

*Exploring Triangle Centers in Euclidean Geometry with the Geometry Explorer*

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**Abstract**

The aim of this study was to have Thai preservice mathematics teachers explore and visualize various centers of a triangle (incenter, orthocenter, centroid, circumcenter and excenter) using the Geometry Explorer program. The participants comprised of 40 Thai preservice mathematics teachers at Rajabhat Rajanagarindra University, Thailand who enrolled in Foundations of Geometry course during the second semester of the 2015 academic year. The research instruments used in this study were activities package exploring properties of triangle centers using the Geometry Explorer program. The results indicated that Thai preservice mathematics teachers could locate centers related to triangle, make conjectures and verify them correctly and rapidly. Therefore, we could use this program in teaching triangle centers topic in Euclidean geometry effectively.

Keywords: triangle centers, Euclidean geometry, Geometry Explorer.

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## **Introduction**

The word “geometry” literally means “earth (geo) measure (metry).” (Smart, 1998, p. 1). In a fundamental sense, geometry is a natural outgrowth of our exposure to the physical universe and in particular to the natural world (Hvidsten, 2005, p.1). Triangles are the simplest two-dimensional shapes that we can construct with segments (Hvidsten, 2005, p.68). Associated with any triangle are four sets of lines, the angle bisectors, the medians, the altitudes, and the perpendicular bisectors. There are also several points associated with a triangle and these lines. These points are all triangle centers in the sense that each of them can claim to be at the center of the triangle in a certain sense. These points were known to the ancient Greeks. These triangle centers form a bridge between elementary and advanced Euclidean geometry (Venema, 2013, p. 23). They also provide an excellent setting in which to develop proficiency with Geometry Explorer.

## **Introduction to Geometry Explorer**

Geometry Explorer is designed as a geometry laboratory where one can create geometric objects (like points, circles, polygons, areas, and the like), carry out transformations on these objects (dilations, reflections, rotations, and translations), and measure aspects of these objects (like length, area, radius, and so on). As such, it is much like doing geometry on paper (or sand) with a ruler and compass. However, on paper such constructions are static—points placed on the paper can never be moved again. In Geometry Explorer all constructions are dynamic. One can draw a segment and then grab one of the endpoints and move it around the canvas with the segment moving accordingly. Thus, one can create a construction and test out hypotheses about construction with numerous variations of the original construction. Geometry Explorer is just what the name implies—an environment to explore geometry (Hvidsten, 2005, p. 411).

Upon starting Geometry Explorer we will see the main Geometry Explorer Euclidean window on the screen (Figure 1)

There are four important areas within this main window (Hvidsten, 2005, p. 412).

1. The Canvas where geometry is created and transformed. This is the large white area on the right side of the main window.
2. The Tool Panel where geometric tools are located. The Tool Panel is directly to the left of the Canvas. It consists of a set of iconic buttons which represent tools used to create and modify geometric figures. The icons (pictures) on the buttons depict the function that the particular button serves. Sometimes this function is quite clear, other times it is harder to figure out, but the pictures serve as reminders as to what the buttons can do. The Tool Panel is split into four sub-panels: Create, Construct, Transform, and Color Palette. Note that the cursor is over the Info tool (the one with the question mark). While not shown in the figure, a small box with the words Get Info on Object will appear below the Info tool when the cursor is help steady over the tool for a second or two. This box is called a Tool Tip. Tool tips are designed to give quick information on a tool’s purpose.

3. The Menu Bar. There are 10 menus shown in the menu bar: File, Edit, View, Measure, Graph, Misc, Turtle, Model, Windows, and Help. Each of these menus will control specific actions. The figure shows the set of menus available when working in Euclidean geometry. Other menus are available in Hyperbolic, Elliptic, and Spherical geometry.

4. The Message Box. This is where detailed information will be shown concerning various tools that one may wish to use. In (Figure 1) the mouse cursor is over the Info tool. In the Message Box we see information concerning how this tool should be used, as well as other information provided by the tool. In the case of the Info tool, we see information regarding memory use for the program. The Message Box is located below the Canvas.

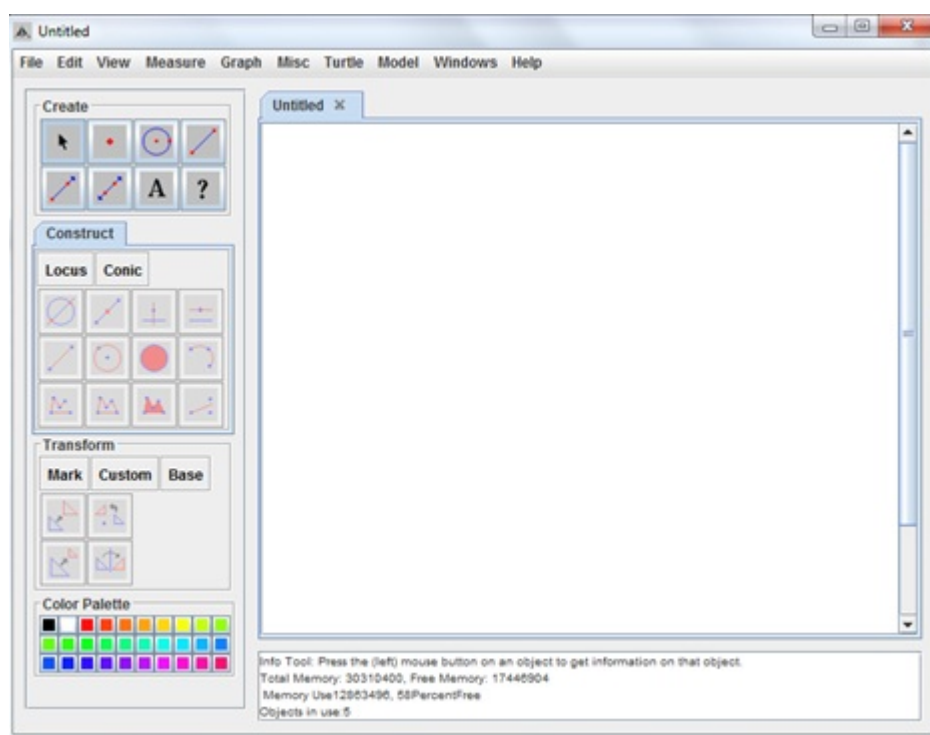


Figure 1: The Geometry Explorer Main (Euclidean) Window

## Centers of Triangle

As mentioned earlier, there are four sets of lines associated with any triangle, the angle bisectors, the medians, the altitudes, and the perpendicular bisectors. The angle bisector is the line that bisects one of the three angles in a triangle. The segment joining a vertex of a triangle to the midpoint of the opposite side is called a median for the triangle. The altitude for a triangle is a line through one vertex that is perpendicular to the opposite sideline of the triangle. The point at which an altitude intersects the opposite sideline of a triangle is called the foot of the altitude. The perpendicular bisector of a side of a triangle is a line which is perpendicular to the side and also passes through its midpoints (Venema, 2013, pp. 23-25). Therefore, there are three angles bisectors, three medians, three altitudes and three perpendicular bisectors in each triangle. On every triangle there are points where these special lines intersect, and those points usually have very interesting geometrical properties. Such

points are called triangle centers. Some examples of triangle centers are incenter, orthocenter, centroid, circumcenter, excenter, Feuerbach point, Fermat points, etc.

### **Concurrent lines:**

Three lines are concurrent if there is a point  $P$  such that  $P$  lies on all three of the lines. The point  $P$  is called the point of concurrency. Three segments are concurrent if they have an interior point in common (Venema, 2013, p.23). Two arbitrary lines will intersect in a point-unless the lines happen to be parallel, which is unusual. Thus concurrency is an expected property of two lines. But it is rare that three lines should have a point in common. One of the surprising and beautiful aspects of advanced Euclidean geometry is the fact that many triples of lines determined by triangles are concurrent. Each of the triangle centers in the following is an example.

### **The inscribed circle and the incenter:**

The bisectors of the interior angles of a triangle are concurrent. The point of concurrency is equidistant from the sides of the triangle. The point of concurrency of the bisectors is called the incenter of the triangle. The distance from the incenter to the sides of the triangle is the inradius. The circle that has its center at the incenter and is tangent to each of the sides of the triangle is called the inscribed circle or the incircle of the triangle (Venema, 2013, p. 42).

### **Altitudes and the orthocenter:**

The line determined by two vertices of a triangle is called a sideline for the triangle. The sidelines play an increasingly important role in the geometry of a triangle. Altitude for a triangle is a line through one vertex that is perpendicular to the opposite sideline of the triangle. The point at which an altitude intersects the opposite sideline of a triangle is called the foot of the altitude. The three altitudes of a triangle are concurrent. The point of concurrency of the three altitudes is called the orthocenter of the triangle (Venema, 2013, pp. 25-26).

### **Medians and the centroid:**

The segment joining a vertex of a triangle to the midpoint of the opposite side is called a median for the triangle. There are three medians of a triangle. The three medians of any triangle are concurrent. The point of concurrency of the medians is called the centroid of the triangle (Venema, 2013, p. 24).

### **Perpendicular bisector and the circumcenter:**

A perpendicular bisector of a side of a triangle is a line which is perpendicular to the side and also passes through its midpoint. There are three perpendicular bisectors of a triangle. The perpendicular bisectors of the three sides of a triangle are concurrent. The point of concurrency of the three perpendicular bisectors of the side of a triangle is the circumcenter of the triangle. A circle that contains all three vertices of the triangle is said to circumscribe the triangle. The circle is called the circumscribed circle or simply the circumcircle of the triangle. The radius of the circumscribed circle is called the circumradius (Venema, 2013, pp. 26-27, 39).

## **The escribed circles and the excenters:**

The external bisectors of two angles of a triangle meet the internal bisector of the third angle at a point. A circle that is outside the triangle and is tangent to all three sidelines of the triangle is called an escribed circle or an excircle. The center of an excircle is called an excenter for the triangles. There are three excircle, one opposite each vertex of the triangle. There are three excircles, one opposite each vertex (Venema, 2013, p. 42).

Many geometry classes were taught in a fairly traditional manner. Definitions were given, theorems stated and proved and problems assigned and corrected. Some classes did do some laboratory type activities using MIRAS, a device use to do reflections in a plane, geoboards, paper folding and standard ruler and compass constructions. The introduction of Sketchpad allowed us to continue these activities in a dynamic manner (Groman, 1996, p. 61). Geometry Explorer is also a dynamic geometry software. It is possible that one can create a triangle and test out hypotheses about its centers with numerous variations of the original construction.

## **Purpose of the study**

The aim of this study was to have Thai preservice mathematics teachers explore and visualize various centers of a triangle (incenter, orthocenter, centroid, circumcenter and excenter) using the Geometry Explorer program.

## **Methodology**

The participants in this study were 40 Thai preservice mathematics teachers at Rajabhat Rajanagarindra University, Thailand who enrolled in the Foundations of Geometry course during the second semester of the 2015 academic year. They were purposively selected.

The research instruments used in this study were activities package exploring properties of triangle centers using the Geometry Explorer program.

## **Findings**

Thai preservice mathematics teachers began their exploration activities in the computer-based environment as follows:

### **Exploration 1: The perpendicular bisectors of the sides of a triangle are concurrent.**

After introducing the concept of a perpendicular bisector of a side of a triangle, the preservice mathematics teachers were asked to construct a triangle, construct the three perpendicular bisectors for the sides of the triangle. Most of the students concluded that these perpendicular bisectors were concurrent. The point of concurrency of the three perpendicular bisectors of the side of a triangle is the circumcenter of the triangle. When the students were asked to move one vertex of the triangle around and observe how the circumcenter changes, they found that for an acute triangle the

circumcenter is inside the triangle, for obtuse triangle it lied outside and for a right triangle it lied at the midpoint of the hypotenuse of the triangle.

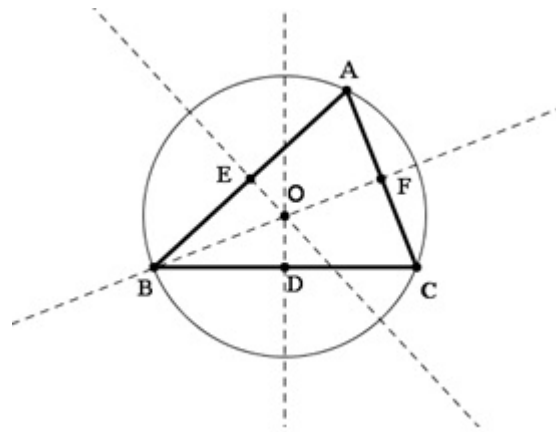


Figure 2: Circumcenter (O) of an acute triangle

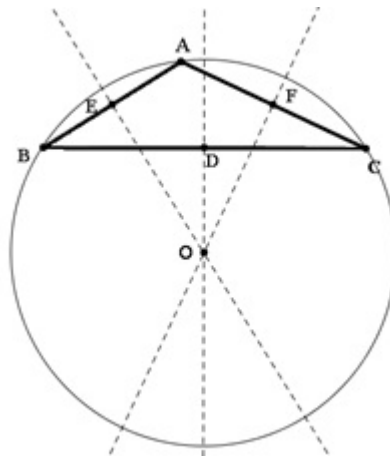


Figure 3: Circumcenter (O) of an obtuse triangle

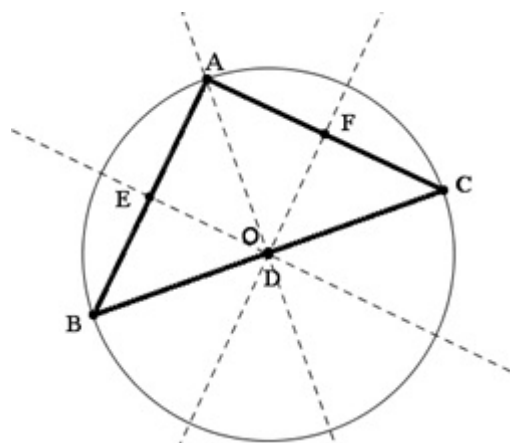


Figure 4: Circumcenter (O) of a right triangle

**Exploration 2: The altitudes of a triangle are concurrent.**

After introducing the concept of the altitudes of a triangle, the preservice mathematics teachers were asked to construct a triangle, construct the three altitudes for the triangle. Most of the students observed that no matter how the vertices of the triangle were moved around in the plane, the three altitudes continued to be concurrent. The point of concurrency of the three altitudes is called the orthocenter of the triangle. When the students were asked to determine by experimentation the shape of triangles for which the orthocenter is outside the triangle, find a shape for triangle so that the orthocenter is equal to one of the vertices of the triangle. They found that for an acute triangle the orthocenter was inside the triangle, for obtuse triangle it lied outside and for a right triangle it lied at the vertex of the triangle where right angle was formed.

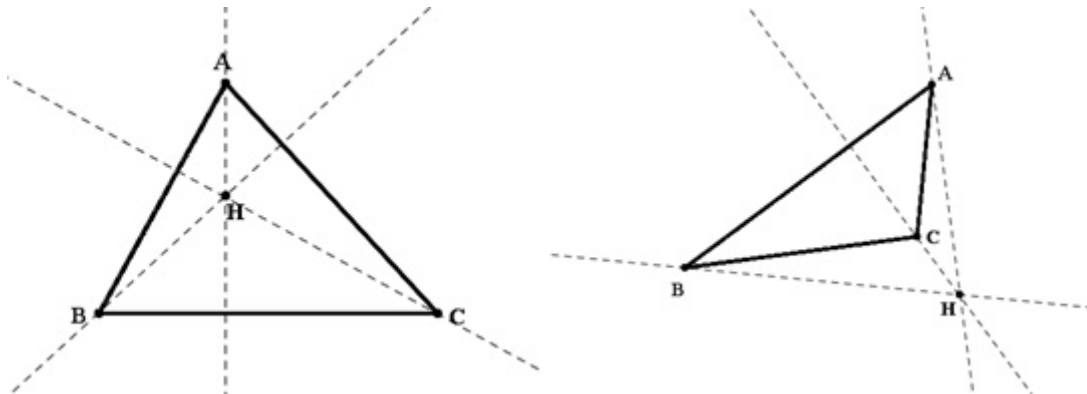


Figure 5: Two triangles with their altitudes and orthocenter

**Exploration 3: The internal bisectors of the angles of a triangle are concurrent.**

After introducing the concept of the bisector of the interior angle of a triangle, the preservice mathematics teachers were asked to construct a triangle, construct the three bisectors of the interior angles of a triangle. Most of the students found that the three angle bisectors were always concurrent. This point of concurrency is equidistant from the sides of the triangle. The point of concurrency of the bisectors was called the incenter of the triangle. The circle that has its center at the incenter and is tangent to each of the sides of the triangle is called the inscribe circle or the incircle of the triangle. The students then were asked to determine whether the incenter can ever be on the triangle or outside the triangle. Most of the students found that the incenter could not be outside the triangle.

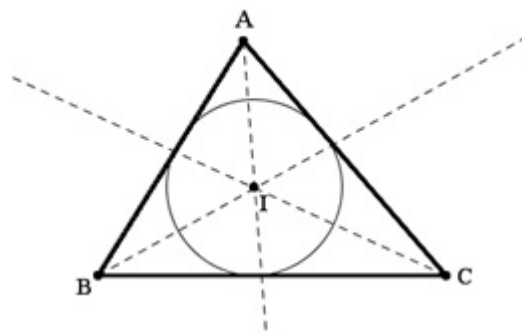


Figure 6: The incenter (I) and the incircle

**Exploration 4: The median of a triangle are concurrent.**

After introducing the concept of the median of a triangle, the preservice mathematics teachers were asked to construct a triangle, construct the three midpoints of the sides of the triangle, and construct the three medians of the triangle. Most of the students could be conclude that these medians were concurrent and also found that this continues to be true when the vertices of the triangle were moved around in the plane. The point of concurrency of the three medians is called the centroid of the triangle. They also found that the centroid of triangle always remains inside the triangle irrespective of its type (scalene, isosceles or equilateral).

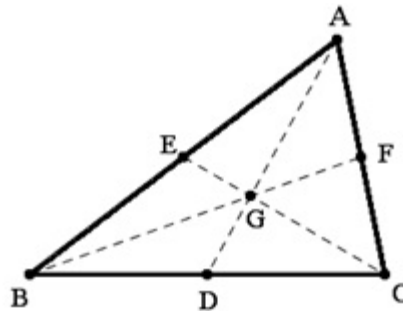


Figure 7: The three medians and the centroid (G)

**Exploration 5: The external bisectors of two angles of a triangle meet the internal bisector of the third angle at a point.**

After introducing the concept of the external bisector of an angle of a triangle and the internal bisector of an angle of a triangle, the preservice mathematics teachers were asked to construct a triangle, construct the two external bisectors of two angles of a triangle and construct the internal bisector of the third angle of the triangle. Most of the students observed that no matter how the vertices of the triangle were moved around in the plane, the external bisectors of two angles of a triangle and the internal bisector of the third angle continued to be concurrent. They also observed that there are three excircles, one opposite each vertex of the triangle.

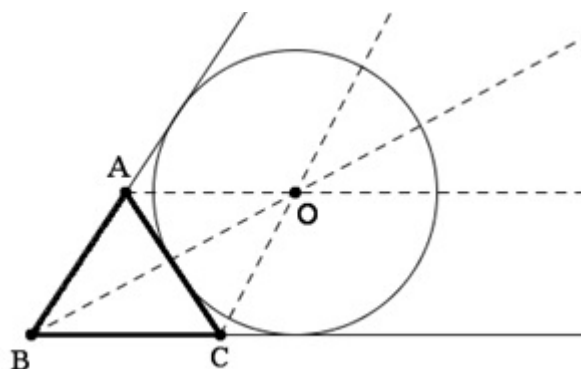


Figure 8: The excircle O



## **Conclusion**

In this study the researcher used Dynamic Geometric Software, Geometry Explorer, to visualize various centers of a triangle. There are many centers of a triangle, the researcher explored five out of them viz Centroid, Circumcenter, Incenter, Orthocenter, and Excenter. The results indicated that Thai preservice mathematics teachers could locate centers related to triangle, explore new relations, make conjectures and verify them correctly and rapidly. Exploring mathematical relations and testing conjectures in this dynamic geometry environment make this type of software a strong learning tool. Therefore, after paper folding we could use this program in teaching triangle centers topic in Euclidean geometry effectively.

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## References

Groman, M. (1996, June). Integrating "Geometer's Sketchpad" into a Geometry Course for Secondary Education Mathematics Majors. In *Association of Small Computer Users in Education (ASCUE)*. Paper presented at the Proceedings of the 29th Summer Conference (pp. 61-65) , North Myrtle Beach, SC: Association of Small Computer Users in Education.

Hvidsten, M. (2005). *Geometry with Geometry Explorer*. New York: McGraw-Hill

Smart, J. (1998). *Modern geometries* (5th ed.). California: Brooks/Cole.

Venema, G.A. (2013). *Exploring advanced Euclidean geometry with GeoGebra*. Washington, DC: The Mathematical Association of America.

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