Probabilistic Thinking Framework in the Context of Indonesian Curriculum: A Literature Study

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Abstract

Probabilistic thinking is an approach to predicting the probability of a situation occurring that utilizes mathematical and logical tools. It is analogous to the ladder of inference, which involves the analysis of relevant prior information, observations, data selection, and reasonable assumptions to facilitate sound decision-making in an uncertain environment. This ability involves both formal and informal knowledge of uncertainty. Formal knowledge, which is acquired in an academic environment such as a school, is a significant component of probabilistic thinking. However, there is a paucity of consensus regarding the materials that build students' probabilistic thinking ability, hereafter referred to as the probabilistic thinking construct. This construct forms the basis for measuring students' thinking ability, termed the probabilistic framework. This research systematically synthesizes the extant literature on the subject. A comprehensive review of recent research reveals a large number of studies from which probabilistic thinking frameworks have been designed can be categorized into two groups based on geographical location: The United States and Australia. This literature study research synthesizes those findings, then adapts them to the content and characteristics of the curriculum in Indonesia, known as the Merdeka Curriculum. The analysis revealed three constructs that facilitate probabilistic thinking skills and align with the curriculum's demands: sample space, probability of an event, and probability comparison.

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Introduction

Uncertainty terms are frequently employed in the context of information acquisition, as evidenced by the utilisation of phrases such as “chance of rain, sunny, cloudy, and so on” by meteorologists, the prediction of the probability of surgical intervention by medical professionals, the estimation of the number of accident victims by age, and the assessment of cancer spread (Batanero et al., 2016). The employment of linguistic expressions such as “unlikely”, “possible”, “certain”, and others, in such contexts, serves to convey the degree of uncertainty prevailing in a given situation. According to the Organization for Economic Co-operation and Development (OECD) (Wijaya et al., 2021), uncertainty can be defined as “a phenomenon at the core of the mathematical analysis of many problem situations”. The ability to engage in probabilistic thinking plays an important role in situations where one faces a phenomenon that is not a definite occurrence, but is still a possibility, albeit a small probability of occurring (Fa’ani et al., 2016).

In the contemporary era, the ability to think probabilistically, to interpret and critically evaluate stochastic phenomena faced by humans in various contexts is considered not only important for scientists, but also for the general public (Gal, 2002). Furthermore, Rumsey (2002) and Utts (2003) posit that these competencies are indispensable for individuals to function as critical and informed citizens in contemporary society. The cultivation of these cognitive abilities is achieved through both formal and informal educational experiences that are related to uncertainty. Formal education is primarily acquired through institutional settings such as schools, while informal education is acquired through games and experiences in social life. In light of the significance attributed to uncertainty, the nation of Indonesia has incorporated uncertainty and randomness as a compulsory component of the curriculum for students, collectively termed “chance” within the domain of mathematics. Nevertheless, there remains considerable debate surrounding the selection of materials and concepts to “cultivate students” probabilistic thinking skills in various countries. This study was conducted to explore and summarise the constructs or concepts associated with probabilistic thinking, and further synthesise a framework adapted into the curriculum in Indonesia, namely the Merdeka Curriculum.

Method

A comprehensive search of the extant literature in mathematics and mathematics education was conducted using the Google Scholar database. The search strategy incorporated the terms “probabilistic thinking”, “probabilistic”, and “framework”. The search was conducted with the inclusion of articles published up to December 2023. Additionally, a manual search of the bibliographies of relevant articles was conducted. The selection criteria comprised articles written in English and indexed in Scopus, which investigated the probabilistic thinking of school students from primary school to university level.

Findings and Discussion

A thorough review of the extant literature was conducted, employing a range of criteria to identify the constructs or concepts underpinning probabilistic thinking. This review revealed a variety of opinions regarding the material constructs that build these abilities. These constructs were then linked to a framework identified using the SOLO taxonomy as a basis for characterising the development of student thinking. The development of this framework was the work of two research groups; one based in the United States led by Graham Jones (Jones et al., 1997; Tarr & Jones, 1997) and the other in Australia led by Jane Watson (Watson & Caney, 2005; Watson et al., 1997; Watson & Moritz, 2003). The development and validation of this framework was achieved through research involving school-aged students, and it describes thinking that generally exhibits the concrete-symbolic mode as described in the SOLO taxonomy (see Table 1) (Chernoff & Sriraman, 2014).

Table 1
Synthesis of Characterizations Across Frameworks

Prestructural Probabilistic Thinking
Student thinking is irrelevant, non-mathematical, or personalized.

Students exhibiting thinking at this level. . .

- Have an intuitive understanding of randomness
- Believe outcomes can be controlled or explained
- Believe consecutive events are always related
- Struggle creating complete sample spaces and distinguishing between fair and unfair situations
- Use idiosyncratic thinking when making predictions

Unistructural Probabilistic Thinking
Student thinking is quantitative and non-proportional.

Students exhibiting thinking at this level. . .

- Have a tendency to revert to subjective probabilistic thinking
- Have a qualitative view of randomness—"anything is possible"
- Make predictions about most or least likely events using quantities
- Create sample spaces for one-stage experiments and some two-stage experiments
- Distinguish between fair and unfair situations
- Compare probabilities
- Determine conditional probabilities
- Become aware that probabilities can change in consecutive events

Multistructural Probabilistic Thinking
Student thinking is quantitative and proportional.

Students exhibiting thinking at this level. . .

- Make use of ratios, counts, probabilities or odds in judging probabilistic situations
- Create samples spaces for one-stage and two-stage experiments systematically
- Provide examples of random situations or methods of random generation
- Recognize changes in probability and independence in without-replacement events
- Make predictions that sometimes rely on representativeness strategies

Relational Probabilistic Thinking
Student thinking shows an interconnection of probabilistic ideas.

Students exhibiting thinking at this level. . .

- Reason with probabilities
- Determine probabilities for complex situations including non-equally likely situations
- Generate sample spaces for two-stage and three-stage experiments systematically
- Connect sample spaces and probabilities
- Examine distribution of outcomes without reverting to representativeness strategies
- Show an appreciation of randomness by not being influenced by order or balance of outcomes

As illustrated in Table 2, the diverse concepts of probability material constitute pivotal components within the probabilistic thinking framework. There is a divergence of opinion regarding the edifice of knowledge that underpins probabilistic thinking. This aspect merits further scholarly investigation to ascertain the fundamental elements or constructs that underpin probabilistic thinking in its entirety. Indeed, research in any field of mathematics education must be underpinned by epistemological reflection on the object under study. In this context, an analysis of textbooks concerning essential knowledge that facilitates the ability of probabilistic thinking is conducted. Endraswara (2021) defines essential knowledge as knowledge that explains the nature of something, which is the core or substance in something. In this study, essential knowledge is defined as scholarly knowledge that is required and serves as the foundation for the formulation of knowledge taught from probabilistic thinking. The research data was analysed in the form of advanced mathematics reference books consisting of, among others: An Introduction to Probability and Statistical Inference Roussas (2015), Introduction to Probability and Mathematical Statistics (Bain & Engelhardt, 1993), and The Nature of Chance and Probability (Batanero et al., 2005). The researcher conducted a re-depersonalization and re-decontextualization process to reconstruct mathematical objects in formal form, so that the construction of mathematical objects in formal form is formed so that the construction of scholarly knowledge will be related to probabilistic thinking. The results of the scholarly knowledge analysis demonstrated that the construction of material related to probabilistic thinking is a stochastic model, randomness, fairness, random experiments, the probability of an event, independent events, conditional probability, basic concepts and results in counting.

Table 2
Key Components of the Prominent Probability Framework

	Concept of Probability Materials	Cognitive level organisation
Jones et al. (1997)	• Sample space • Odds of an event • Comparison of odds • Conditional probability • Event independence	• *Subjective* • *Transitional* • *Informal quantitative* • *Numerical*
Tarr & Jones (1997)	• Conditional probability • Event independence	• *Subjective* • *Transitional* • *Informal quantitative* • *Numerical*
Watson & Moritz (1997)	Measurement of probability (simple events, likelihood, comparison of events)	• *Iconic* • *Unistructural (U)* • *Multistructural (M)* • *Relational (R)*
Watson & Moritz (2003)	*Fairness*	• *Iconic* • *Unistructural (U)* • *Multistructural (M)* • *Relational (R)*
Watson & Caney (2005)	Random Process (definition of randomness, language, luck, equal-likelihood)	• *Iconic* • *Unistructural (U)* • *Multistructural (M)* • *Relational (R)*

As illustrated in Table 2, certain researchers employed disparate terminology when distinguishing between the cognitive levels of students. In general, the iconic mode of functioning is demonstrated by students in all framework development studies and is reflected in the inclusion of the prestructural level (subjective, prestructural/iconic) in each framework. Jones (Jones et al., 1997; Tarr & Jones, 1997) employs the term “Transitional”, which characterises the transition from unstructured probabilistic thinking to quantitative thinking in students. Nevertheless, students’ quantitative thinking is often disproportionate and occasionally flawed. It has been observed that students frequently revert to subjective thinking (Tarr & Jones, 1997). In a similar manner to the term “informal quantitative”, when associated with SOLO, the stronger use of quantitative thinking is indicative of multistructural probabilistic thinking. The final level of this taxonomy is relational probabilistic thinking, which is based on students’ ability to reason with probability capabilities. Students have the capacity to utilise or assign numerical probabilities in complex situations, encompassing both those with equal and unequal possibilities (Jones et al., 1997). Consequently, it can be posited that the numerical level delineated by Jones is congruent with the relational level as conceptualised within Watson’s framework.

In order to respond to the following research question - namely, the adaptation of probabilistic thinking abilities to the content and characteristics of the curriculum in Indonesia, known as the Merdeka Curriculum - a thorough analysis of the minimum learning outcomes stipulated by the curriculum must be conducted. During the 2022/2023 and 2023/2024 academic years, the Merdeka Curriculum is available as one of the options that educational institutions can choose from. The Merdeka Curriculum has been meticulously designed to empower educators, granting them greater autonomy to craft learning experiences that are tailored to the individual needs of students, with a pronounced emphasis on character development. The curriculum is organised into six learning phases, with probabilistic material starting at the junior high school level in phase D, which is the main focus of this article.

The learning objectives stipulated in the Merdeka Curriculum for mathematics in phase D, which focuses on chance material, are that students can explain and use the notion of chance and relative frequency to determine the expected frequency of an event in a simple experiment (all experimental results can appear equally) (Kepala BSKAP Kemendikbudristek, 2024). It can thus be posited that the curriculum demands students to have the ability to think probabilistically. The curriculum delineates the materials to be taught as encompassing the definition of probability, relative frequency, and expected frequency. When considered in conjunction with preceding expert opinion, these materials can be further elaborated through the incorporation of constructs that foster probabilistic thinking, including the sample space, the probability of an event, and probability comparison. The sample space construct can be further enriched by introducing concepts of randomness and fairness, along with an introduction to stochastic events, which form the focal point of the probability material.

Conclusions

The present study is founded on the prior existence of a probabilistic thinking framework that utilises the SOLO taxonomy with cognitive thinking levels that are prestructural, unistructural, multistructural and relational. The material constructs (for junior high school) that have been adapted to the Merdeka curriculum are incorporated, namely the sample space (randomness and fairness and an introduction to stochastic events), the probability of an event, and probability comparison. The framework that has been obtained as a result is as follows:

Tabel 3
A Framework for Assessing Students' Level of Probabilistic Thinking

Thinking Levels	Construction			
	Sample Space	**Probability of an Event**		**Comparison of Probability**
Subjective (Level 1)	The list of members of the set that may occur in a single event is not yet complete	Predicting the most/least likely event to occur based on subjective judgment	Distinguish “certain”, “impossible” and “possible” events subjectively based on the sample space	Comparing the probability of an event in two different sample spaces, usually based on various subjective considerations
Transition (Level 2)	Complete in terms of registering members of the set that may occur in one event, but still relying on subjective opinion	Predicting the most/least likely event based on quantitative judgment but can return to subjective judgment	Distinguishing “certain”, “impossible” and “possible” events within reasonable parameters	Making probability comparisons based on inconsistent quantitative assessments
Informal Quantitative (Level 3)	Complete in listing the members of the set that may occur in a single occurrence	Predict the most/least likely event based on quantitative judgments including situations involving contiguous outcomes	Distinguishing “certain, “impossible” and “possible” events and justifying them quantitatively	Making probability comparisons based on quantitative assessments related to the concept of opportunity that is consistently justified by valid quantitative reasoning but in a limited manner
Numeric (Level 4)	Complete in terms of registering members of the set that may occur in a single event and able to use the strategy in two or more events	Predict the most/least likely event based on the numerical probability of an event	Distinguishing “certain,” “impossible” and “possible” events based on the membership of the overall sample space	Provides numerical probability measures and compares them

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Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The article under consideration was written using an AI-driven writing tool, specifically DeepL Write, which has been found to assist in refining grammar and enhancing its quality.

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