A Simplified Mathematical Model to Predict the Thermal Physiological Responses of a Sleeping Person in Steady-state and Uniform Conditions

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Abstract

This paper reports on the simplified mathematical model to predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions. By modifying Gagge's two-node model and coupled with a mathematical model of predicting the total insulation value of a bedding system, a mathematical model for predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions was firstly developed. This is followed by comparing the predicted values using the mathematical model with the previous experimental data.

Keyword: Thermoregulation model; sleeping person; total insulation value of a bedding system

1 Introduction

A human being spends approximately one-third of his / her life in sleep. Sleep is not simply a state of rest, but has its own specific, positive functions [1]. Sleep can help people overcome tiredness and is very important to one's memory. For decades, numerous medical researchers have investigated various factors that affected the quality of sleep [2−6]. It was commonly acknowledged that the quality of sleep was mainly determined by mental-physical factors of a sleeping person and environmental factors in a bedroom. Although the latter one covered lighting, noise and thermal environment, the influence of thermal parameters in a sleeping environment on the quality of sleep was gradually understood [7−10]. Previous experimental results have demonstrated that when the thermal environment in a bedroom deviated greatly from the so-called 'thermal comfort zone', there were a remarkable increase in numbers and duration of wakefulness and a decrease in Rapid Eye Moment (REM) stage [8−10]. Although thermoregulatory responses were present across different sleep stages, they would be partly depressed in REM stage [11−15]. Hence, an increase in wakefulness and a decrease in REM stage reflected thermoregulatory need, and suggested that there was a competition between sleep maintenance and thermoregulation [15, 16]. Therefore, sleep quality became disturbed or even deteriorated as soon as the thermoregulatory responses were present. Consequently, it was necessary to investigate the thermoregulatory responses of a sleeping person in order to improve sleep quality.

A human being's thermoregulation is very complicated due to numerous variables involved in many control loops [17]. Using a thermoregulation model to predict human thermal physiological responses is a very useful tool in investigating the thermoregulation capability of a human body. Therefore, a large number of models for thermoregulation have been developed [18−26]. From early 40's to late 60's, human thermoregulation was studied primarily using analogue simulation [18−20]. In 1970s, there was a transition from the use of analogue simulation to digital simulation which was powerful in computational capability. Since then, a number of thermoregulation models have been developed, including the Gagge, the Stolwijk and the Wissler models [21−23] which were the most commonly used. With the further advancement of computer technology, Smith-Fu finite element thermoregulation model [24, 25] and a 65-node thermoregulation model [26] were developed to predict three-dimensional body temperature distributions, local sweat rates, and latent and sensible heat losses from a human body surface.

However, all above mentioned thermoregulation models developed were for a person who was awake. For a sleeping person, his/her thermoregulatory process can be different from that of an awaking person because of two reasons. Firstly, the total insulation value of a bedding system was different from that of clothing and would play an important role in people's thermal sensation [27, 28]. Secondly, the set points for skin temperature and core temperature were different [29−31]. Therefore, thermoregulation models previously developed should be modified in order to predict thermal physiological responses of a sleeping person.

This paper reports on the simplified mathematical model to predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions. By modifying Gagge's two-node model and coupled with a mathematical model of predicting the total insulation value of a bedding system, a simplified mathematical model for predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions was firstly developed. This is followed by comparing the predicted values using the mathematical model with the previous experimental data.

2 The development of the mathematical model

Based on Gagge's two-node model [21], the mathematical model was developed by considering the following features for a sleeping person:

- The total insulation value of a bedding system would significantly affect the thermal comfort of a sleeping person, instead of the clothing insulation.
- The skin set-point temperature of a sleeping person would be larger than that of an awake person, and core set point temperature would be less. The set point skin and core temperature were set at 34.6°C and 36.8°C, respectively [14, 18].

2.1 Controlled system of the mathematical model

The whole body was subdivided into two concentric compartments. The outer compartment represents the skin, and the inner compartment the core. The heat is transferred between skin and core by conduction. The body and the environment exchange heat by conduction, radiation, convection, evaporation and respiration.

These equations represented were as follows:
(1) Heat storage for skin and core compartments

$$S_{cr} = (M + M_{shiv} - Q_{res}) - K(T_{cr} - T_{sk}) - c_{p,bl}(T_{cr} - T_{sk}) = 0 \qquad (1)$$

$$S_{sk} = K(T_{cr} - T_{sk}) + c_{p,bl} m_{bl}(T_{cr} - T_{sk}) - E_{sk} - (C + R) = 0 \qquad (2)$$

(2) Heat loss from respiration Q_{res}

$$Q_{res} = 0.014M(34 - t_a) + 0.0173M(5.87 - P_a) \qquad (3)$$

(3) Sensible heat loss from skin $C+R$

$$C + R = \frac{t_{sk} - t_o}{R_t} \qquad (4)$$

(4) Evaporative heat loss from skin E_{sk}

$$E_{sk} = wE_{\max} \qquad (5)$$

$$E_{\max} = \frac{P_{sk,a} - P_a}{R_{e,t}} \qquad (6)$$

$$i_m L_R = \frac{R_t}{R_{e,t}} \tag{7}$$

$$w = w_{rsw} + 0.06(1 - w_{rsw}) = 0.06 + 0.94\frac{E_{rsw}}{E_{max}} \tag{8}$$

$$w_{rsw} = \frac{E_{rsw}}{E_{max}} \tag{9}$$

$$E_{rsw} = m_{rsw} h_{fg} \tag{10}$$

2. 2 Control system of the mathematical model

Based on Gagg's two node model, the controlling of the mathematical model was divided into two parts. In the first part, which recognize the thermal state of the controlled system, the error signal was equal to the difference temperature between the actual temperature in two compartments between the set point temperature, so that the outputs of warm or cold can be determined, it can be expressed by following equations:

$$WSIG_{cr} = \max((T_{cr} - T_{cr,n}),0) \tag{11}$$

$$WSIG_{sk} = \max((T_{sk} - T_{sk,n}),0) \tag{12}$$

$$CSIG_{cr} = \max((T_{cr,n} - T_{cr}),0) \tag{13}$$

$$CSIG_{sk} = \max((T_{sk,n} - T_{sk}),0) \tag{14}$$

$$WSIG_b = \max((T_b - T_{b,n}),0) \tag{15}$$

$$CSIG_b = \max((T_{b,n} - T_b),0) \tag{16}$$

Where,

$$T_b = \xi T_{sk} + (1-\xi)T_{cr} \tag{17}$$

$$T_{b,n} = \xi_n T_{sk,n} + (1-\xi_n)T_{cr,n} \tag{18}$$

In the second part, based on the temperature signals, the effector outputs of the controlling system were determined, respectively. The controller equations are as follows:

(1) Vasomotor regulation:

$$m_{bl} = \frac{(6.3 + 200WSIG_{cr})}{(1 + 0.1CSIG_{sk})3600} \tag{19}$$

$$\xi = 0.042 + \frac{0.745}{3600m_{bl} + 0.585} \tag{20}$$

(2) Sweating regulation:

$$m_{rsw} = 4.72 \times 10^{-5} WSIG_b \exp(WSIG_{sk}/10.7) \tag{21}$$

(3) Shivering regulation:

$$Mshiv = 19.4CSIG_{sk}CSIG_{cr} \tag{22}$$

2.3 The mathematical model of predicting the total insulation value of a bedding system

The total insulation value of a bedding system would be affected by many factors: beddings, the percentage coverage of body surface by beddings and bed with mattress, the insulation of mattress and so on.

The mathematical model of predicting the total insulation value of a bedding system was developed as follows [28]:

$$\frac{1}{R_t}=\frac{\frac{3}{5}\times\left(A_c-0.233\right)}{\left(\left(0.03984\times H_{fab}\right)+\frac{1}{h_c+h_r}\right)}+\left(\left(\frac{2}{5}\times\left(A_c-0.233\right)\right)\cdot\times h_c\right)\times\left[\frac{\left(\frac{2}{\left(0.03984\times H_{fab}\right)+\frac{1}{h_c+h_r}}+\frac{\sqrt{3}}{r_m}\right)}{\frac{2}{\left(0.03984\times H_{fab}\right)+\frac{1}{h_c+h_r}}+\frac{\sqrt{3}}{r_m}+h_c}\right]+\frac{0.233}{r_m}+\frac{\left(1-A_c\right)}{\frac{1}{h_c+h_r}} \tag{23}$$

To make sure the thermal comfort of a sleeping person, the air velocity around the person would be less than 0.25 m/s, and Air velocity in all the experiments was controlled at not greater than 0.15 m/s. Therefore, the convective heat transfer coefficient was assumed at 5.1 W/(m^2·K), then, with a value of 3.235 W/(m^2·K) for the radiant heat transfer coefficient, Equation (23) could be simplified to:

$$\frac{1}{R_t}=\frac{\frac{3}{5}\times\left(A_c-0.233\right)}{\left(\left(0.03984\times H_{fab}\right)+\frac{1}{8.335}\right)}+\left(\left(\frac{2}{5}\times\left(A_c-0.233\right)\right)\cdot\times 5.1\right)\times\left[\frac{\left(\frac{2}{\left(0.03984\times H_{fab}\right)+\frac{1}{8.335}}+\frac{\sqrt{3}}{r_m}\right)}{\frac{2}{\left(0.03984\times H_{fab}\right)+\frac{1}{8.335}}+\frac{\sqrt{3}}{r_m}+5.1}\right]+\frac{0.233}{r_m}+\frac{\left(1-A_c\right)}{\frac{1}{8.335}} \tag{24}$$

3 Validation of the mathematical model

Combining the above equations described in Section 2, the simplified mathematical model was developed and would be used to simulate the thermal physiological responses of a sleeping person in steady-state and uniform conditions.

Figure 1 shows the relationship between air temperature and skin temperature for a naked sleeping person under three humidity levels. It can be that the humidity did not affect significantly the skin temperature, compared with the air temperature. The increase in humidity from 30 % to 70 % resulted only in the increase in skin temperature, which was less than 0.7°C.

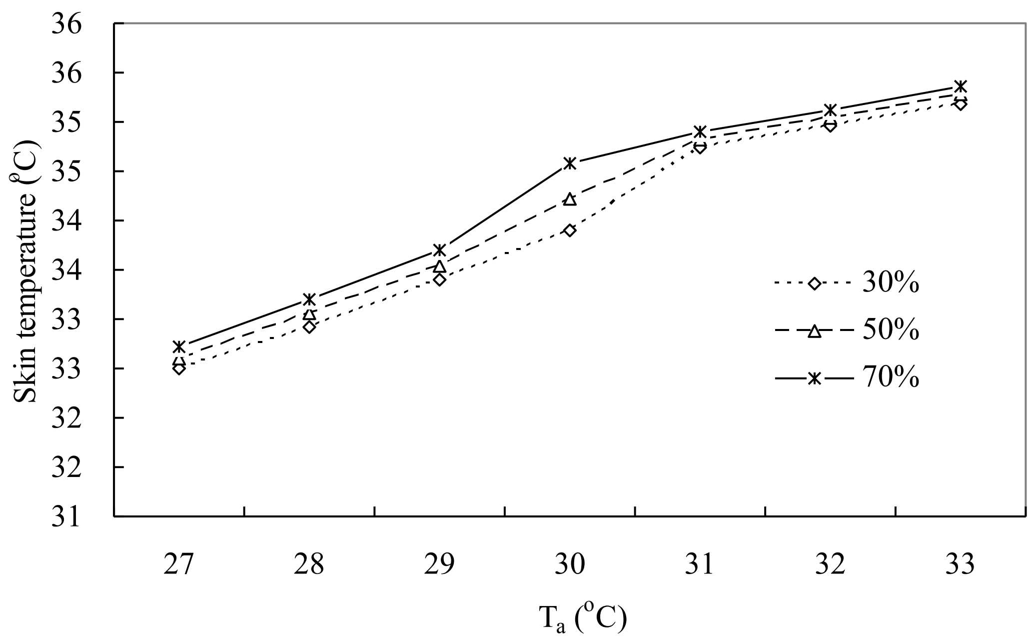

Figure 1 The relationship between air temperature and skin temperature for a naked sleeping person under three humidity levels

For the humidity did not affect significantly the skin temperature, the humidity was assumed to be 50% in the next results analysis. Figure 2 shows the relationship between air temperature and the skin and core temperature for a naked sleeping person. It can be seen that the higher the air temperature, the higher the skin temperature. This suggested that the skin temperature was affected significantly the air temperature. However, the core temperature kept nearly the same with the change in air temperature.

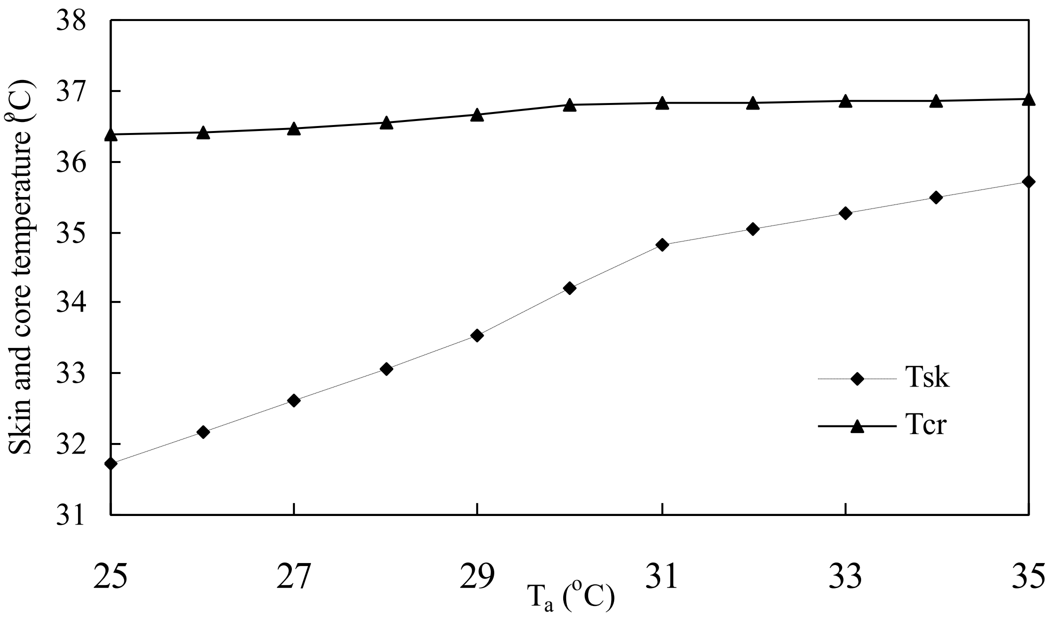

Figure 2 The relationship between air temperature and the skin and core temperature for a naked sleeping person

Figure 3 shows the relationship between air temperature and heat losses for a naked sleeping person. It can be seen that when the air temperature was more than 30°C, the heat loss resulted from evaporation increased, and heat production by shivering became zero. And the heat loss from evaporation kept nearly the same, and heat production by shivering increased with the decrease in the air temperature, when the air temperature was less than 30°C. This suggested that the air temperature at 30°C may be a neutral air temperature for a naked sleeping person, which agreed well with the previous experimental results [16, 22]. For sensible heat loss from skin, *C*+*R*, it decreased with the increase in the air temperature. Since sensible heat loss was proportionally related to the temperature difference between operative temperature and skin temperature.

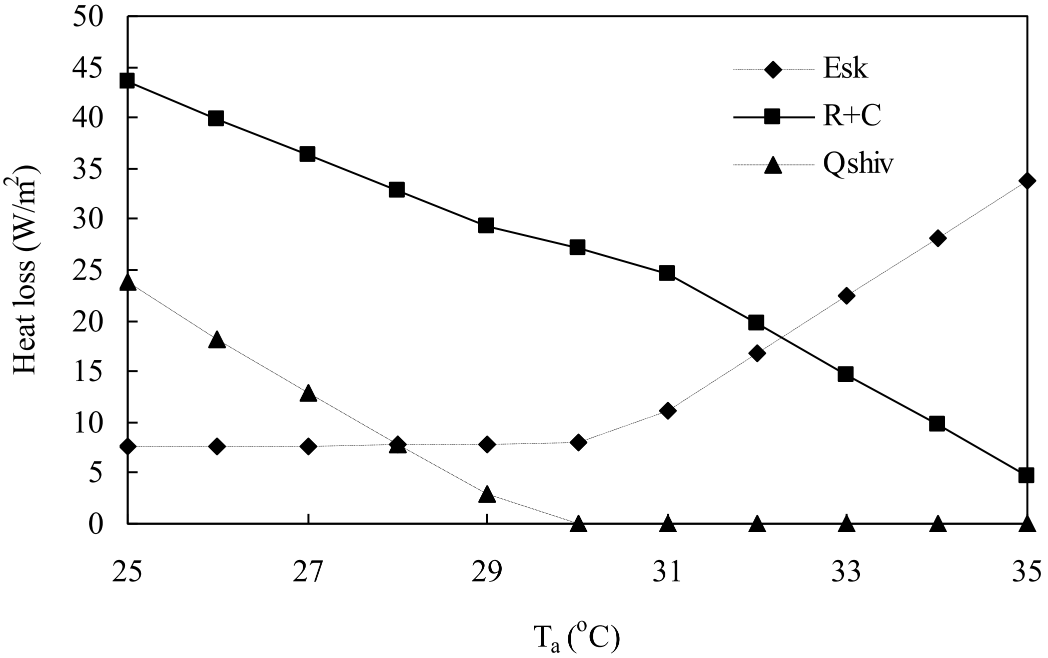

Figure 4 Relationship between air temperature and heat loss for a naked sleeping person

To investigate the total insulation of a bedding system on the thermal physiological responses of a sleeping person, the air temperature was set at 26°C. The details of the beddings used to simulate the mathematical model were described in Section 2. Figure 5 shows the relationship between A_C and skin temperature for a sleeping person covered with three different beddings. It can be seen that the skin temperature was affected significantly by A_C. The increase in A_C from 30 % to 70 % for three beddings B, Q2 and Q1 led to the increase in skin temperature, 0.24°C, 0.32°C and 0.42°C, respectively. However, the increase in A_C from 30 % to 40 % for three beddings B, Q2 and Q1 resulted in the increase in skin temperature, 0.5°C, 0.74°C and 1°C, respectively. This suggested that for a higher A_C, or a thicker bedding, the resultant total insulation of a bedding system was larger, and the heat loss resulted from evaporation would be decrease remarkably. This would lead to a remarkable increase in skin temperature, since the heat could not be effectively transferred by evaporation.

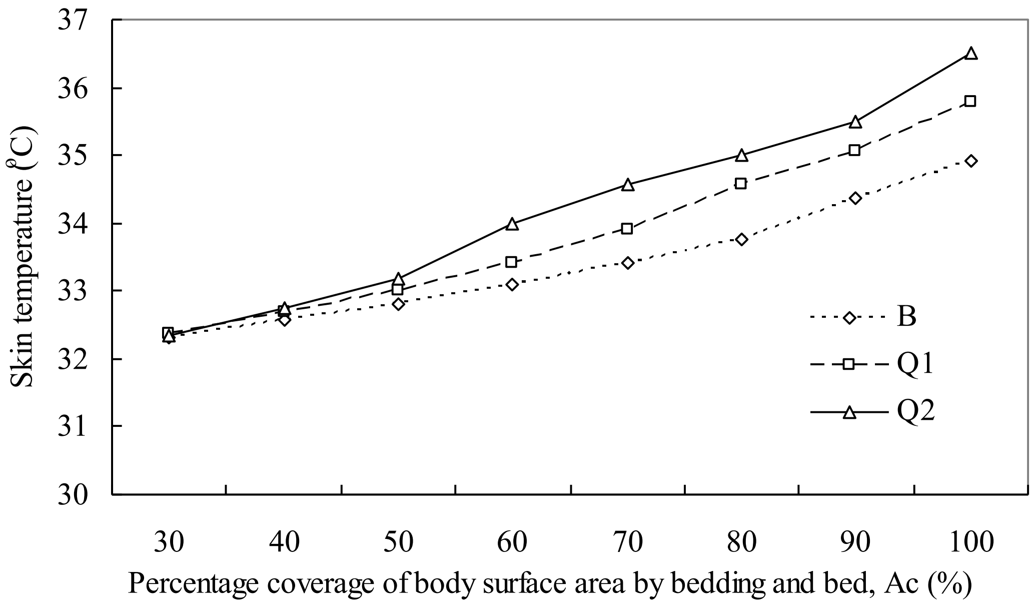

Figure 5 The relationship between A_C and skin temperature for a sleeping person covered with three different beddings

Table 1 summarized the comparisons between the experimental and simulation values for skin and core temperature under various environmental conditions.

Table 1 Comparisons between the experimental and simulation results for skin and core temperature

Case	Researchers	T_a(°C)/h(%)	Experimental Results		Simulation Results	
			$T_{sk,ex}$ (°C)	$T_{cr,ex}$ (°C)	$T_{sk,si}$ (°C)	$T_{cr,si}$ (°C)
1	Okamato-Mizuno (10)	29/50	34.2	36.49	34.15	36.8
2	Okamato-Mizuno (10)	29/75	34.4	36.59	34.6	36.81
3	Tsuzuki (32)	32/80	35.3	37.1	35.31	36.85
4	Okamato-Mizuno (10)	35/50	35.5	36.91	35.78	36.89
5	Okamato-Mizuno (10)	35/75	35.9	37.32	36.17	36.96

Figure 6 shows the comparisons between the experimental and simulation results for skin and core temperature for five cases. It can be seen that the simulation results agreed well with the experimental values. The differences between the simulation and experimental results were relatively small, and the errors were within ±1%.

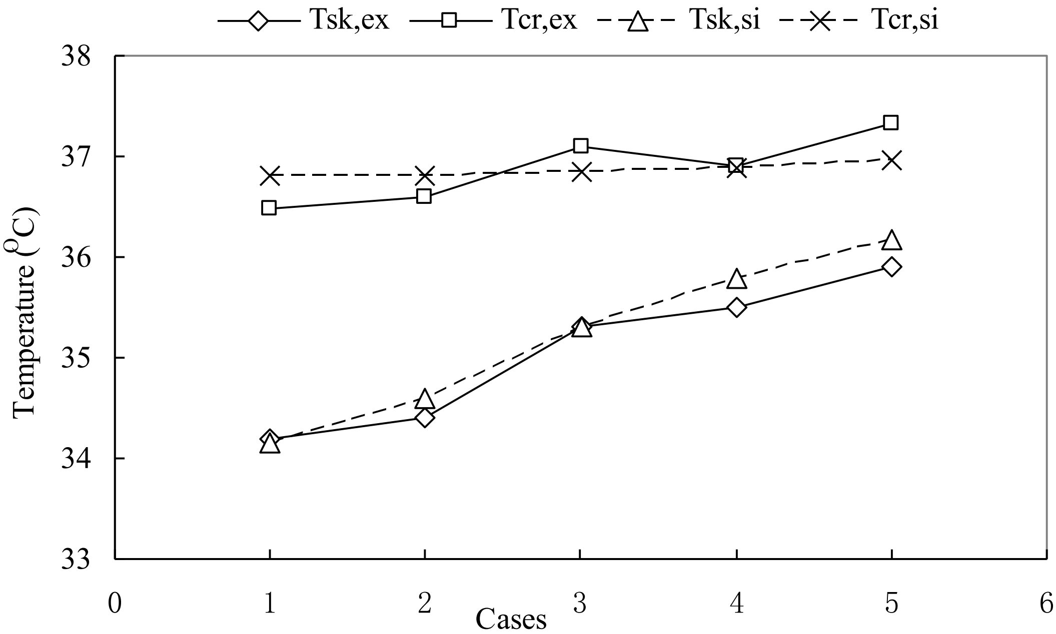

Figure 6 Comparisons between experimental and simulation results for skin and core temperatures

A mathematical model was developed in this section. It was used to predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions.

4 Conclusions

A simplified mathematical model to predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions has been developed and is reported in this paper. The model was validated by comparing the predicted values with the experimental data available in open literature. The comparison results demonstrated that the simplified mathematical model developed could be used to predict the thermal physiological responses of a sleeping person in steady-state and uniform conditions with an acceptable accuracy.

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