac{S0}{\pi}}$ with coordinates $(0, r0)$. The same is done for the remaining points. After identifying the coordinates of the 13 points, it is possible to define the function *f(x)* that interpolates these points (red function).

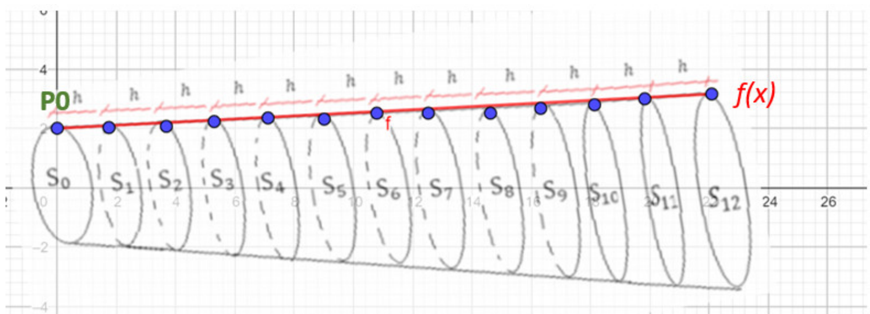

Figure 6: Equally spaced circular sections.

With the function *f(x)*, it was then possible to create a solid of revolution by rotating the plane region defined in GeoGebra around the *X*-axis. The solid created represents the tunnel. Figure 7 shows two examples 3D tunnel created in GeoGebra performed by two groups of students.

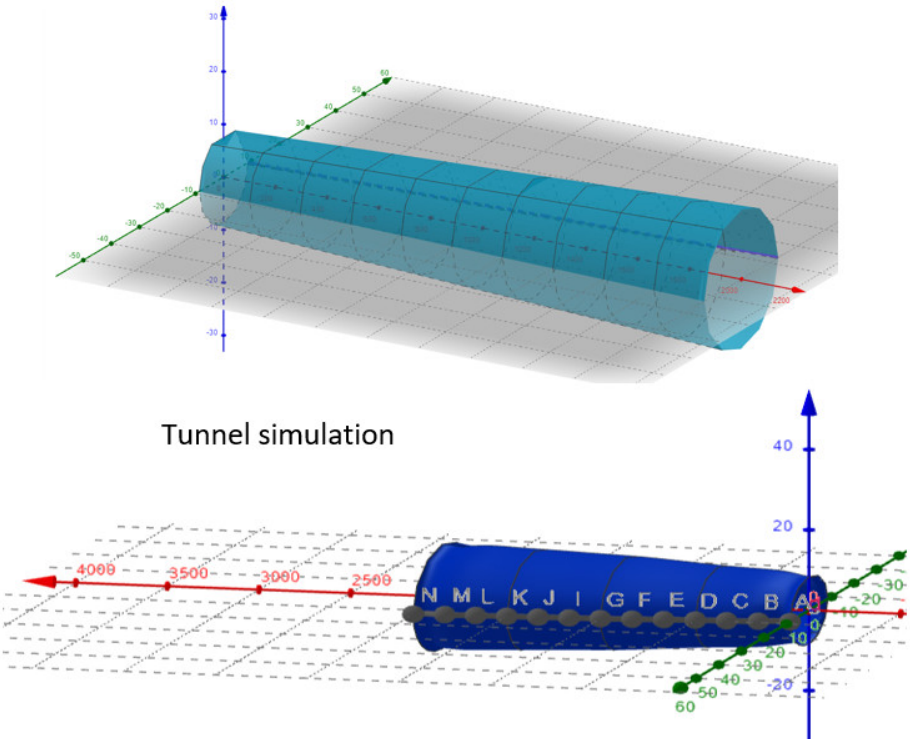

Figure 7: 3D representation of the tunnel in GeoGebra (2 examples).

Finally numerical integration was applied again through Simpson's rule to calculate the volume of this solid, as the example shown in Figure 8. In this example, the 13 points are spaced 154 *units* apart, the first point being *P0(12,f(12))* and the last *P12(1860,f(1860)*), with a volume of 70135600 *units*.

Volume – Simpson's rule

$$V = \frac{154}{3} * \Big(\pi * (f(12))^2 + 4*\pi*\big(f(166)\big)^2 + 2*\pi*\big(f(320)\big)^2 + 4*\pi * (f(474))^2 + 2*\pi*\big(f(628)\big)^2 + 4*\pi*\big(f(782)\big)^2 + 2*\pi * \big(f(936)\big)^2 + 4*\pi*\big(f(1090)\big)^2 + 2*\pi*\big(f(1244)\big)^2 + 4*\pi * \big(f(1398)\big)^2 + 2*\pi*\big(f(1552)\big)^2 + 4*\pi*\big(f(1706)\big)^2 + \pi * \big(f(1860)\big)^2\Big) = 701356$$

for the scale 1:100

701356*100=70135600 *units*

Figure 8: Calculations for the tunnel volume using Simpson's rule.

Results and Discussion

To verify the students' interest, motivation, and learning, two questionnaires were carried out, one before the classroom experience and the other at the end of the experience (Figure 1).

The initial questionnaire formed by only 4 questions intended to analyse the students' knowledge about GeoGebra, and the use of PBL-CE activities in the classroom. Of the 35 students who answered the questionnaire, 40% of the students had never carried out PBL activities in the classroom, 71.4% had never carried out activities in GeoGebra and 80% had never carried out PBL activities in GeoGebra. Regarding the content to be explored in the classroom, 85.7% of students have never used GeoGebra to apply the Trapezoidal rule or Simpson's rule. This reinforces the authors' view that PBL activities are still few implemented in higher education.

During the experience implemented in the classroom, it was found that students initially felt lost and not used to PBL-CE activities, where learning is carried out by solving the proposed problem, in this case a story. The CE improved as the class went on, since the interaction and learning between the group elements was being adjusted. The direct observation, made by the teachers, during supervision in the classroom, allowed the analysis of the students' reactions and the understanding and learning that they developed through the experience lived in the story. Doubts were frequent, at the beginning and in some situations questioning the teachers about the follow-up to give to the experience. Afterwards, they went through the pages of the guide and learned what they really had to solve, always supported by the considerations and comments of the teachers. In this way, they also began to have a clearer view of the objectives of the experience. The environment became more creative and motivating as the tasks were being carried out, however some students showed a shallow knowledge of both the contents to be acquired and the technologies to be used, relying on teachers and other peers. At the end of the experience, they showed satisfaction for reaching the end of the story – a sign that they had successfully achieved their objectives – but, at the same time, comments such as “Is it over yet? We could do another one!” What a pleasant surprise.

The final questionnaire was carried out only by 26 students out of the 35 students who participated in this experience. In the questionnaire, 100% of the students say that their contribution was valuable for the group during the classroom experience and that they were a fundamental element for the group regarding the aspects presented in Table 1. Suggest ways of solving and boosting the development of the work (76.5%); structure the work to submit it (61.5%); justify the reasoning developed (57.7%); apply different mathematical procedures (57.7%); complete the work (57.7%); mobilizing prerequisites (knowledge already acquired through previous learning - 53.8%) were the aspects selected by more than half of the students.

Student's contribution to the group	# students
Mobilize prerequisites (knowledge already acquired with previous learning)	14(53.8%)
Involve the different elements of the group	7(26.9%)
Lead the group	8(30.8%)
Suggest forms of resolution and boost the development of the work	20(76.9%)
Justify the reasoning developed	15(57.7%)
Apply the different mathematical procedures	15(57.7%)
Mobilize GeoGebra tools	13(50%)
Organize knowledge, ideas, and reasoning to build new knowledge	12(46.2%)
Complete the work	15(57.7%)
Structure the work to submit it	16(61.5%)

Table 1: Contribution of the student as an element of the group.

In the Figure 9 are represented the expressions chosen by the students that best classify their performance in carrying out the group work. Cooperation (80.8%), responsibility (69.2%), commitment (61.5%), knowledge (57.7%) are the expressions with a percentage above 50%. It should be noted that all expressions have a positive connotation, except for "Disinterest", which was only chosen by one student.

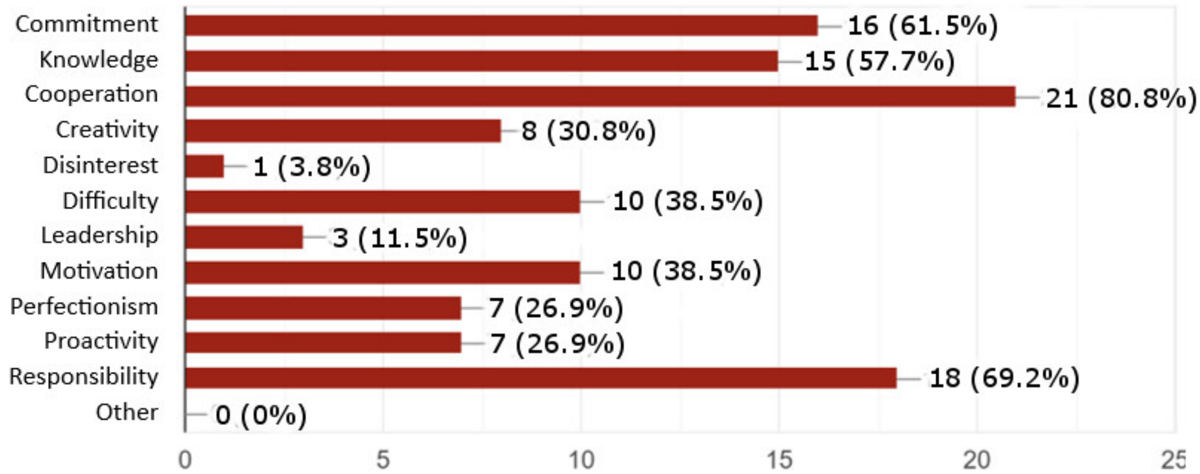

Figure 9: Expressions that best classify the student's performance in carrying out group work.

In the following 3 questions with answer (yes or no): 88.5% answered "yes" and 11.5% "no" (see Figure 10). Students feel confident in what they have learned about numerical integration (Trapezoid and Simpson rules), they enjoyed using GeoGebra and consider that the use of GeoGebra allows a greater perception of reality in relation to the syllabus and gives greater expression to their applicability in a real context.

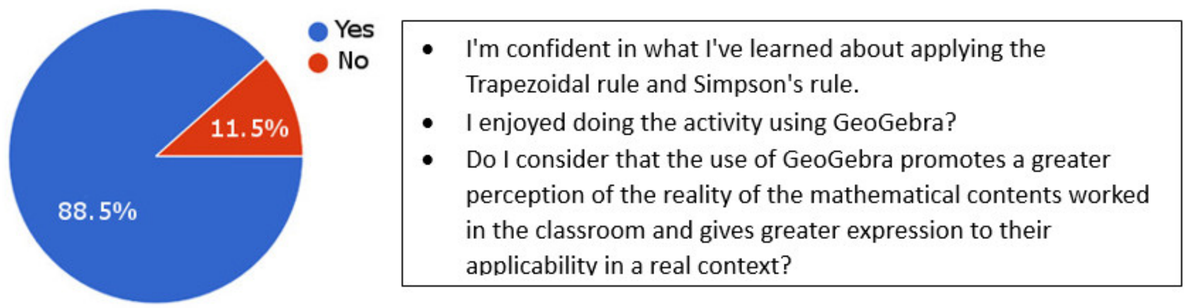

Figure 10: "Yes" or "no" questions.

The expressions that the students identify as being the ones that best define the way in which the use of GeoGebra was related to the acquisition of new knowledge are represented in the Figure 11. Allow building new knowledge in a conscious and reflective way (84.8%); Facilitate calculations (61.5%); Favouring the development of reasoning (53.8%) are the three expressions most chosen by students. This demonstrates that the students' perception of the use of GeoGebra is in line with the teachers' perception: the use of this tool as a means of learning and understanding Mathematics allows students to obtain a greater and varied number of skills.

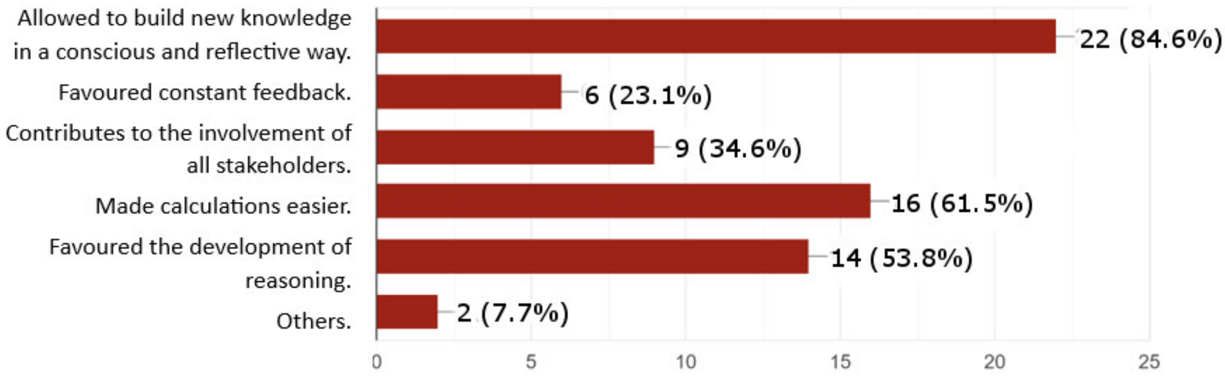

Figure 11: The expressions chosen by the students, which best define the way in which the use of GeoGebra was related to the acquisition of new knowledge.

Regarding the use of innovative learning activities in the classroom, all students liked it and 92.5% consider that the work methodology used in this experience contributes to the development of skills inherent to the students' academic (engineering) and professional choices.

The importance of mediation between teacher/student and between student/student was confirmed by the students when they answered that they were essential to overcome the stages of work design (96.2% - teacher/student and 100% student/student).

In the open answers, the students indicate that what they liked most about the experience was the use of GeoGebra (14 answers), working in groups (3 answers), developing knowledge about calculating perimeters, areas, and volumes (3 answers) and how functions can be related with maps (1 answer). However, the biggest challenge during the experience was GeoGebra (15 answers) and relating the new knowledge of the syllabus with previous knowledge (7 answers). As for the suggestions for future classes, the students proposed to work more often with GeoGebra, an introduction class to the basic functions of GeoGebra, continue to implement this type of activities and explore these tools in the classroom.

Conclusion

In this paper, an experience of PBL-CE was presented, where the student is actively involved in an enriched learning process, allowing him to acquire hard and soft skills. The experience consists of the story "An adventure on treasure island" where each group (2 or 3 students) is inserted and is invited to participate according to a set of tasks described in a small guide. The experience was presented to 35 students of Mathematical Analysis of Electrical and Computer Engineering on numerical integration. Students were encouraged to use different technologies, both computational and geometric (GeoGebra).

To assess the students' interest and acquired skills, two questionnaires were carried out. The first before the experience, to identify the students' knowledge about GeoGebra and PBL-CE learning. 35 students answered the questionnaire, where it was verified that the students had not previously used GeoGebra (71.4%) nor had PBL-CE experiences with GeoGebra in the classroom (80%). The second questionnaire, after the experience, to verify the interest and motivation of the students, as well as the skills that were acquired. 26 students answered the questionnaire, indicating that their contributions were valuable and that they were a fundamental element for the group. That cooperation (80.0%), responsibility (69.2%) and commitment (61.5%) define their performance in the group and that they had confidence in what they learned (88.5%). All students liked the use of PBL-CE and 92.5% stated that it contributed to the development of skills inherent to the engineering course and professionals.

With this study, it was possible to verify that, despite the difficulties presented, teachers and students see that PBL-CE has great potential for promoting autonomy, learning and motivation of students in higher education. It was also possible to verify that the classes held contributed to the dynamization of pedagogical practices, creating more collaborative educational spaces in the context of teaching Mathematics. Thus, teachers and students began to act together to build learning, sharing information and knowledge, which favored peer learning and student creativity. This new dynamic of Mathematics classes allowed the students involved to develop skills and competences more efficiently for personal, academic, and professional life (soft and hard skills). Another aspect that also became quite evident from the study carried out was the gradual development of students' autonomy (individual and in groups), since the teachers in the classroom played the role of mediators and facilitators of the entire learning process.

The difficulties that occurred throughout this study, more specifically in relation to the experience, through the citation of the students were: the use of GeoGebra and relating the new knowledge of the syllabus with the previous knowledge. This goes hand in hand with the normally existing difficulties when starting to work with new technologies and different learning environments.

This case study, based on the results obtained, demonstrated the effectiveness of implementing the PBL-CE in the classroom context with a view to developing soft and hard skills in students. The results obtained were in line with the advantages of PBL learning in an CE environment mentioned in the literature, which enhance the improvement of students' academic performance and the development of essential social skills to train citizens capable of interacting in a healthy way in different contexts like family, academic life and professional. In relation to PBL-CE in learning, it will be necessary to use a summative assessment to confirm the effectiveness of learning. However, it is recognized that it is possible to develop a wide range of skills in students, encouraging them to engage in constructive learning. Through the direct observation of the teachers and the results of the surveys, the enthusiasm and dedication of the students, always present, was recognized.

After this study, the teachers feel that the experience has enriched their work, aware, however, that they still have a way to go, in the more frequent use and in the evaluation of the competences acquired with PBL-CE methodologies in the classroom. As future work, it is intended to apply this experience again in the next academic year and evaluate, in addition to the acquired skills, the knowledge obtained, through a small individual and group diagnostic test.

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