Classroom Interaction Analysis in Mathematics Grounding Activities: A Case Study of Decimal Concepts

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Abstract

In recent years, Taiwanese students have excelled in international mathematics assessments, yet reports reveal a gap between academic achievement and learning interest. Addressing this discrepancy has become a key goal for elementary mathematics education in Taiwan. This study analyzes teacher–student interaction patterns during the implementation of Mathematics Grounding Activities (MGA) focused on decimal concepts. Using the IRE (Initiation–Response–Evaluation) and IRF (Initiation–Response–Feedback) frameworks, five mathematics lessons were examined. Findings show that IRF interactions were more prevalent than IRE. While IRE structures often involved rapid checks of computational accuracy, IRF interactions were more frequent in game-based activities, fostering strategic reasoning, conceptual understanding, and error reflection. The study highlights that IRF-supported dialogues enhance students' logical thinking and engagement, underscoring the value of game-based, hands-on activities in promoting meaningful mathematical learning and active classroom participation.

Keywords: Mathematics Grounding Activities (MGA), classroom interaction, decimal concepts, Conversation Analysis (CA)

Introduction

Taiwanese students have consistently demonstrated strengths in mathematical knowledge and skills in international assessments such as TIMSS and PISA. However, they face affective challenges, such as a lack of interest, low confidence, and mathematics anxiety (Ministry of Education, 2022). According to the TIMSS 2019 report, only 46% of Taiwanese students reported enjoying mathematics (Ministry of Education, 2022). To address this issue, educators and researchers have promoted various strategies, including curriculum reform, innovative instructional approaches, and technology-assisted learning. Approaches utilizing concrete real-life contexts (Hunter & Anthony, 2003) or game-based learning have been advocated to help students develop mathematical concepts.

Since 2014, the Ministry of Education has promoted Mathematics Grounding Activities (MGA), an instructional approach that integrates gameplay and guided hands-on activities to construct mathematical understanding. MGA uses manipulative and strategic games to guide students through concrete experiences, facilitating the cognitive processes of assimilation and accommodation. For teachers, MGA not only triggers student curiosity but also provides tangible experiences for students to progressively link gameplay to mathematical concepts, encouraging exploration and mathematical problem-solving.

Decimal concepts form an essential foundation for mathematics learning, frequently applied in daily life and serving as a basis for advanced topics such as ratios, fractions, and algebra. However, students often struggle with understanding place value, comparing magnitudes, and performing operations involving decimals. Common misconceptions include judging 0.5 as smaller than 0.45 (Lin, 2021), reflecting typical conceptual errors (Resnick et al., 1989; Steinle & Stacey, 2004).

Thus, exploring effective instructional designs to enhance decimal learning is a pressing issue. Integrating MGA into classroom instruction can boost students' motivation, cognitive engagement, and flexible thinking, while deepening abstract conceptual understanding through concrete experiences (Abdul Jabbar & Felicia, 2015; Moon & Ke, 2020). Research has indicated that game-based learning fosters cognitive and affective engagement, enriching students' learning experiences (Liang et al., 2014; Lin et al., 2018).

Nevertheless, few studies have systematically examined classroom interaction processes during MGA activities, particularly at the micro-level linking teacher–student dialogues to conceptual development. Therefore, this study uses conversation analysis to explore how interaction structures support learning processes in a case study of decimal-related MGA lessons in an elementary school context. The research questions guiding this study are:

(1) What are the characteristics of teacher–student verbal interactions during MGA lessons?
(2) What teaching features emerge when teachers adopt the IRE (Initiation–Response–Evaluation) structure?
(3) What teaching features emerge when teachers employ the IRF (Initiation–Response–Feedback) structure?

Literature Review

Mathematics Grounding Activities and the Decimal Decomposition Game

Mathematics Grounding Activities (MGA) focus on “building foundational mathematical knowledge” through structured game-based learning. Taiwanese mathematics education researchers developed MGA drawing upon Piaget's (1962) concepts of “assimilation” and “accommodation,” combined with Dienes' (1973) theory of the “six stages of learning mathematics,” advocating for knowledge construction through play (Lin & Hsieh, 2014). MGA modules align with mathematics concepts across primary and secondary school curricula.

The “Decimal Decomposition Game” is one such MGA module designed for elementary students. It emphasizes hands-on activities and games to help students understand decimal place value, the base-10 structure, and operations of addition and subtraction, while cultivating a sense of quantity and number sense. The instructional design highlights three core principles: First, introducing decimal concepts through fractional contexts to help students understand the meaning of 0.1 and 0.01, framing decimals as an extension of the integer system; Second, utilizing concrete manipulatives such as cards, rods, and hundred boards to visually and physically reinforce abstract concepts; Third, incorporating game elements like dice rolling and puzzle assembly to enhance student engagement and motivation. Overall, the module guides students to build decimal concepts through “learning by doing” and “thinking through games,” thereby strengthening their mathematical foundational skills.

Decimal Concepts

Research over the past decade has highlighted common difficulties elementary students face in learning decimals, particularly the abstract nature of decimal notation and the invisibility of denominators, which often leads to misconceptions (Gorman, 2024). Decimal concepts are a core component of mathematical learning, closely tied to daily life and foundational to understanding ratio, proportion, and algebra. Children frequently encounter difficulties when attempting to grasp the abstract nature of decimals, leading to misunderstandings (Steinle & Stacey, 2004).

Common challenges include grasping place value, comparing magnitudes, and performing decimal operations (Resnick et al., 1989). Some students may become proficient in decimal calculations without developing a solid understanding of place value and relative magnitude, indicating an overemphasis on procedural fluency at the expense of conceptual understanding (Žakelj & Klančar, 2024). Without effective decimal conceptualization, students' future mathematics learning can be adversely affected. Thus, identifying appropriate instructional strategies to strengthen students' understanding and application of decimal concepts remains a crucial concern in mathematics education.

IRE and IRF Interaction Patterns

Classroom discourse analysis is a critical approach to understanding teaching and learning processes, with IRE/IRF frameworks recognized as valuable tools for uncovering how teachers and students co-construct knowledge through dialogue. Within classroom interaction research, the IRE (Initiation–Response–Evaluation) and IRF (Initiation–Response–Feedback) structures, first introduced by Mehan (1979) and Sinclair and Coulthard (1975), have been widely applied to analyze classroom language and teacher–student exchanges. These frameworks help researchers elucidate the pragmatic structures and functions of instructional discourse, shedding light on how different interaction patterns influence student engagement and conceptual development (Li & Lam, 2022).

In traditional mathematics teaching, IRE structures typically dominate teacher-led questioning and evaluation sequences. While IRE can assist in managing classroom flow and confirming student responses, it may also restrict opportunities for extended mathematical reasoning (Wood, 1998) or prematurely close interactions after initial student responses, limiting deeper conceptual exploration (Nystrand, 2006). By contrast, the IRF structure emphasizes elaborative feedback following student responses, encouraging further articulation, correction, and reflective thinking. Research by Mercer and Howe (2012) indicates that IRF patterns can significantly enhance students' language participation and error correction in mathematics classrooms. Moreover, IRF interactions promote richer mathematical understanding and communication skills (Boaler & Greeno, 2000; Chapin et al., 2009).

Therefore, analyzing IRE/IRF structures not only allows researchers to identify interaction styles and linguistic features but also deepens the understanding of students' learning behaviors and participation dynamics. Based on this context, the present study aims to clarify the application of interaction structures in mathematics classrooms and provide empirical evidence and practical recommendations for designing effective dialogue strategies to promote deeper learning.

Through this exploration, the study seeks to enhance the understanding of teacher interaction styles and student engagement dynamics via IRE/IRF analysis, contributing to the evidence base for future instructional design and practice.

Methodology

This study adopted a qualitative case study approach, focusing on detailed observations and analyses of teacher–student interactions across five mathematics lessons. Data were collected from natural classroom settings through video recordings and teaching records to reconstruct the authentic discourse features during Mathematics Grounding Activities (MGA) lessons.

Sample Selection

The study was conducted in a third-grade mathematics classroom at a public elementary school in Taipei, involving 28 mixed-ability students (16 boys and 12 girls, approximately nine years old). The instructor, referred to as Teacher Jane, is an experienced educator with over 20 years of teaching experience and eight years of involvement in the MGA project, demonstrating expertise in designing and implementing Mathematics Grounding Activities. Teacher Jane's familiarity with MGA modules ensured the smooth execution of lessons and provided a stable instructional setting for observation. Over two weeks (from March 24 to April 7, 2023), Teacher Jane conducted five lessons focusing on single-digit decimal concepts using MGA modules.

Teaching Activities

The lessons were adapted from the Kang Hsuan version of Taiwan's Grade 3 mathematics textbook, incorporating MGA activities. The unit was divided into four instructional activities: (1) Understanding single-digit decimals, (2) Recognizing tenths-place decimals, (3) Comparing decimal magnitudes, and (4) Addition and subtraction of decimals. Although the original "Decimal Decomposition Game" module was designed for two-digit decimals, Teacher Jane, with the consent of the module's developers, modified the content to suit the instruction of single-digit decimal concepts.

Data Collection

Data sources included video recordings (approximately 40 minutes per lesson) of the five MGA decimal lessons and semi-structured interviews with the teacher and selected students after class. All video recordings were transcribed verbatim for conversation analysis (CA).

Data Analysis

To systematically analyze classroom interactions, this study adopted coding procedures based on the dialogue structures proposed by Sinclair and Coulthard (1975) and Mehan (1979). An interaction coding scheme (Table 1) was developed to distinguish between IRE (Initiation–Response–Evaluation) and IRF (Initiation–Response–Feedback) patterns, with teacher utterances coded sentence-by-sentence and categorized accordingly.

The analysis process was as follows: First, the researcher and a second rater, both with backgrounds in mathematics education, independently coded the teacher's speech in the first lesson ("Understanding Single-Digit Decimals") according to the operational definitions provided in Table 1. Discrepancies between coders were resolved through discussion to reach consensus. Subsequently, the primary researcher completed the coding of the remaining lessons. Interrater reliability was assessed, yielding a Cohen's Kappa value of 0.77, indicating an acceptable level of agreement.

Table 1
Interaction Coding Scheme

Type	Definition	Example
I-R-E (Initiation–Response–Evaluation)	1. The teacher initiates a question or instruction. 2. The student responds. 3. The teacher evaluates the student's response, usually confirming correctness.	T: "So, what is two-tenths?" SS: "0.2" T: "Is one-tenth equal to 0.1?" SS: "Yes."
I-R-F (Initiation–Response–Feedback)	1. The teacher initiates a question or instruction. 2. The student responds. 3. The teacher provides elaborative feedback, prompts further thinking, or scaffolds reasoning rather than simply evaluating correctness.	T: "How do you know that each part is equal, S1?" S1: "Because there are lines to divide them." T: "Right, and how does that prove that each part is the same size? S2?" S2: "Because if you draw the lines, you can see there are exactly 10 parts."

Note. T = Teacher; Sx = Individual Student; SS = Students responding in unison.

Findings

This chapter presents the preliminary findings based on the conversation analysis (CA) of teacher–student interactions observed during the five MGA decimal lessons, focusing on interaction patterns and discourse features.

Interaction Patterns in the Classroom

Based on the video recordings of the five MGA lessons (L1–L5), teacher–student interactions were analyzed and categorized into two primary structures: IRE (Initiation–Response–Evaluation) and IRF (Initiation–Response–Feedback).

At the beginning of the lessons (L1), the teacher predominantly employed the IRE structure to rapidly assess students' basic understanding of decimal concepts, as illustrated in Table 2 (a).

Table 2
Example of IRE Interaction: Confirming Decimal Reading

Speaker	Dialogue
T	So, what is two-tenths?
SS	0.2
T	Is one-tenth equal to 0.1?
SS	Yes.

Note. T = Teacher; SS = Students responding in unison.

This type of interaction shows the teacher using direct questioning and evaluation to swiftly gauge students' prior knowledge. As the lessons progressed (L2–L3), the proportion of IRF structures increased, particularly during game-based activities. The teacher utilized follow-up questions to guide discussions on strategies, reasoning, and conceptual understanding, as illustrated in Table 3.

Table 3
Example of IRF Interaction: Confirming Equal Division

Speaker	Dialogue
T	How do you know that each part is equal, S1?
S1	Because there are lines drawn on it.
T	Right, and how do you know each part is really equal? S2, how do you know?
S2	Because when you draw it out, you can see exactly 10 parts.

Note. T = Teacher; Sx = Individual Student.

Observations from Lessons L4–L5 indicated a more frequent and in-depth use of the IRF structure, where the teacher engaged students in multi-turn dialogues that extended reasoning and conceptual elaboration. Overall, a trend emerged wherein the teacher gradually shifted from predominantly IRE interactions to more IRF-based interactions, demonstrating flexible adaptation of discourse strategies according to different instructional stages.

Characteristics of IRE Interactions

This study further analyzed the IRE (Initiation–Response–Evaluation) interactions observed across the five MGA lessons, identifying the following instructional functions and features: First, the IRE structure enabled the teacher to quickly verify students' foundational understanding during the early stages of instruction. Typically, the teacher posed closed questions, eliciting single correct answers, followed by immediate evaluative confirmation, as seen in Table 2. IRE structures frequently appeared in whole-class questioning or rapid checks with individual students, facilitating efficient monitoring of conceptual readiness and maintaining the instructional pace, as shown in Table 4.

Table 4
Example of IRE Interaction: Whole-Class Check

Speaker	Dialogue
T	Is writing 1.0 acceptable?
SS	Yes.
T	Is writing zero point ten acceptable?
SS	No.

Note. T = Teacher; SS = Students responding in unison.

From a longitudinal perspective, the highest frequency of IRE interactions occurred in the first lesson (L1), primarily for checking prior knowledge of decimal concepts. As the lessons progressed into game activities and strategic discussions (L2–L5), the frequency of IRE interactions declined, gradually being replaced by more exploratory IRF structures.

In summary, the IRE structure is characterized by its simplicity, directness, and efficiency, making it suitable for knowledge confirmation and error diagnosis. However, it provides limited opportunities for deeper reasoning or conceptual exploration and may not adequately support students' active mathematical understanding.

Characteristics of IRF Interactions

The study also analyzed the characteristics and effects of IRF (Initiation–Response–Feedback) interactions observed during the lessons: First, follow-up questioning (“Follow-up”) within the IRF structure emerged as a key strategy for extending students’ thinking. Through reflective prompts and requests for elaboration, the teacher fostered deeper conceptual understanding and enhanced students’ language articulation skills. An example from a discussion on ensuring equal division is shown in Table 5.

Table 5
Example of IRF Interaction 1: Ensuring Equal Division

Speaker	Dialogue
T	How do you know that each part is equal, S1?
S1	Because there are lines that you can draw.
T	Since there are lines, how do you know they divide it equally? S2?
S2	Because if you draw the lines, it splits into exactly 10 parts.

Note. T = Teacher; S = Individual Student.

Through such follow-up questioning, the teacher encouraged students to not only answer but also reason and support their explanations with observational evidence.

Second, the teacher employed diverse responsive strategies within the IRF structure, including further questioning, providing additional information, challenging students' reasoning, and offering extended examples. For instance, during an activity comparing decimal magnitudes, the teacher used sustained probing to deepen students' understanding of place value, as shown in Table 6.

Table 6
Example of IRF Interaction 2-Understanding the Importance of Place Value

Speaker	Dialogue
T	Why is it that if the whole number is larger, the tenths-place comparison doesn't matter? Like between 3.8 and 4.3—is 3 smaller than 4?
SS	Yes.
T	So, 4 must be larger! Then why don't we need to compare the tenths if the whole number is different? S1?
S1	Because ten tenths make one whole number.
T	Very good. Now, what if the tenths digit is 9? Could it be larger than the whole number? S2?
S2	No, because even 9 tenths would not exceed 10, so the whole number is still more important.

Note. T = Teacher; Sx = Individual Student.

Such interactions demonstrate how the teacher scaffolded layered reasoning and deepened students' understanding of the decimal place value system. Overall, the use of the IRF structure created an open and inquiry-oriented classroom atmosphere. Students were not only expected to answer questions but also to explain their reasoning, correct misconceptions, and construct new understandings. In the context of game-based learning, IRF interactions effectively supported students' self-directed exploration and knowledge construction.

Summary

Through conversation analysis (CA) of the five MGA decimal lessons (L1–L5), the following major findings were identified: First, the interaction pattern in the classroom showed a developmental shift from IRE (Initiation–Response–Evaluation) to IRF (Initiation–Response–Feedback) structures. In the early phase (L1), the teacher primarily used the IRE structure with closed-ended questions and immediate evaluation to quickly check students' grasp of fundamental decimal concepts. However, the depth of interaction in IRE was limited, often constrained to confirming correct answers rather than fostering higher-order reasoning.

As the lessons progressed into game-based activities and conceptual applications (L2–L5), the use of IRF structures increased. Through follow-up questioning, the teacher extended student thinking, promoted reasoning, and facilitated deeper conceptual elaboration. The use of IRF not only increased students' opportunities for verbal expression and strategy explanation but also nurtured a more open and inquiry-driven classroom environment.

Overall, the teacher demonstrated flexibility in adapting discourse strategies according to instructional phases and learning objectives, showcasing proficient language scaffolding techniques. The MGA modules, with their game-based learning design, further facilitated students' transition from passive responders to active explorers, enhancing both conceptual depth and engagement in decimal learning. The findings from this chapter provide empirical evidence on how teacher–student interactions can support conceptual development in mathematics classrooms and offer practical implications for optimizing classroom dialogue strategies.

Conclusions

Research Summary

This research adopted a qualitative case study methodology and applied conversation analysis (CA) to explore the dynamics of teacher–student discourse and its influence on third-grade students' understanding of decimal concepts during the implementation of Mathematics Grounding Activities (MGA). The key findings are summarized as follows:

First, the progression of classroom interaction patterns revealed a noticeable shift from the IRE (Initiation–Response–Evaluation) structure to the IRF (Initiation–Response–Feedback) structure. In the early stages of instruction, the teacher predominantly utilized IRE sequences to pose closed-ended questions followed by swift evaluations. This pattern proved efficient in gauging students' baseline comprehension of decimals and maintaining instructional momentum. Nevertheless, such exchanges often limited students' opportunities to engage in extended reasoning or construct their own understanding.

Second, as the lessons transitioned into phases emphasizing game-based exploration and strategic discussion, the teacher increasingly adopted the IRF pattern. Through probing follow-up questions and responsive feedback, students were encouraged to explain their strategies, examine alternative perspectives, and refine their conceptual frameworks. This shift fostered a more dialogic and inquiry-driven learning environment, supporting the development of students' mathematical language, logical reasoning, and deeper conceptual insight. Learners engaged through IRF interactions demonstrated greater autonomy in meaning-making and showed more active participation in collaborative problem-solving.

Third, the teacher's ability to adapt interactional strategies according to instructional intent and student response demonstrated pedagogical responsiveness and a keen awareness of students' evolving learning needs. The design of MGA—centered around game-based tasks and hands-on manipulation—further enriched the nature of classroom dialogue. These interactive elements supported students' transformation from passive recipients of information to active inquirers, facilitating the internalization of abstract decimal concepts through embodied and meaningful learning experiences.

In conclusion, this study highlights several implications for mathematics instruction: While IRE exchanges can be instrumental in assessing foundational knowledge, over-reliance on such patterns may constrain deeper student engagement. In contrast, the IRF structure offers a more generative space for learners to articulate reasoning, explore strategies, and consolidate conceptual understanding. Moreover, aligning flexible discourse strategies with interactive, game-based pedagogies can enhance student engagement, foster conceptual growth, and support the development of mathematical thinking in elementary classrooms. These findings provide practical insights for designing discourse-rich mathematics lessons and contribute empirical evidence to support the integration of inquiry-based teaching practices.

Practical Implications for Teaching

Based on the analysis of classroom interaction structures during Mathematics Grounding Activities (MGA) in a third-grade mathematics class, the following practical suggestions are proposed for instructional practices and classroom interaction design:

Appropriate Use of IRE Structures to Confirm Basic Conceptual Understanding

At the early stages of a lesson or when introducing new concepts, teachers can appropriately utilize the IRE (Initiation–Response–Evaluation) structure to conduct closed-ended questioning, allowing for a rapid assessment of students' foundational knowledge. Through efficient questioning and immediate evaluation, teachers can adjust instructional pacing to ensure that lesson progression is built on a solid conceptual foundation. However, it is recommended that teachers avoid relying excessively on unidirectional questioning and monitor student engagement to prevent interactions from becoming superficial.

Active Development of IRF Structures to Foster Deep Thinking and Conceptual Construction

As instruction moves into hands-on activities, strategic discussions, or conceptual application stages, teachers should consciously shift toward the IRF (Initiation–Response–Feedback) interaction structure. By posing follow-up questions, teachers can guide students in reasoning, explanation, and conceptual elaboration. Designing inquiry-based questions, encouraging multiple rounds of student responses, and offering supplementary information or challenging prompts can help students connect concrete experiences with abstract mathematical concepts. This approach supports the development of mathematical language, critical thinking, and autonomous learning skills.

Integrating Game-Based Learning to Promote Natural and Rich Interactions

The findings suggest that incorporating game elements into mathematics instruction, such as the MGA Decimal Decomposition Game, can effectively enhance students' learning motivation and interaction engagement. Teachers are encouraged to design activities that combine concrete manipulation, rule-based gameplay, and challenging scenarios, creating opportunities for natural discussion and deeper thinking. Through active participation, students can progressively construct mathematical concepts in an engaging and meaningful context.

Strengthening Teacher Training on Interaction Strategies to Improve Classroom Quality

The study also highlights that teachers' sensitivity to interaction structures and their strategic adjustments play a crucial role in promoting student learning. It is recommended that schools and educational institutions incorporate training on interaction structures (e.g., IRE/IRF analysis) into professional development programs. Such training can help teachers understand the impact of different interaction models on learning processes and cultivate their ability to flexibly apply questioning and feedback strategies, thereby enriching classroom discourse and enhancing instructional effectiveness.

Research Limitations and Future Directions

Research Limitations

This study focused on a single case in a third-grade public elementary school classroom in Taipei, examining the characteristics and progression of teacher–student interaction structures during Mathematics Grounding Activities (MGA). While the qualitative data collected offer rich and in-depth insights, several limitations must be acknowledged:

First, the research sample was limited to one class and one teacher, with specific student demographics. As such, the generalizability of the findings to other regions, grade levels, or instructional cultures remains limited.

Second, the instructional content was confined to decimal concepts and based on a specific MGA module—the Decimal Decomposition Game. Therefore, the observed interaction structures and student behaviors may be influenced by the unique nature of the mathematical content and instructional design, and may not be directly transferable to other mathematical topics such as geometry or algebraic reasoning.

Third, the study adopted qualitative conversation analysis as its primary method. While this approach emphasizes the detailed portrayal and interpretation of interactive processes, it did not incorporate quantitative measures of student learning outcomes (e.g., pre- and post-tests). Consequently, the direct correlation between interaction structures and student achievement should be interpreted with caution.

Future Research Directions

In light of the above limitations, several directions for future research are proposed:

First, future studies could expand the sample to include diverse school settings, grade levels, and teacher backgrounds, enabling cross-case comparisons that reveal both commonalities and differences in teacher–student interactions. Such research would enhance the generalizability and practical relevance of the findings.

Second, researchers may explore interaction structures across various mathematical domains—such as fractions, ratios, or algebraic reasoning—as well as in different instructional formats, including inquiry-based or project-based learning. This could clarify how content-specific features influence interaction patterns and discourse strategies.

Third, future studies are encouraged to integrate both qualitative and quantitative methods. For example, analyzing interaction frequency, administering conceptual assessments, and tracking learning outcomes can provide more comprehensive evidence of how specific interaction structures impact reasoning skills, conceptual understanding, and achievement.

Finally, further research could investigate the development of teachers' interactive strategies over time, particularly how they transition from IRE to IRF patterns. This line of inquiry could shed light on the relationship between teacher professional development and long-term student learning outcomes, highlighting the role of teacher discourse expertise in fostering meaningful classroom interactions.

Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

In the preparation of this manuscript, generative artificial intelligence (AI) tools and AI-assisted technologies were employed to support language refinement, improve readability, and ensure clarity of expression. All intellectual contributions, including the research design, data interpretation, and conceptual development, were made independently by the author.

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