*Beyond Kolmogorov Philosophy*

Seifedine Kadry

American University of the Middle East, Kuwait

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Abstract

The Kolmogorov's probability philosophy is based on set of axioms that can be extended to encompass the imaginary set of numbers and this by adding to the original five axioms of Kolmogorov an additional three axioms. Hence, any experiment can thus be executed in what is now the complex set C which is the sum of the real set R with its corresponding real probability, and the imaginary set M with its corresponding imaginary probability. Whatever the probability distribution of the random variable in R is, the corresponding probability in the whole set C is always one, so the outcome of the random experiment in C can be predicted totally. Hence chance and luck in R is replaced by total determinism in C. This is the consequence of the fact that the probability in C is got by subtracting the chaotic factor from the degree of our knowledge of the system.

***keywords***–Kolmogorov's axioms, random variable, probability, real set, imaginary set, complex set, complex number, probability norm, degree of knowledge of the system, chaotic factor, Bernoulli experiment, Binomial distribution, Gaussian or normal distribution, density function, Young's modulus.

**Original Kolmogorov's Set of Axioms:**

The simplicity of *Kolmogorov's* system of axioms may be surprising. Let E be a collection of elements $\{E_1, E_2, \ldots\}$ called elementary events and let F be a set of subsets of E called random events. The five axioms for a finite set E [2] are:

1. F is a field of sets.
2. F contains the set E.
3. A non-negative real number $P_{rob}(A)$, called the probability of A, is assigned to each set A in F.
4. $P_{rob}(E)$ equal 1.
5. If A and B have no elements in common, the number assigned to their union is $P_{rob}(A \cup B) = P_{rob}(A) + P_{rob}(B)$ ; hence, we say that A and B are disjoint; otherwise, we have $P_{rob}(A \cup B) = P_{rob}(A) + P_{rob}(B) - P_{rob}(A \cap B)$.

And we say also that $P_{rob}(A \cap B) = P_{rob}(A) \times P_{rob}(B|A) = P_{rob}(B) \times P_{rob}(A|B)$ which is the conditional probability. If both A and B are independent, then $P_{rob}(A \cap B) = P_{rob}(A) \times P_{rob}(B)$.

An example of probability would be the game of coin tossing. Let $p_1$ denote the probability of getting head $H$ and $p_2$ denote the probability of getting tail $T$. Then we have:

$E = \{H, T\}$

$F = \{\Phi, \{H\}, \{T\}, \{H, T\}\}$

$P_{rob}(\Phi) = 0$, $P_{rob}(E) = P_{rob}(\{H, T\}) = 1$

$P_{rob}(\{H\}) = p_1$ and $P_{rob}(\{T\}) = p_2$

$P_{rob}(\{H\} \text{ or } \{T\}) = p_1 + p_2 = 1$

$P_{rob}(\{H\} \text{and } \{T\}) = 0$

And this according to the original Kolmogorov's set of axioms.

**Adding the Imaginary Part M:**

Now, if we can add to this system of axioms an imaginary part such that:

6. Let $P_m = i(1-P_r)$ be the probability of an associated event in M (the imaginary part) to the event A in R (the real part). It follows that $P_r + P_m/i = 1$ where $i^2 = -1$ (the imaginary number).
7. We construct the complex number $Z = P_r + P_m = P_r + i(1-P_r)$ having a norm $|Z|^2 = P_r^2 + (P_m / i)^2$
8. Let Pc denote the probability of an event in the universe C where C = R + M

   We say that Pc is the probability of an event A in R with its associated event in M such that: $Pc^2 = (P_r + P_m / i)^2 = |Z|^2 - 2iP_rP_m$ and is always equal to 1.

We can see that the system of axioms defined by Kolmogorov could be hence expanded to take into consideration the set of imaginary probabilities.

**Example: Coin Tossing (Bernoulli experiment)**
If we return to the game of coin tossing, we define the probabilities as follows:

| ***Output*** | **R** | **M** |
|---|---|---|
| Getting Head | $\Pr_1 = p_1$ | $Pm_1 = q_1 = i(1 - p_1)$ |
| Getting Tail | $\Pr_2 = p_2$ | $Pm_2 = q_2 = i(1 - p_2)$ |
| **Sum** | $\sum_i p_i = 1$ | $\sum_i q_i = i$ |

If we calculate $|Z_1|^2$ for the event of getting Head, we get:

$$|Z_1|^2 = \Pr_1{}^2 + (Pm_1 / i)^2$$

$$= p_1{}^2 + (q_1 / i)^2 = p_1{}^2 + (1 - p_1)^2 = 1 + 2p_1(p_1 - 1) = 1 - 2p_1(1 - p_1)$$

This implies that:

$$1 = |Z_1|^2 + 2p_1(1 - p_1) = |Z_1|^2 - 2.i.p_1.i(1 - p_1) = |Z_1|^2 - 2i\Pr_1 Pm_1$$

$$= \Pr_1{}^2 + (Pm_1 / i)^2 - 2i\Pr_1 Pm_1 = (\Pr_1 + Pm_1 / i)^2 = Pc_1{}^2$$

where $i^2 = i \times i = -1$ and $\frac{1}{i} = -i$

This is coherent with the axioms already defined and especially axiom 8.

Similarly, if we calculate $|Z_2|^2$ for the event of getting Tail, we get:

$$|Z_2|^2 = \Pr_2{}^2 + (Pm_2 / i)^2$$

$$= p_2{}^2 + (q_2 / i)^2 = p_2{}^2 + (1 - p_2)^2 = 1 + 2p_2(p_2 - 1) = 1 - 2p_2(1 - p_2)$$

This implies that:

$$1 = |Z_2|^2 + 2p_2(1 - p_2) = |Z_2|^2 - 2.i.p_2.i(1 - p_2) = |Z_2|^2 - 2i\Pr_2 Pm_2$$

$$= \Pr_2{}^2 + (Pm_2 / i)^2 - 2i\Pr_2 Pm_2 = (\Pr_2 + Pm_2 / i)^2 = Pc_2{}^2$$

This is also coherent with the axioms already defined and especially axiom 8.

1. Role of the Imaginary Part:

It is apparent from the set of axioms that the addition of an imaginary part to the real event makes the probability of the event in C always equals to 1. In fact, if we begin to see the universe as divided into two parts, one real and the other imaginary, understanding will follow directly. The event of tossing a coin and of getting a head occurs in R (in our real laboratory), its correspondent probability is $P_r$. One may ask directly what makes that we ignore the output of the experiment (e.g. tossing the coin). Why should we use the probability concept and would not be able to determine surely the output? After reflection one may answer that: if we can know all the forces acting upon the coin and determine them precisely at each instant, we can calculate their resultant which will act upon the coin, according to the well known laws of dynamics and determine thus the output of the experiment:

$$\sum_i F_i = ma \text{, where } F \text{ is the force, } m \text{ the mass, and } a \text{ the acceleration}$$

Hence, taking into consideration the effect of all hidden (i.e. unknown and undetermined) forces or variables, the experiment becomes deterministic, that is, it becomes possible to know the output with a probability equals to 1. This is plausible if we consider the simple experiments of dynamics like of a falling apple or a rolling body experiments where the hidden variables are totally known and determined and which are: gravitation, air resistance, friction and resistance of the material. But when the hidden variables become difficult to determine totally like in the example of lottery – where the ball has to be chosen mechanically by the machine in an urn of hundred moving bodies!!!– we are not able in the latter case to determine precisely which ball will be chosen since the number of forces acting on each ball are so numerous that the kinematics study is very difficult indeed. Consequently, the action of hidden variables on the coin or the ball makes the result what it is. Hence the complete knowledge of the set of hidden variables makes the event deterministic; that is, it will occur surely and thus the probability becomes equal to one, [3].

Now, let M be the universe of the hidden variables and let $|Z|^2$ be the degree of our knowledge of this phenomenon. $P_r$ is always, and according to Kolmogorov's axioms, the probability of an event. A total ignorance of the set of variables in **M** makes:
$P_r = P_{rob}$(getting Head) = 1/2 and $P_{rob}$(getting Tail) = 1/2 (if they are equiprobable).
$|Z_1|^2$ in this case is equal to $1-2p_1(1-p_1)=1-(2\times1/2)\times(1-1/2)=1/2$.
Conversely, a total knowledge of the set in R makes: $P_{rob}$(getting Head) = 1 and $P_m$ = $P_{rob}$(imaginary part) = 0;
Here we have $|Z_1|^2=1-(2\times1)\times(1-1)=1$ because the phenomenon is totally known, that is, its laws are determined, hence; our degree of our knowledge of the system is 1 or 100%.
Now, if we can tell for sure that an event will never occur i.e. like 'getting nothing' (the empty set), in the game of Head and Tail, $P_r$ is accordingly = 0, that is the event will never occur in R. $P_m$ will be equal to $i(1-P_r)$ = $i(1-0)$ = $i$, and $|Z_1|^2=1-(2\times0)\times(1-0)=$ 1, because we can tell that the event of getting nothing surely will never occur thus our degree of knowledge of the system is 1 or 100%.
It follows that we have always: $1/2\le|Z|^2\le1$ since $|Z|^2=P_r^{\,2}+(P_m/i)^2$ and $0\le P_r, P_m\le1$. And in all cases we have: $Pc^2=(P_r+P_m/i)^2=|Z|^2-2iP_rP_m=1$

2. Meaning of the Last Relation:

According to an experimenter tossing the coin in R, the game is a game of luck: the experimenter doesn't know the output in the sense already explained. He will assign to each outcome a probability $P_r$ and say that the output is not deterministic. But in the universe C = R + M, an observer will be able to predict the outcome of the game since he takes into consideration the contribution of M, so we write:

$$Pc^2=(P_r+P_m/i)^2$$

So in C, all the hidden variables are known and this leads to a deterministic experiment executed in an eight dimensional universe (four real and four imaginary; where three for space and one for time in R, and three for space and one for time in M) [1]. Hence $Pc$ is always equal to 1. In fact, the addition of new dimensions to our experiment resulted to the abolition of ignorance and non-determination. Consequently, the study of this class of phenomena in C is of great usefulness since we will be able to predict with certainty the outcome of experiments conducted. In

fact, the study in R leads to non-predictability and uncertainty.
So instead of placing ourselves in R, we place ourselves in C then study the phenomena, because in C the contributions of M are taken into consideration and therefore a deterministic study of the phenomena becomes possible. Conversely, by taking into consideration the contribution of the hidden forces we place ourselves in C and by ignoring them we restrict our study to non-deterministic phenomena in R.

**Conclusion:**
The degree of our knowledge in the real universe R is unfortunately incomplete, hence the extension to the complex universe C that includes the contributions of both the real universe R and the imaginary universe M. Consequently, this will result in a complete and perfect degree of knowledge in C. This hypothesis is verified in this paper by the mean of many examples encompassing both discrete and continuous domains. Moreover, we have proved a linear proportional relation between the degree of knowledge and the chaotic factor. In fact, in order to have a certain prediction of any event it is necessary to work in the complex universe C in which the chaotic factor is quantified and subtracted from the degree of knowledge to lead to a probability in C equal to one. Thus, the study in the complex universe results in replacing the phenomena that used to be random in R by deterministic and totally predictable ones in C.

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