The Effects of Instructional Scaffolding in Students' Conceptual Understanding, Proving Skills, Attitudes, and Perceptions Towards Direct Proofs of Integers

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Abstract
Students find mathematical proving a challenging task and often perform poorly in proving despite its importance in developing students' critical thinking and reasoning skills. The purpose of the study is to determine if instructional scaffolding can improve students' conceptual understanding, proving skills, and attitudes and perceptions towards proving. The instructional scaffolding strategies used were providing hints, examples, and questions for the students to develop ideas, showing how to perform a task, and letting the students provide feedback, ask questions, and show support to their fellow peers. Foundations or preliminaries prior to proving integers were also tackled first. The study used mixed methods quasi-experimental design where twenty-six Grade 11 STEM students participated in surveys involving attitudes and perceptions on proving, an odd/even concept test, and a proving test. Students generally had positive attitudes and perceptions towards proving even prior to the intervention and these further improved due to the intervention as the t-test result shows a significant improvement. A rubric was used to score students' proofs. Nine students were able to progress from the beginning level to developing, approaching proficiency, proficiency, and advanced levels in their proving skills although fifteen of them retained their levels. Students' difficulties in proving were due to improper representations of the integers as arbitrary values and errors in performing operations in simplifying algebraic expressions. Nonetheless, it can be deduced that instructional scaffolding is effective in improving students' conceptual understanding of integers and proving skills, and attitudes and perceptions towards proving.

Keywords: Instructional Scaffolding, Conceptual Understanding, Proving Skills, Direct Proofs of Integers

Introduction

Proving is regarded as an important activity in mathematics by many if not all mathematics educators and mathematicians (Baştürk, 2010; Ersen, 2016). The National Council of Teachers of Mathematics (NCTM, 2000) espoused the integration of proving into the curriculum since it improves mathematical thinking and reasoning skills across age levels. Thus, it comes as no surprise that concepts involving logic including direct proofs are now included as competency skills required of Senior High School students to learn in the Philippines (Department of Education, 2016).

Leddy (2001, p. 13) defined proof as “a reasoned argument from acceptable truths.”. De Villiers (1999) described proving to involve exploration, analysis, and creating new results which plays an important role in mathematics knowledge generation through deduction. In proving statements, students could use various strategies such as using examples to perform illustrations, disproving false statements, and using definitions, properties, and theorems (Ersen, 2016).

However, there are learners who struggle in proving (Weber, 2001). Weber (2001) identified students’ difficulties in starting a proof, the lack of mathematics concepts and how to use these concepts in the proof. Due to its abstract nature, proving has always been considered a challenging skill to learn due to its complex processes which often than not, teachers are avoiding to teach (Güler, 2016; Varghese, 2017).

Varghese (2017) expressed that effective classroom mathematics teaching can bring about any desired improvement in students’ mathematics education by providing opportunities for students to interact, propose mathematical ideas and conjectures, evaluate their thinking, and develop reasoning skills. Instructional scaffolding has been used in teaching mathematics to students in order to develop certain mathematical skills, that can serve as a factor in their achievement. Instructional scaffolding strategies involve collaboration between the teachers and students when learning a certain lesson. Students initially need support from the teacher, but through gradual release of responsibility, they have to do tasks individually once the purpose of the instructional scaffolding strategies has been achieved. Ihechukwu (2020) have shown instructional scaffolding strategies to be effective in developing students’ critical thinking skills in mathematics, especially in problem-solving. Therefore, this study posits that instructional scaffolding strategies can also be applied in teaching students in proving.

The purpose of the study is to determine the effects of instructional scaffolding to students’ conceptual understanding, attitudes and perceptions towards direct proof on integers. Specifically, it sought to answer the following research questions:

1. Is there a significant difference in the attitudes of students on proving theorems on integers before and after instruction?
2. Is there a significant difference in the perceptions of students on proving theorems on integers before and after instruction?
3. Is there a significant difference in the levels of students on proving skills before and after instruction?
4. What was the students’ conceptual understanding of direct proving of integers?

Proving Skills

Tall (1999) states mathematical proof is following a logical way to explain why and how the conjecture has been reached. Varghese (2017) explains that proving is a complex task because it covers a wide range of student competencies such as identifying assumptions, identifying relevant properties and structures which may be definition of terms, postulates, corollaries or theorems, and organizing these in logical arguments. In the study of Güler (2016), academicians experience difficulties teaching proof because the proofs mostly focus on the nature of mathematics being incremental, has an abstract structure and uses symbolic representations. Because of these difficulties, Varghese (2017) suggests that students may initially be exposed to proofs using illustrative examples for its explanatory function, but should gradually progress to communicating mathematical ideas using symbolic representations using arbitrary values as a mathematical language of proof. Academicians in the study of Güler (2016) further remarked that the lack of understanding of the logic of proof methods might cause students to mistakenly think that proofs can be solved using trial and error method and by illustrating examples. They suggest students understand the different types of proofs and to internalize logical proof methods representation.

Direct Proof is one of the most fundamental proving strategies to be studied along with proofs by contradiction, proofs by contraposition, and proofs by induction. According to Doruk (2019), the different ways to prove consist of the following: using counterexamples, providing mathematical induction, contradicting statements, direct proving, and indirect proving. In direct proving, the hypothesis is usually treated as the given and then gaps are filled in to reach the conclusion of the conditional statement. This entails that students first understand (1) the parts of a conditional statements, the hypothesis or premise and the conclusion; (2) the different forms of a conditional statement, converse, inverse, and contrapositive; (3) discerning which of the forms is logically equivalent to the conditional statement; (4) translating a statement in the conditional statement form; and (5) understanding mathematical symbols such as “$\in$” (“is an element of”) and “$\Longrightarrow$” (“if… then...) (Laili & Siswono, 2020).

Conceptual Understanding of Proof

Stavrou (2014) indicated that there are misconceptions about direct proofs. One of these is students use specific examples instead of applying properties, axioms, definitions, and theorems in proving various statements. He cited students using numbers instead of arbitrary constants in proving a number theory-related statement as one of the examples in situations in proving. Aside from this error, students use the conclusion as a basis of assumption in proving the conclusion of the given statement. Students also do not use both conditions of a biconditional statement in proving biconditional statements. Lastly, students also lacked understanding and analysis of the definitions they would be using in proving statements. There were instances when students were able to complete proving statements although there were some mistakes in parts of their proofs. This implied that understanding definitions, properties, axioms, and theorems serve as one of the most fundamental and important steps in proving statements (Sari, et. al., 2018).

In ruling out the misconceptions in proofs, Buchbinder and McCrone (2020a) devised teaching strategies using their MKT-P, also known as Mathematical Knowledge for Teaching, Framework. In this framework, they used KLAP (Knowledge of the Logical Aspects of Proof) in addressing their misconceptions about basic terminologies, definitions, and theorems in proving. This aimed at teaching students in using proper mathematical vocabulary and notations in proving statements since these serve as the first steps in proving. This also targeted correcting their logical reasoning skills, which are required in proving. It was also suggested that students be encouraged to develop their skills in explicating conditional statements, using mathematical language, and reasoning logically (Buchbinder & McCrone, 2020b).

Attitudes and Perceptions

Attitudes in mathematics in general is focused on the following aspects: liking, value, and confidence. In the liking aspect, attitudes were based on how the students like and show their interest in mathematics, in general wherein proofs are integrated in the lessons. These highlighted enjoyment as one of the determinants of liking mathematics. In the value aspect, attitudes indicated the need to learn proving and problem-solving in mathematics and the purposes of learning mathematics in real-life situations and everyday life. These also emphasized how important mathematics concepts, including proving, are. The confidence aspect is composed of self-esteem and independence in doing mathematical problems and even proving mathematical statements (Giannoulas & Stampoltzis, 2021; Khine et al., 2015). Aside from liking, value, and confidence, Laili & Siswono (2021) considered motivation as one of the indicators of attitudes in proofs. Motivation involved the willingness to prove statements independently. In proving mathematical statements, attitudes focused on interest, enjoyment, and appreciation towards proofs and their relevance. It was found that proving is important not just in learning mathematics and its concepts, but also in its application in everyday life. Additionally, proving builds critical thinking and other higher-order thinking skills in the students and enhances confidence in mathematical concepts (Lee, 2022). Proving has also been perceived by teachers as a way of communicating in mathematics and a guide in providing logical and valid explanations (Lesseig et al., 2018; Ersen, 2016).

On the other hand, based on the study of Ersen (2016), teachers perceive students have to memorize theorems, properties, and definitions and consider it as a requirement in understanding mathematics in proving, which can cause some students to have negative perceptions that proving is difficult, time-consuming, and unnecessary.

Methods

The research study utilized a quasi-experimental research design with mixed methods approach to analyzing data. Quantitative data constitutes students' responses from survey questionnaires on attitudes, perceptions, and scores from students' proving tests. Qualitative data consist of students' solutions and answers. There were twenty-six (26) Grade 11 students taking up Science, Technology, Engineering, and Mathematics (STEM), from a private school located in Quezon City, who participated in the study. They were chosen through convenience sampling method. The selection was based on the availability of their schedules and willingness to participate.

The following research instruments were administered: surveys focusing on attitudes and perceptions on proofs, diagnostic tests, comprehensive tests, and instructional scaffolding worksheets. Permission to conduct the study were secured from the school principal, class adviser, and the students.

First, the participants took the diagnostic test to measure their prior knowledge on related concepts in proofs and answered the surveys on attitudes and perceptions of proof before the intervention. Then, the researcher implemented a series of scaffolding interventions focusing on the fundamentals of direct proofs. The instructional scaffolding strategies used were providing hints, examples, and questions for the students to develop ideas, showing how to perform a task, and letting the students provide feedback, ask questions, and show support to their fellow peers. Foundations or preliminaries prior to proving integers were also tackled first. The researcher conducted the comprehensive test and the survey questions on attitudes and perceptions of proofs after the intervention. A rubric was used to score students' proofs. Students' scores were used to determine their proving skills level. The paired t-test was used to determine if there is improvement in students' attitudes and perceptions on proof brought by the intervention. Students' solutions and answers were analyzed using narrative analysis in order to draw out students' difficulties in proving. There were 4 class sessions allotted for the intervention and each intervention session consisted of 60 minutes.

Findings

Attitudes and Perceptions of Proof

Table 1: Comparative Analysis of the Attitudes Towards Proofs
Before and After Instructional Scaffolding

	Pre-Intervention Attitudes	Post-Intervention Attitudes
Mean	3.7276	4.2115
Number of Participants	26	26
Standard Deviation	0.4500	0.5428
Standard Error Mean	0.0883	0.1064
Pairwise Comparison		
Mean Difference	0.4839	
Standard Deviation	0.6663	
Standard Error Mean	0.1307	
95% Confidence Interval of the Mean Difference	Lower 0.2149 Upper 0.7531	
t-value	3.704	
Degrees of freedom	25	
p-value (2-tailed)	0.001	

Based on Table 1, the students already showed positive attitudes during the pre-intervention with a mean of 3.7276. After the interventions, their attitudes improved with a difference of 0.4839, leading their attitudes to have a mean of 4.2115, which implies that their attitudes were very positive. The level of significance for this data was 0.05. The computed t-statistic was 3.704, which is greater than the critical value for a 2-tailed hypothesis, 2.060. The p-value, 0.001 is less than the significant level, a = 0.05. With these, the null hypothesis was rejected and we conclude that there is a significant difference between the attitudes of the students toward proof before and after the instructional scaffolding interventions.

Table 2: Comparative Analysis of the Perceptions Towards Proofs Before and After Instructional Scaffolding

	Pre-Intervention Perceptions	Post-Intervention Perceptions
Mean	3.6731	3.8237
Number of Participants	26	26
Standard Deviation	0.3681	0.4333
Standard Error Mean	0.7219	0.0848
Pairwise Comparison		
Mean Difference	0.1506	
Standard Deviation	0.3652	
Standard Error Mean	0.0716	
95% Confidence Interval of the Mean Difference	Lower Upper	0.0032 0.2981
t-value	2.104	
Degrees of freedom	25	
p-value (2-tailed)	0.046	

Based on Table 2, the students showed positive perceptions during the pre-intervention with a mean of 3.6731. After the interventions, their attitudes improved with a difference of 0.1506, leading their attitudes to have a mean of 3.8237. The computed t-statistic is 2.104, which is greater than the critical value for a 2-tailed hypothesis, 2.060. The p-value, 0.046 is less than the significant level, a = 0.05. With these, the null hypothesis was rejected. Thus, there is a significant difference between the perceptions of the students toward proof before and after the instructional scaffolding interventions.

Proving Skills

Table 3: Frequency Distribution of Students based on their Proving Skills Levels Before and After the Interventions

Level of Proving Skills	**Advanced**	**Proficiency**	**Approaching Proficiency**	**Developing**	**Beginning**	**Total**
Before the Intervention	0	1	0	1	24	**26**
After the Intervention	6	2	1	2	15	**26**

In can be gleaned from Table 3 that most of the students (24) were at the beginning level in proving direct proofs, while only one of the students received a developing level and another received an approaching proficiency level before the instructional scaffolding interventions. After the interventions, there are less students in the beginning proficiency level and there are more students who reached higher proficiency levels. To be more specific, nine students were able to progress from the beginning level to developing, approaching proficiency, proficiency, and advanced levels in their proving skills. Nonetheless, fifteen (15) out of the twenty-four (24) students remained in the beginning proficiency level.

Conceptual Understanding of Proof

In the diagnostic test, the areas of conceptual understanding focused on the following: identifying the hypothesis and conclusion of a conditional statement and providing illustrations (Stavrou, 2014; Sari et. al., 2018). Here are samples of the answers given by the students in the diagnostic test:

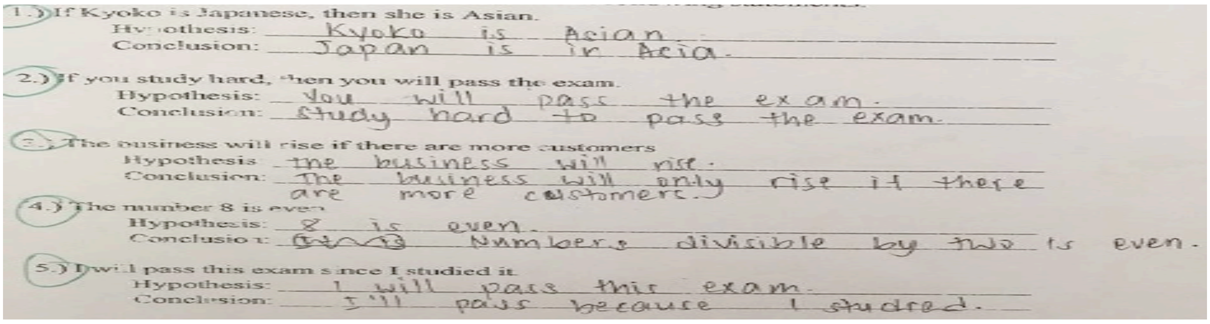


Figure 1: Answers of the Student 1 in Identifying the Hypothesis and Conclusion

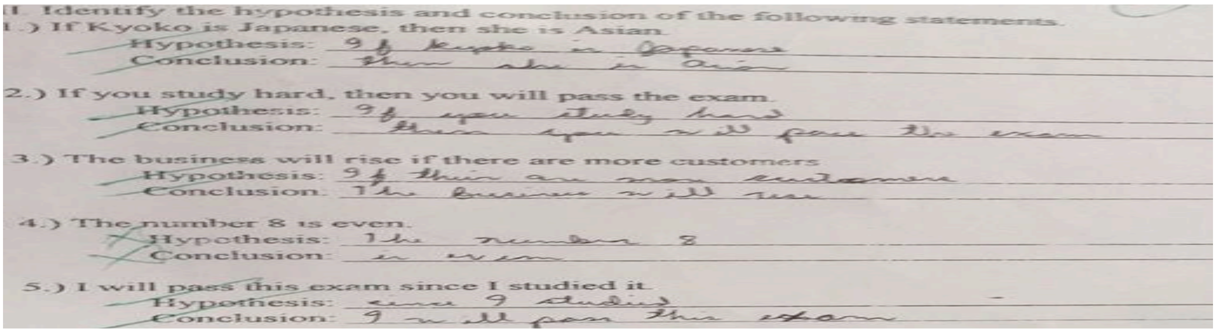


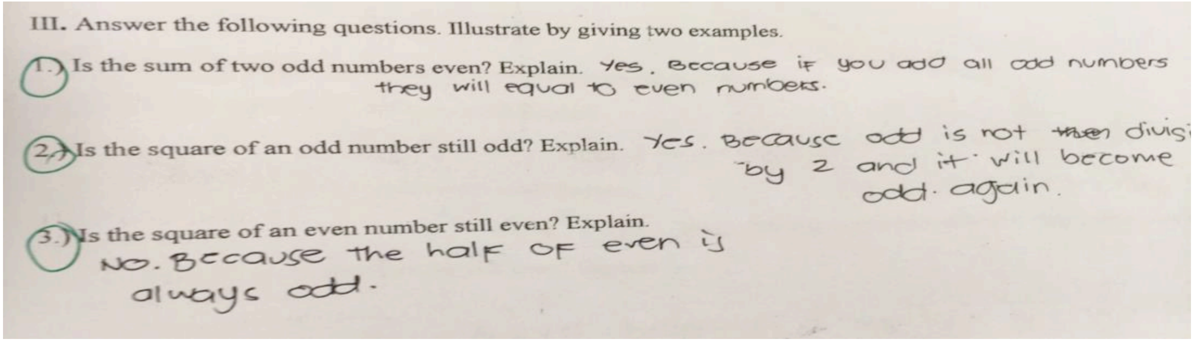
Figure 2: Answers of Student 2 in Identifying the Hypothesis and Conclusion

Based on Figure 1, Student 1 was not able to identify the hypothesis and conclusion properly since he was not able to understand what the hypothesis and conclusion are. While in Figure 2, Student 2 was able to identify the hypothesis and conclusion of a conditional statement.

Table 4: Students' Scores in Identifying Hypothesis and Conclusion of Given Conditional Statements

Score (Out of 10 points)	Number of Students (N=26)
9	2
8	5
7	1
6	3
4	2
3	1
2	2
0	10

The results (see Table 4) show that there are still many students (10 out of 26) who were unable to distinguish the two different parts of a conditional statement and very few, only 2 students got a score of 9 points. No student got a perfect score.



III. Answer the following questions. Illustrate by giving two examples.

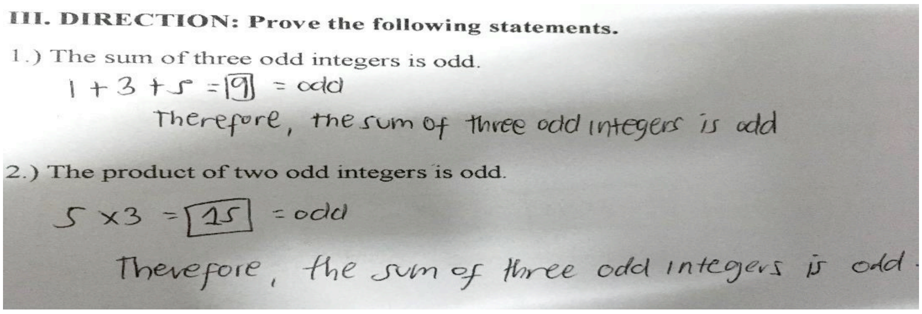
1.) Is the sum of two odd numbers even? Explain. Yes, Because if you add all odd numbers they will equal to even numbers.

2.) Is the square of an odd number still odd? Explain. Yes. Because odd is not ~~then~~ divisi by 2 and it will become odd. again.

3.) Is the square of an even number still even? Explain.
No. Because the half of even is always odd.

Figure 3: Answers of Student 3 in Providing Illustrations

Another set of items were given to test if students could explore and observe patterns by giving examples in order to formulate conjectures prior to its direct proving skills. Figure 3 shows Student 3 failed to illustrate by giving numerical examples to explain their answers. Instead, they reiterated the statement and gave a verbal explanation without any computational basis. Figure 4 shows the correct answer by Student 4.



III. DIRECTION: Prove the following statements.

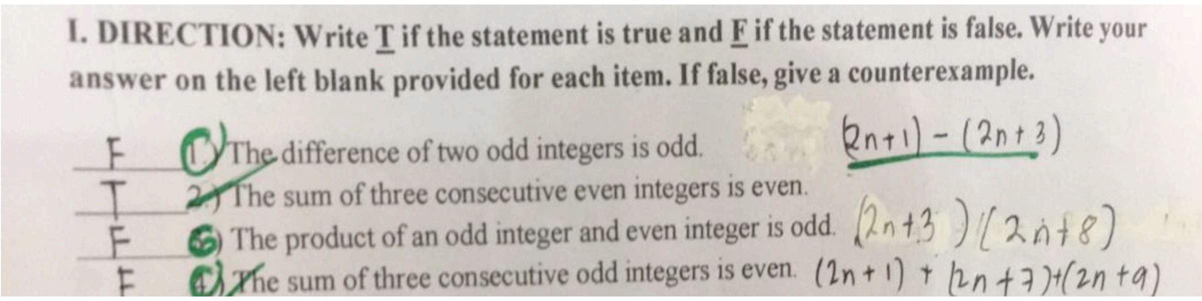
1.) The sum of three odd integers is odd.
$1 + 3 + 5 = \boxed{9}$ = odd
Therefore, the sum of three odd integers is odd

2.) The product of two odd integers is odd.
$5 \times 3 = \boxed{15}$ = odd
Therefore, the sum of three odd integers is odd.

Figure 4: Answers of Student 4 in Proving Statements

The figures presented so far illustrate students' prior knowledge before the implementation of instructional scaffolding strategies.

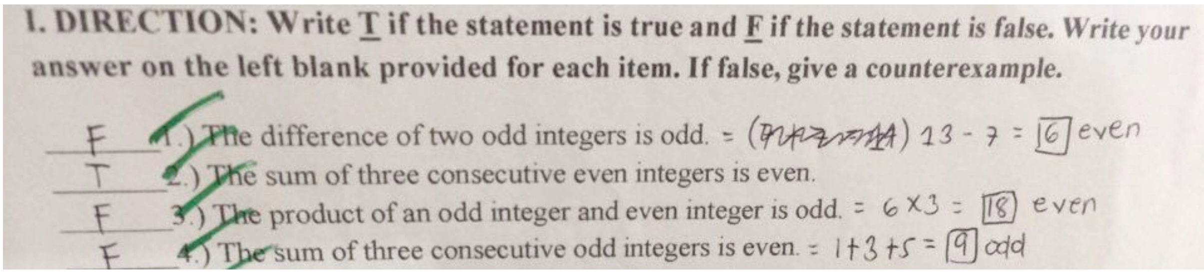
In the comprehensive test, the key areas for conceptual understanding focused on the following: representing integers using arbitrary constants, proving counterexamples, and direct proving. Here are examples of students' answers in the comprehensive test:



I. DIRECTION: Write T if the statement is true and F if the statement is false. Write your answer on the left blank provided for each item. If false, give a counterexample.

F 1.) The difference of two odd integers is odd. $(2n+1) - (2n+3)$
T 2.) The sum of three consecutive even integers is even.
F 3.) The product of an odd integer and even integer is odd. $(2n+3)(2n+8)$
F 4.) The sum of three consecutive odd integers is even. $(2n+1) + (2n+7) + (2n+9)$

Figure 5: Answers of Student 5 in Providing a Counterexample


I. DIRECTION: Write T if the statement is true and F if the statement is false. Write your answer on the left blank provided for each item. If false, give a counterexample.

F 1.) The difference of two odd integers is odd. = ( ) 13 - 7 = $\boxed{6}$ even

T 2.) The sum of three consecutive even integers is even.

F 3.) The product of an odd integer and even integer is odd. = 6 x 3 = $\boxed{18}$ even

F 4.) The sum of three consecutive odd integers is even. = 1+3+5 = $\boxed{9}$ odd

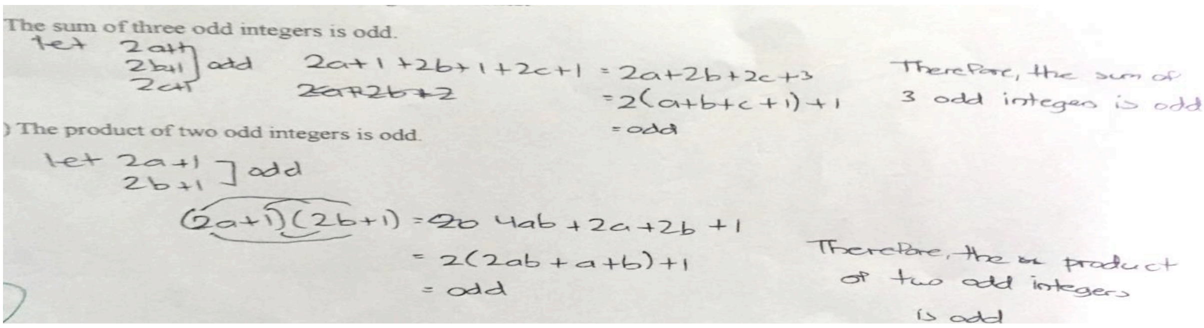
Figure 6: Answers of Student 6 in Providing a Counterexample

As can be seen in Figure 5, Student 5 used arbitrary constants in providing counterexamples. This means the student has not understood what a counterexample is. While in Figure 6, Student 6 used numbers in providing counterexamples. Student 6 was able to disprove statements by using numbers as means of counterexamples.

Table 5: Students' Scores in True or False and Counterexample Items

Score (Out of 5 points)	Number of Students (N=26)
4	2
3	6
2	4
1	14

In Table 5, many students (14 out of 26) scored only one point, and few students (only 2) scored 4 points. No student got a perfect score.


The sum of three odd integers is odd.

let 2a+1, 2b+1, 2c+1] odd

2a+1 + 2b+1 + 2c+1 = 2a+2b+2c+3

= 2(a+b+c+1)+1

= odd

Therefore, the sum of 3 odd integers is odd

) The product of two odd integers is odd.

let 2a+1, 2b+1] odd

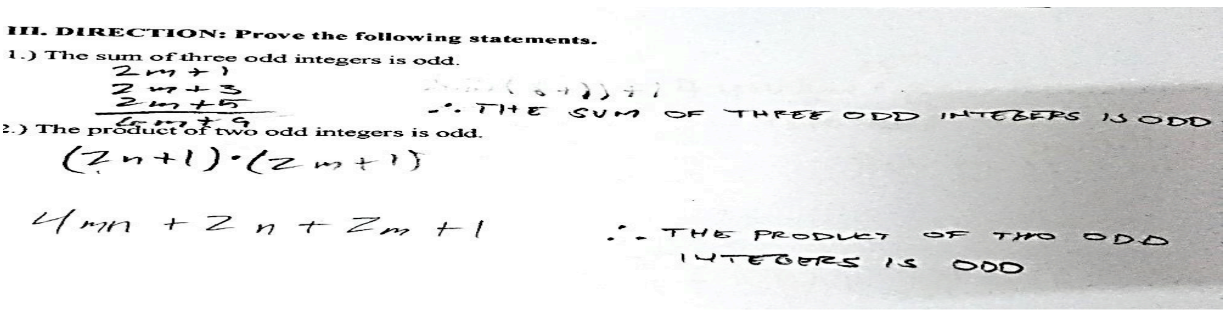
(2a+1)(2b+1) = 4ab + 2a + 2b + 1

= 2(2ab + a + b) + 1

= odd

Therefore, the product of two odd integers is odd

Figure 7: Answers of Student 7 in Proving Statements


III. DIRECTION: Prove the following statements.

1.) The sum of three odd integers is odd.

2m+1

2m+3

2m+5

6m+9

∴ THE SUM OF THREE ODD INTEGERS IS ODD

2.) The product of two odd integers is odd.

(2n+1)·(2m+1)

4mn + 2n + 2m + 1

∴ THE PRODUCT OF TWO ODD INTEGERS IS ODD

Figure 8: Answers of Student 8 in Proving Statements

Students 7 and 8 were able to represent arbitrary constants as given for odd and even integers. Moreover, both of them were able to use different variables in representing different odd integers. However, only Student 7 was able to represent the product as an arbitrary form of an odd integer, such that "2(a + 2ab + b) + 1, where a and b are integers". Student 8 failed to show that the product is an odd integer.

Discussion

As students go to higher levels in mathematics, critical thinking is a necessary skill in problem-solving and proving. Since they are at higher levels in mathematics, they need to explore other skills, particularly in proving. Based on the results of the diagnostic test, which was before the implementation of the instructional scaffolding strategies, students lack the preliminary skills, such as using arbitrary constants, in proving direct proofs. Because of the misconceptions they had, they needed to undergo a series of instructional scaffolding interventions. During the interventions, they were given worksheets with guided examples and were allowed to collaborate with their peers for them to develop ample prerequisite knowledge and skills in proving direct proofs involving integers. With the help of instructional scaffolding strategies, the students developed some skills in proving direct proofs regarding integers. They were able to represent integers using arbitrary constants and provide counterexamples to disprove false statements involving integers. Many improved in representing integers using arbitrary constants, which is one of the first steps in proving direct proofs involving integers. Observing how the frequency count of students from different proficiency levels, the instructional scaffolding proved to be effective. Consistent to students' performance, t-test results show students' attitudes and perceptions significantly improved after instructional scaffolding intervention.

Conclusion and Recommendation

This study focuses on how the students perceive and behave towards direct proofs and prove and understand direct proofs. Proofs are one of the lessons students should learn in order to understand mathematics. These are being studied by Grade 8 students only in geometry. However, direct proofs involving integers are being studied in Grade 11 General Mathematics under Logic. However, students exhibit misconceptions about proving direct proof. Based on the findings, the respondents had problems identifying the hypothesis and conclusion of conditional statements, representing arbitrary constants in terms of odd and even integers, and stating the given in proving direct proofs involving integers. In the diagnostic test, many struggled with their conceptual understanding of the basic concepts prior to direct proving. While, in the comprehensive test that was administered after the interventions, there were still students who struggled in proving albeit some students have shown improvement in skills in proving integers. All these show that instructional scaffolding has helped the students in learning how to prove integers directly.

This study promotes instructional scaffolding interventions in proving direct proofs involving integers. It is recommended that teachers be more intentional in utilizing instructional scaffolding in proving so the students can be able to maximize their potential in proving.

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