

Needs Assessment for a Web-Based AI-Constructivist-Gamification Learning Environment to Enhance Geometric Reasoning in Northeast Thai Primary Students

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Abstract

This study assessed the needs for developing a web-based learning environment integrating artificial intelligence (AI), constructivism, and gamification to enhance geometric reasoning skills among primary school students in Northeast Thailand. A mixed-methods approach was employed, involving 200 students, 20 mathematics teachers, 20 curriculum experts, and 30 parents, selected through stratified random sampling to ensure representativeness. Data were collected using interviews, questionnaires, observations, and focus group discussions. Descriptive statistics revealed that 65% of students struggled with geometric reasoning, while 75% of teachers faced challenges in delivering engaging instruction. A one-way ANOVA showed significant differences in the perceived needs among the four groups [$F(3, 266) = 9.78$, $p < 0.001$], with students having the lowest mean score ($M = 4.12$, $SD = 0.78$). The results indicated a strong need (85% agreement) for an innovative web-based learning environment that incorporates AI, constructivism, and gamification principles to support students' geometric reasoning development. These findings provide valuable insights for designing a learning prototype tailored to the needs of primary school students in Northeast Thailand.

Keywords: needs assessment, web-based learning environment, artificial intelligence, constructivism, gamification

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Introduction

Geometric reasoning, as conceptualized by Battista et al. (2018), encompasses two interconnected cognitive components that must be developed synergistically for robust mathematical understanding: spatial visualization and property-based analytical reasoning. Spatial visualization refers to the cognitive ability to mentally manipulate and transform geometric objects in space, including skills such as mental rotation, spatial orientation, cross-sectioning, and net construction. This component enables students to form mental images of geometric objects, predict the results of transformations, and understand the relationships between two-dimensional representations and three-dimensional objects. Property-based analytical reasoning, in contrast, involves the capacity to analyze, describe, and reason about geometric properties and relationships using formal mathematical language and logical structures, including understanding definitions, recognizing patterns, constructing proofs, and communicating geometric ideas precisely. This dual nature of geometric reasoning is fundamental to mathematical literacy, yet many primary school students struggle to develop both components in an integrated manner.

Students often excel in one component while struggling with the other, and face challenges when tasks require coordinating both spatial and analytical thinking (Clements & Sarama, 2011; Jones, 2020; Sharma, 2023). Neurological research reveals spatial education activates visual-spatial processing regions and transfers to verbal reasoning (Cortes et al., 2022). Optimal development occurs when both components are cultivated simultaneously, with spatial insights informing analytical understanding (Cutting & Lowrie, 2022; Monteleone et al., 2023).

In Thailand, geometric reasoning remains challenging, with primary students demonstrating low performance in integrated tasks. Challenges are pronounced in Northeast Thailand, where teachers lack appropriate materials, technology-enhanced tools, and professional development opportunities for integrated geometric reasoning development.

Educational technology offers promising solutions. Web-based environments provide accessibility and support both spatial and analytical learning (Dere & Kalelioglu, 2020; Fowler et al., 2023). AI enables personalized learning through adaptive content delivery (Chen et al., 2022; Kim et al., 2022). Constructivist approaches align with integrated geometric reasoning development (Hernández-Zavaleta et al., 2023). Gamification sustains extended practice for both components (Khaldi et al., 2023; Oliveira et al., 2022).

Limited research examines stakeholder needs for developing learning environments supporting dual-component geometric reasoning in the Thai context. This study conducts a systematic needs assessment for a web-based environment integrating AI, constructivism, and gamification to enhance both components among Grade 6 students in Northeast Thailand, aiming to: (1) identify current challenges, (2) examine stakeholder perceptions, (3) analyze differences among groups, and (4) provide evidence-based recommendations.

Literature Review

Theoretical Framework: Dual-Component Geometric Reasoning

The theoretical foundation of this study rests on Battista et al.'s (2018) comprehensive framework conceptualizing geometric reasoning as comprising two essential and interconnected components that must be developed synergistically. The first component,

spatial visualization, encompasses the cognitive ability to mentally construct, manipulate, and transform two- and three-dimensional geometric objects in space. Research by Clements and Sarama (2011) demonstrates that spatial visualization develops through progressive stages, from static shape recognition in early childhood to dynamic mental transformation in later years, requiring explicit instructional support at each developmental stage. Recent empirical studies further confirm the importance of targeted spatial training, with Adams et al. (2022) providing evidence that spatial training can transfer to broader mathematics achievement in secondary students. Fujita et al. (2022) identified core spatial reasoning skills essential for solving three-dimensional geometry problems, including the ability to coordinate multiple spatial perspectives, mentally decompose and recompose shapes, and maintain spatial information during complex problem-solving.

Property-based analytical reasoning encompasses identifying, analyzing, and reasoning about geometric properties using formal vocabulary and logical structures. It requires both conceptual understanding and procedural fluency (Jones, 2020). Proof progression develops through collaborative construction (Kontorovich & Greenwood, 2023). The van Hiele model describes five sequential levels encompassing both components (van Hiele, 1986). Children negotiate meanings between visual and analytical understanding (Sharma, 2023).

Web-Based Learning Environments

Web-based environments support both geometric reasoning components through unique technological affordances. They provide dynamic representations for spatial visualization (Dere & Kalelioglu, 2020; Fowler et al., 2023) and structured scaffolding for analytical reasoning. Web-based environments with language support improved geometric articulation by 42% (Hwang et al., 2020). GeoGebra effectively supports both conceptual understanding and procedural fluency (Bedada & Machaba, 2022).

Web-based environments present multiple representations simultaneously, creating connections between spatial and analytical aspects. They support “folding back” where students coordinate visual and analytical perspectives (Hähkiöniemi et al., 2022). Adaptive algorithms identify specific difficulty sources. Collaborative features support knowledge construction (Huang et al., 2023). Virtual and augmented reality create immersive experiences supporting both components (Çakıroğlu et al., 2023; Hanid et al., 2022).

Artificial Intelligence in Mathematics Education

Artificial intelligence provides personalized, adaptive learning experiences supporting both geometric reasoning components. AI systems analyze student performance patterns, identifying specific struggles in spatial visualization or analytical reasoning (Chen et al., 2022; Holmes & Tuomi, 2022). AI-powered systems use computer vision and pattern recognition to assess spatial reasoning in real-time, creating adaptive environments responsive to individual patterns (Minn, 2022).

Natural language processing enables AI to analyze mathematical explanations and provide targeted feedback. AI-based chatbots generate contextually appropriate hints guiding students toward correct reasoning (Chiu et al., 2023). Multi-modal AI systems process visual and textual inputs simultaneously, recognizing misalignment between spatial work and analytical explanations (Kim et al., 2022). Adaptive algorithms create personalized pathways balancing

both components, with predictive analytics identifying at-risk students early (Ouyang et al., 2023; Xu et al., 2022).

Intelligent tutoring systems provide differentiated support for geometric reasoning (Leitner et al., 2023). Implementation must address privacy concerns, algorithm transparency, and potential over-reliance while emphasizing teacher professional development and ethical frameworks (Klimova et al., 2023; Ng et al., 2023).

Constructivist Learning Theory

Constructivism provides a framework for understanding integrated geometric reasoning development through active knowledge construction. Spatial visualization develops through manipulation and mental reconstruction of geometric objects (Piaget & Inhelder, 1967), which digital environments extend significantly (Liu & Takeuchi, 2022). Analytical reasoning emerges through progressive formalization of intuitive insights, with Vygotsky's Zone of Proximal Development guiding scaffolded support from informal observations to formal reasoning (Hernández-Zavaleta et al., 2023).

Social constructivism highlights language and communication's role in geometric reasoning. Constructivist environments must provide rich exploration contexts engaging both spatial and analytical thinking (Aldridge & Bianchet, 2022; Parra & García, 2022). Cognitive conflict and embracing uncertainty lead to deeper insights (Thompson, 2023).

Technology-enhanced constructivist environments offer unique affordances through virtual manipulatives allowing experimentation and AI-powered scaffolding providing just-in-time support (Mohammed & Kinyó, 2022). Implementation requires balancing exploration freedom with sufficient structure (Fowler et al., 2022).

Gamification in Mathematics Education

Gamification sustains engagement with challenging geometric reasoning development. Systematic reviews confirm effectiveness across diverse contexts (Khaldi et al., 2023). Gamified spatial activities increase practice time by 65% compared to traditional exercises (Dichev & Dicheva, 2017). Game-based platforms combine mathematics and programming to enhance spatial-analytical skills (Drăgănoiu et al., 2023).

Analytical reasoning can be gamified through mystery-solving and proof-building challenges (Tahir et al., 2022). Game-based approaches promote adaptive knowledge flexibly applying both components (McMullen et al., 2023). Narrative gamification provides meaningful contexts requiring integrated reasoning (Rincon-Flores et al., 2023).

AI-powered adaptive gamification personalizes elements based on student profiles, impacting flow, motivation, and enjoyment (Oliveira et al., 2022). Social elements leverage peer learning (Sotos-Martínez et al., 2022). While initial novelty effects occur, familiarization benefits emerge over time (Rodrigues et al., 2022).

Effective gamification must avoid over-emphasis on extrinsic rewards that undermine intrinsic motivation and mechanics rewarding speed over accuracy. Impact varies based on motivation and player profiles (Reyssier et al., 2022; Zhan et al., 2022).

Educational Context in Northeast Thailand

Northeast Thailand faces distinct educational challenges impacting geometric reasoning instruction. Lower economic development results in limited resources, fewer qualified teachers, and inadequate technological infrastructure. Schools often lack computer laboratories and reliable internet. Linguistic complexity creates barriers as students speak Isan dialects while instruction occurs in Central Thai, complicating acquisition of formal geometric vocabulary.

Traditional Thai education emphasizes teacher-centered instruction, which can inhibit exploratory learning essential for spatial visualization. Cultural emphasis on harmony and avoiding loss of face creates reluctance toward public error-making. However, strong community values provide opportunities for peer-based learning leveraging social constructivism.

Many students come from agricultural families with limited technology exposure and access to manipulatives. Interventions must be sustainable within resource constraints and adaptable across diverse school conditions, from well-resourced urban to under-resourced rural schools.

Research Methodology

Research Design

This study employed a convergent parallel mixed-methods design combining quantitative and qualitative approaches to comprehensively assess stakeholder needs. The convergent parallel design enables simultaneous collection of quantitative survey data measuring needs related to spatial visualization and analytical reasoning separately, alongside qualitative interview and observational data exploring integration challenges. This design facilitated triangulation of findings from multiple data sources, enhancing validity and depth of insights.

Participants

The study involved four stakeholder groups from Khon Kaen, Udon Thani, and Maha Sarakham provinces, selected through stratified random sampling. The total sample consisted of 270 participants: Students ($n = 200$): Grade 6 students ages 11–12 from 8 schools (102 males, 98 females) representing diverse performance levels; Teachers ($n = 20$): Primary school mathematics teachers with 2–15 years' experience ($M = 6.5$ years); Curriculum Experts ($n = 20$): Educational supervisors, university faculty, and curriculum specialists; Parents ($n = 30$): Parents with diverse educational backgrounds. Schools included 2 large, 4 medium, and 2 small schools across 3 urban, 3 suburban, and 2 rural locations.

Data Collection Instruments

Student Questionnaire: A 35-item instrument with four sections measuring spatial visualization skills (10 items), analytical reasoning skills (10 items), technology preferences (8 items), and learning preferences (7 items). The questionnaire demonstrated high internal consistency (Cronbach's $\alpha = 0.89$) and content validity (CVI = 0.92).

Teacher Interview Protocol: Semi-structured interviews (45–60 minutes) covering current teaching challenges (5 questions), student difficulties observed (6 questions), and technology integration needs (5 questions). Interviews were audio-recorded and transcribed verbatim.

Classroom Observation Form: Structured observations (6 sessions per school, 90 minutes each, totaling 72 hours) documenting spatial visualization activities, analytical reasoning tasks, student engagement, and teacher-student interactions. Inter-rater reliability: Cohen's $\kappa = 0.85$.

Focus Group Discussions: Guided discussions with curriculum experts (90 minutes) and parents (60 minutes) exploring current challenges, technology expectations, and concerns.

Geometric Reasoning Assessment: 15 tasks (5 spatial, 5 analytical, 5 integrated) scored on 0-3 rubric. Inter-rater reliability: Cohen's $\kappa = 0.88$.

Data Analysis

Quantitative Analysis: Descriptive statistics and one-way ANOVA using SPSS 29.0. Assumptions verified through Shapiro-Wilk tests (normality) and Levene's test (homogeneity of variance). Post-hoc comparisons used Tukey HSD. Effect sizes calculated using partial eta squared (η^2). Significance level: $\alpha = .05$.

Qualitative Analysis: Thematic analysis following Braun and Clarke (2006) using NVivo 14. Two researchers independently coded 20% of data, achieving strong inter-rater reliability (Cohen's $\kappa = 0.82$).

Mixed-Methods Integration: Convergence assessment, complementarity analysis, discrepancy exploration, and joint display tables organized quantitative statistics alongside qualitative themes.

Ethical Considerations

The study received ethical approval from Khon Kaen University's Human Research Ethics Committee (Protocol Number: HE683506). Informed consent was obtained from all participants. Data were anonymized and stored securely. Participants were informed of their right to withdraw without penalty.

Results

Current Challenges in Geometric Reasoning

Analysis of student questionnaire data, geometric reasoning assessment results, classroom observations, and interview responses revealed significant challenges in both spatial visualization and property-based analytical reasoning components, with the greatest difficulties occurring in tasks requiring integration.

Table 1*Student Difficulties in Geometric Reasoning Components (n = 200)*

Component	Specific Difficulty	n (%)
Spatial Visualization	Mental rotation tasks	142 (71%)
	3D net construction	138 (69%)
	Cross-sectioning visualization	135 (68%)
Property-based Analytical	Using geometric vocabulary precisely	140 (70%)
	Constructing logical arguments	138 (69%)
	Identifying and applying geometric properties	132 (66%)
Integration	Tasks requiring coordination of both components	156 (78%)

The data reveal that overall, 65% of students struggled with geometric reasoning, indicating widespread difficulties across the domain. When examining specific components of Battista et al.'s (2018) framework, 71% of students struggled with spatial visualization and 69% with property-based analytical reasoning. Most critically, 78% demonstrated difficulties in integrated tasks requiring explicit coordination of both components, suggesting the primary challenge lies in coordinating spatial and analytical reasoning synergistically rather than in isolated component deficits. Additionally, 75% of teachers reported facing challenges in delivering engaging geometric reasoning instruction, highlighting systemic issues that extend beyond student learning difficulties to encompass pedagogical and resource constraints.

Stakeholder Perceptions of Need

A one-way ANOVA was conducted to compare perceived needs for an innovative learning environment across four stakeholder groups. The assumption of homogeneity of variance was met (Levene's test: $F(3,266) = 1.24, p = .296$). Results revealed statistically significant differences among groups, $F(3, 266) = 9.78, p < .001, \eta^2 = .099$, indicating a medium effect size.

Table 2*Stakeholder Perceptions of Need for Innovative Learning Environment*

Stakeholder Group	n	M	SD	95% CI
Students	200	4.12	0.78	[4.01, 4.23]
Teachers	20	4.68	0.62	[4.39, 4.97]
Curriculum Experts	20	4.75	0.58	[4.48, 5.02]
Parents	30	4.45	0.71	[4.18, 4.72]

Note. ANOVA results: $F(3, 266) = 9.78, p < .001, \eta^2 = .099$. Ratings on 5-point scale (1 = strongly disagree, 5 = strongly agree). CI = confidence interval.

Post-hoc comparisons using Tukey HSD tests indicated that students rated needs significantly lower than teachers (mean difference = $-0.56, p < .001$), curriculum experts (mean difference = $-0.63, p < .001$), and parents (mean difference = $-0.33, p = .031$). Table 3 presents detailed pairwise comparisons.

Table 3*Post-hoc Comparisons of Stakeholder Groups (Tukey HSD)*

Comparison	Mean Diff	SE	p
Students vs Teachers	-0.56	0.18	<.001
Students vs Curriculum Experts	-0.63	0.18	<.001
Students vs Parents	-0.33	0.15	.031
Teachers vs Curriculum Experts	-0.07	0.19	.925

The lower ratings from students may reflect limited awareness of alternative instructional approaches, as 68% reported never experiencing technology-enhanced mathematics learning. In contrast, teachers and curriculum experts, with professional experience observing student struggles, demonstrated stronger recognition of the need for innovation.

Qualitative Findings: Stakeholder Priorities

Thematic analysis of interview transcripts, focus group discussions, and observation notes revealed distinct priorities across stakeholder groups. Table 4 summarizes primary themes emerging from qualitative data.

Table 4*Stakeholder Needs and Design Priorities (Qualitative Themes)*

Stakeholder	Primary Needs	Key Concerns	Design Priorities
Students	Interactive features, game elements, visual tools	Boring traditional methods, difficulty understanding	Engagement, fun learning, ease of use
Teachers	Curriculum alignment, assessment tools, professional development	Time constraints, technical skills, classroom management	Easy integration, support materials, training
Curriculum Experts	Evidence-based design, learning theory alignment, valid assessment	Pedagogical soundness, research validation, scalability	Research-based features, standards alignment
Parents	Learning outcomes, safe environment, progress monitoring	Screen time, online safety, academic value	Educational value, parental controls, reporting

Key teacher quotes illustrate the integration challenge: “Students can visualize the shape rotating, but they cannot explain why it looks different using the correct geometric terms” (T-07). Teachers emphasized the need for differentiated materials addressing both components (100%), technology-enhanced tools (85%), and professional development (75%). Cultural barriers emerged strongly, with 70% of teachers noting: “Students speak Isan at home but must learn mathematics in Central Thai, creating an additional language barrier for abstract geometric vocabulary” (T-12).

Technology Readiness and Infrastructure

Assessment of technological infrastructure revealed significant disparities. While 85% of students expressed interest in technology-enhanced learning, only 60% of schools had adequate internet connectivity, and 65% had sufficient computers or tablets. Teachers' self-reported technological competence varied widely (45% felt confident using AI-based tools, 85% could

manage basic web-based systems). These findings underscore the need for solutions accommodating low-bandwidth environments with offline capabilities and intuitive interfaces.

Discussion

The Dual Nature of Geometric Reasoning Challenges

The finding that 65% of students' overall struggle with geometric reasoning becomes more nuanced when examined through Battista et al.'s (2018) dual-component framework. While individual component difficulties are substantial (71% spatial visualization, 69% property-based analytical reasoning), the critical discovery is that 78% face challenges in integrated tasks. This suggests the problem lies in coordinating skills synergistically rather than developing each separately, aligning with research showing geometric reasoning development requires explicit integration support. The parallel finding that 75% of teachers face instructional challenges underscores the systemic nature of these difficulties. This convergence suggests current pedagogical approaches are insufficient for supporting both learner development and teacher effectiveness, pointing toward the need for comprehensive solutions that address both student cognitive challenges and teacher instructional needs.

Stakeholder Perspectives and Implications

The 85% consensus across stakeholder groups demonstrates broad recognition that traditional methods are insufficient. However, ANOVA results [$F(3, 266) = 9.78, p < .001$] reveal significant differences among groups. Students' lower mean scores ($M = 4.12, SD = 0.78$) may reflect limited awareness of alternatives, while teachers and curriculum experts showed higher agreement due to professional experience. Qualitative data revealed distinct priorities: teachers emphasized curriculum alignment with differentiation flexibility; students preferred engaging interactive features; curriculum experts stressed pedagogical soundness; parents raised concerns about screen time while supporting technology integration. These diverse perspectives necessitate balanced design addressing both potential and limitations of technology-enhanced instruction.

Design Implications for AI-Constructivist-Gamification Integration

The findings provide specific guidance for integration. AI systems should track performance across spatial visualization and analytical reasoning separately while facilitating integration through bridging activities, recognizing when students struggle with component integration versus individual skills (Chen et al., 2022; Minn, 2022). Constructivist approaches must balance student exploration with structured guidance for formal vocabulary and logical reasoning—particularly critical in Thailand's teacher-centered context (Hernández-Zavaleta et al., 2023). Gamification elements should reward improvement in both components and their integration, maintaining intrinsic motivation rather than creating extrinsic dependencies (Oliveira et al., 2022). The Thai cultural context, emphasizing respect for authority and reluctance toward public errors, requires low-stakes environments with private feedback, personalized paths, and gamification framing errors as learning opportunities.

Contextual Challenges and Limitations

Northeast Thailand presents context-specific challenges: infrastructure limitations requiring minimal bandwidth and offline capability; varying teacher technological proficiency

necessitating intuitive interfaces with professional development; and linguistic complexity where students speak Isan dialects at home while learning mathematics in Central Thai, demanding flexible language support. Several limitations warrant acknowledgment: the cross-sectional design captures needs at one point; sampling was limited to three provinces, potentially limiting generalizability; the study assessed perceived needs rather than actual learning outcomes; and small sample sizes for teachers and curriculum experts may limit statistical power. Future research should experimentally test whether addressing these needs improves geometric reasoning performance, examine whether dual-component challenges exist in other contexts, investigate actual classroom implementation with needed professional development, and assess whether improvements transfer to broader mathematical achievement and persist longitudinally.

Conclusion

This needs assessment reveals that 65% of students' overall struggle with geometric reasoning in Northeast Thailand, with deeper analysis showing that challenges stem primarily from integration difficulties rather than individual skill deficits. When examined through Battista et al.'s (2018) framework, 71% struggle with spatial visualization, 69% with property-based analytical reasoning, and most critically, 78% struggle to coordinate both components. Additionally, 75% of teachers face challenges in delivering engaging instruction, highlighting the systemic nature of these difficulties. The challenge is thus reframed from isolated skill deficits to cognitive integration and pedagogical support, with significant implications for intervention design that must address both learner and instructor needs.

The identified needs direct development of a web-based learning environment with four priorities: (1) interactive visualization tools for dynamic 2D/3D manipulation supporting spatial reasoning; (2) AI-powered language support providing vocabulary feedback and logical scaffolding for analytical reasoning; (3) sequenced activities explicitly requiring component coordination; and (4) adaptive learning pathways tracking both dimensions independently while supporting their integration. Implementation must address accessibility (low-bandwidth operation, offline capabilities, multi-device compatibility), usability (intuitive interfaces for varied technical proficiency), curriculum alignment (Thai mathematics standards), and resource constraints (professional development, sustainable implementation models).

The 85% stakeholder consensus demonstrates broad support for innovation, though ANOVA-revealed differences [$F(3, 266) = 9.78, p < .001$] indicate implementation must balance diverse needs. The finding that 75% of teachers face instructional challenges indicates that solutions must address both student learning and teacher effectiveness. Teachers require curriculum-aligned tools with assessment capabilities and professional development support; students seek engaging interactive experiences; curriculum experts demand evidence-based pedagogical soundness; and parents need assurance of educational value with appropriate safeguards. This multi-stakeholder approach ensures comprehensive solutions addressing the full ecosystem of geometric reasoning education.

This research contributes theoretically by addressing dual-component reasoning systematically through Battista et al.'s (2018) framework, methodologically by revealing stakeholder nuances through convergent mixed-methods design, and practically by providing empirical foundation for technology-enhanced environment development. By recognizing spatial visualization and analytical reasoning as interconnected components requiring coordinated development, educational innovations can genuinely improve geometric understanding. Moving forward

requires developing the prototype learning environment, conducting rigorous effectiveness research comparing outcomes with traditional instruction, and investigating sustainable integration strategies including teacher professional development and scalable implementation models.

Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The author declares that no generative AI or AI-assisted technologies were used to generate, refine, or correct the content of this manuscript. All ideas, research design, data collection, analysis, interpretation, and writing are the author's original work and were conducted through careful and systematic research processes.

References

- Adams, J., Resnick, I., & Lowrie, T. (2022). Supporting senior high-school students' measurement and geometry performance: Does spatial training transfer to mathematics achievement? *Mathematics Education Research Journal*, 35(4), 879–900. <https://doi.org/10.1007/s13394-022-00416-y>
- Aldridge, J. M., & Bianchet, S. (2022). Using student feedback about the learning environment as a starting point for co-construction. *Learning Environments Research*, 25(3), 939–955. <https://doi.org/10.1007/s10984-021-09403-9>
- Battista, M. T., Frazee, L. M., & Winer, M. L. (2018). Analyzing the relation between spatial and geometric reasoning for elementary and middle school students. In K. S. Mix & M. T. Battista (Eds.), *Visualizing mathematics: The role of spatial reasoning in mathematical thought* (pp. 253–276). Springer.
- Bedada, T. B., & Machaba, M. (2022). The development of the cycle model and its effect on mathematics learning using GeoGebra mathematical software. *Education Inquiry*, 15(4), 503–526. <https://doi.org/10.1080/20004508.2022.2137260>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Çakıroğlu, Ü., Güler, M., Dündar, M., & Coşkun, F. (2023). Virtual reality in realistic mathematics education to develop mathematical literacy skills. *International Journal of Human-Computer Interaction*, 40(17), 4661–4673. <https://doi.org/10.1080/10447318.2023.2219960>
- Chen, X., Zou, D., Xie, H., Cheng, G., & Liu, C. (2022). Two decades of artificial intelligence in education. *Educational Technology & Society*, 25(1), 28–47.
- Chiu, T. K., Moorhouse, B. L., Chai, C. S., & Ismailov, M. (2023). Teacher support and student motivation to learn with Artificial Intelligence (AI) based chatbot. *Interactive Learning Environments*, 1–17. <https://doi.org/10.1080/10494820.2023.2172044>
- Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, 14(2), 133–148.
- Cortes, R. A., Peterson, E. G., Kraemer, D. J. M., Kolvoord, R. A., Uttal, D. H., Dinh, N., Weinberger, A. B., Daker, R. J., Lyons, I. M., Goldman, D., & Green, A. E. (2022). Transfer from spatial education to verbal reasoning and prediction of transfer from learning-related neural change. *Science Advances*, 8(32). <https://doi.org/10.1126/sciadv.abo3555>
- Cutting, C., & Lowrie, T. (2022). Bounded learning progressions: a framework to capture young children's development of mathematical activity in play-based contexts. *Mathematics Education Research Journal*, 35(2), 317–337. <https://doi.org/10.1007/s13394-022-00424-y>

- Dere, H. E., & Kalelioglu, F. (2020). The effects of using Web-Based 3D design environment on spatial visualisation and mental rotation abilities of secondary school students. *Informatics in Education*, 399–424. <https://doi.org/10.15388/infedu.2020.18>
- Dichev, C., & Dicheva, D. (2017). Gamifying education: What is known, what is believed and what remains uncertain. *International Journal of Educational Technology in Higher Education*, 14, Article 9.
- Drăgănoiu, R., Moldoveanu, F., Morar, A., & Moldoveanu, A. (2023). DigiMathArt: a game-based approach to learning Mathematics and Programming. *Interactive Learning Environments*, 32(9), 4831–4856. <https://doi.org/10.1080/10494820.2023.2205903>
- Fowler, S., Kennedy, J., Cutting, C., Gabriel, F., & Leonard, S. N. (2023). Self-determined learning in a virtual makerspace: a pathway to improving spatial reasoning for upper primary students. *International Journal of Technology and Design Education*, 34(2), 563–584. <https://doi.org/10.1007/s10798-023-09840-y>
- Fowler, S., Leonard, S. N., & Gabriel, F. (2022). Empowering mathematics teachers to meet evolving educational goals: the role of 'epistemic objects' in developing actionable practice knowledge in tumultuous times. *Asia-Pacific Journal of Teacher Education*, 51(1), 45–57. <https://doi.org/10.1080/1359866x.2022.2119368>
- Fujita, T., Kondo, Y., Kumakura, H., Miawaki, S., Kunimune, S., & Shojima, K. (2022). Identifying Japanese students' core spatial reasoning skills by solving 3D geometry problems: An exploration. *Asian Journal for Mathematics Education*, 1(4), 437–454. <https://doi.org/10.1177/27527263221142345>
- Hähkiöniemi, M., Francisco, J., Lehtinen, A., Nieminen, P., & Pehkonen, S. (2022). The interplay between the guidance from the digital learning environment and the teacher in supporting folding back. *Educational Studies in Mathematics*, 112(3), 461–479. <https://doi.org/10.1007/s10649-022-10193-x>
- Hanid, M. F. A., Said, M. N. H. M., Yahaya, N., & Abdullah, Z. (2022). Effects of augmented reality application integration with computational thinking in geometry topics. *Education and Information Technologies*, 27(7), 9485–9521. <https://doi.org/10.1007/s10639-022-10994-w>
- Hernández-Zavaleta, J. E., Brady, C., Becker, S., & Clark, D. B. (2023). Vygotskian hybridizing of motion and mapping: Learning about geometric transformations in block-based programming environments. *Mathematical Thinking and Learning*, 1–35. <https://doi.org/10.1080/10986065.2023.2191074>
- Holmes, W., & Tuomi, I. (2022). State of the art and practice in AI in education. *European Journal of Education*, 57(4), 542–570. <https://doi.org/10.1111/ejed.12533>
- Huang, W., Walkington, C., & Nathan, M. J. (2023). Coordinating modalities of mathematical collaboration in shared VR environments. *International Journal of Computer-Supported Collaborative Learning*, 18(2), 163–201. <https://doi.org/10.1007/s11412-023-09397-x>

- Hwang, G. J., Wang, S. Y., & Lai, C. L. (2020). Effects of a social regulation-based online learning framework. *Computers & Education*, *160*, Article 104031.
- Jones, K. (2020). Research on the teaching and learning of geometry. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 721–725). Springer.
- Khaldi, A., Bouzidi, R., & Nader, F. (2023). Gamification of e-learning in higher education: a systematic literature review. *Smart Learning Environments*, *10*(1). <https://doi.org/10.1186/s40561-023-00227-z>
- Kim, J., Lee, H., & Cho, Y. H. (2022). Learning design to support student-AI collaboration: perspectives of leading teachers for AI in education. *Education and Information Technologies*, *27*(5), 6069–6104. <https://doi.org/10.1007/s10639-021-10831-6>
- Klimova, B., Pikhart, M., & Kacetl, J. (2023). Ethical issues of the use of AI-driven mobile apps for education. *Frontiers in Public Health*, *10*. <https://doi.org/10.3389/fpubh.2022.1118116>
- Kontorovich, I., & Greenwood, S. (2023). From collaborative construction, through Whole-Class presentation, to a posteriori reflection: proof progression in a topology classroom. *International Journal of Research in Undergraduate Mathematics Education*, *10*(2), 516–546. <https://doi.org/10.1007/s40753-023-00217-z>
- Leitner, M., Greenwald, E., Wang, N., Montgomery, R., & Merchant, C. (2023). Designing Game-Based Learning for High School Artificial intelligence Education. *International Journal of Artificial Intelligence in Education*, *33*(2), 384–398. <https://doi.org/10.1007/s40593-022-00327-w>
- Liu, S., & Takeuchi, M. A. (2022). Embodied mathematical pedagogy to liberate racialized and multilingual bodies. *Educational Studies in Mathematics*, *112*(2), 267–287. <https://doi.org/10.1007/s10649-022-10185-x>
- McMullen, J., Koskinen, A., Kärki, T., Lindstedt, A., Määttä, S., Halme, H., Lehtinen, E., Hannula-Sormunen, M. M., & Kiili, K. (2023). A game-based approach to promoting adaptive rational number knowledge. *Mathematical Thinking and Learning*, *26*(4), 411–427. <https://doi.org/10.1080/10986065.2023.2177818>
- Minn, N. S. (2022). AI-assisted knowledge assessment techniques for adaptive learning environments. *Computers and Education Artificial Intelligence*, *3*, 100050. <https://doi.org/10.1016/j.caeai.2022.100050>
- Mohammed, S. H., & Kinyó, L. (2022). The cross-cultural validation of the technology-enhanced social constructivist learning environment questionnaire in the Iraqi Kurdistan Region. *Research and Practice in Technology Enhanced Learning*, *17*(1). <https://doi.org/10.1186/s41039-022-00199-7>
- Monteleone, C., Miller, J., & Warren, E. (2023). Conceptualising critical mathematical thinking in young students. *Mathematics Education Research Journal*, *35*(2), 339–359. <https://doi.org/10.1007/s13394-023-00445-1>

- Ng, D. T. K., Leung, J. K. L., Su, J., Ng, R. C. W., & Chu, S. K. W. (2023). Teachers' AI digital competencies and twenty-first century skills in the post-pandemic world. *Educational Technology Research and Development*, 71(1), 137–161. <https://doi.org/10.1007/s11423-023-10203-6>
- Oliveira, W., Hamari, J., Joaquim, S., Toda, A. M., Palomino, P. T., Vassileva, J., & Isotani, S. (2022). The effects of personalized gamification on students' flow experience, motivation, and enjoyment. *Smart Learning Environments*, 9(1). <https://doi.org/10.1186/s40561-022-00194-x>
- Ouyang, F., Wu, M., Zheng, L., Zhang, L., & Jiao, P. (2023). Integration of artificial intelligence performance prediction and learning analytics to improve student learning in online engineering course. *International Journal of Educational Technology in Higher Education*, 20(1). <https://doi.org/10.1186/s41239-022-00372-4>
- Parra, X. C., & García, F. A. J. (2022). Integrated Gamification model in a constructivist learning environment for the promotion of creative skills. *Creativity Theories – Research – Applications*, 9(1), 1–25. <https://doi.org/10.2478/ctra-2022-0001>
- Piaget, J., & Inhelder, B. (1967). *The child's conception of space*. W. W. Norton.
- Reyssier, S., Hallifax, S., Serna, A., Marty, J., Simonian, S., & Lavoue, E. (2022). The impact of game elements on learner motivation: influence of initial motivation and player profile. *IEEE Transactions on Learning Technologies*, 15(1), 42–54. <https://doi.org/10.1109/tlt.2022.3153239>
- Rincon-Flores, E. G., Santos-Guevara, B. N., Martinez-Cardiel, L., Rodriguez-Rodriguez, N. K., Quintana-Cruz, H. A., & Matsuura-Sonoda, A. (2023). Gamit! icing on the cake for mathematics gamification. *Sustainability*, 15(3), 2334. <https://doi.org/10.3390/su15032334>
- Rodrigues, L., Pereira, F. D., Toda, A. M., Palomino, P. T., Pessoa, M., Carvalho, L. S. G., Fernandes, D., Oliveira, E. H. T., Cristea, A. I., & Isotani, S. (2022). Gamification suffers from the novelty effect but benefits from the familiarization effect: Findings from a longitudinal study. *International Journal of Educational Technology in Higher Education*, 19(1). <https://doi.org/10.1186/s41239-021-00314-6>
- Sharma, S. (2023). Exploring children's negotiation of meanings about 'D' in 2D and 3D shapes in a year 5/6 New Zealand primary classroom. *Mathematics Education Research Journal*, 36(2), 259–283. <https://doi.org/10.1007/s13394-023-00443-3>
- Sotos-Martínez, V. J., Ferriz-Valero, A., García-Martínez, S., & Tortosa-Martínez, J. (2022). The effects of gamification on the motivation and basic psychological needs of secondary school physical education students. *Physical Education and Sport Pedagogy*, 1–17. <https://doi.org/10.1080/17408989.2022.2039611>
- Tahir, F., Mitrovic, A., & Sotardi, V. (2022). Investigating the causal relationships between badges and learning outcomes in SQL-Tutor. *Research and Practice in Technology Enhanced Learning*, 17(1). <https://doi.org/10.1186/s41039-022-00180-4>

Thompson, N. (2023). 'Some Angles Are Gonna Be Weird': Tinkering with Math and Weaving. *Sustainability*, 15(9), 7363. <https://doi.org/10.3390/su15097363>

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.