Cultivating Intuition for Mathematical Modelling in an Interdisciplinary STEM Lesson: A Case Study in Singapore

Lim Kim Yong, National University of Singapore, Singapore
Ng Mei Ting, National University of Singapore, Singapore

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Abstract
This study presents the rationale and execution of an interdisciplinary STEM lesson on mathematical modelling for secondary and pre-tertiary students in Singapore. Instead of directly introducing differential equations, which are typically employed in such models, an alternative computational thinking approach is utilized. The lesson employs relatable contexts such as a zombie apocalypse to foster understanding of concepts in infectious diseases. A key objective is the acquiring of the mathematical reasoning behind the susceptible-infected-recovered (SIR) model for infectious disease spread. In addition, this approach is also accessible to students unacquainted with calculus, without compromising the rigor and accuracy of the model. Aligned with Singapore's strategic focus on educational technology in the education system, the lesson supports students in developing foundational data competencies and computational thinking skills with the use of spreadsheet software. These readily available digital tools facilitate the automation of calculations and modelling of disease spread. Predicted outcomes by the model can be compared with real-world data e.g. COVID-19 infection numbers. This enables students to see how mathematics and biology intersect, fostering a better appreciation of the interdisciplinary nature of real-world problems.

Keywords: Mathematical Modelling, Epidemic Modelling, STEM, Lesson Design

Introduction

In Singapore's Mathematics curriculum framework, mathematical modelling is regarded as a process of translating real-world problems into mathematical terms in order to find potential solutions for it (Ministry of Education, 2020). A real-world problem is first simplified and represented by a mathematical model. The model can then be solved to obtain a solution which would be interpreted in the real-world context (Ang, 2001). The model then undergoes iterations of refinement to reflect the real-world problem more accurately (Figure 1).

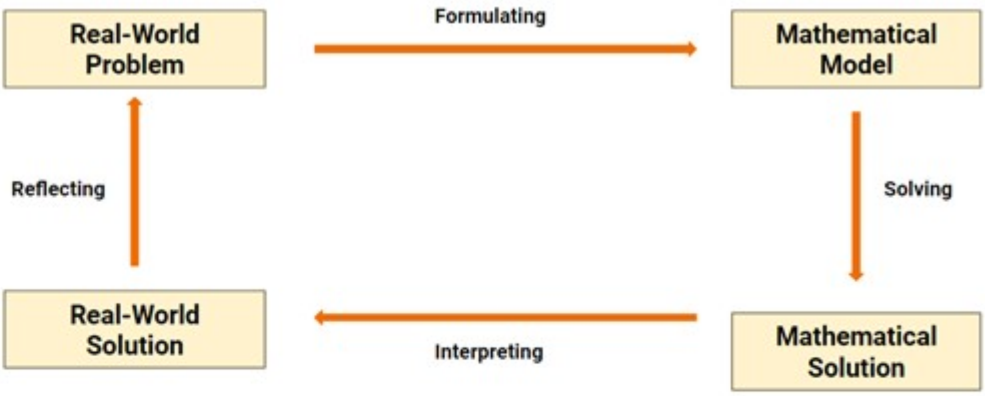


Figure 1: A Simplified View of the Mathematical Modelling Process

Mathematical modelling process has widespread relevance in many real-world applications, including in the study of infectious diseases. Through mathematical reasoning, epidemic models are constructed and become invaluable tools used to predict the spread of diseases and support governments in making informed public health interventions.

Many epidemic models are traditionally driven by a system of ordinary differential equations to estimate epidemiological parameters, such as the transmission rate and reproduction number. Epidemic models, which are often based on various assumptions, can be classified into stochastic or deterministic models. Two common examples of the latter include the Susceptible-Infected-Recovered (SIR) model and Susceptible-Exposed-Infected-Recovered (SEIR) model (Frontiers in Public Health, 2020).

In teaching mathematical modelling, Lofgren et al. (2016) highlighted that making mathematical modelling concepts accessible to allow students to build an intuitive understanding and appreciation for the subject remains a key challenge. Additionally, the conventional use of differential equations in these models can be daunting for students with little or no prior exposure to calculus. In Singapore, introductory courses on coupled differential equations are only formally introduced in the third or fourth year of undergraduate studies.

In the aftermath of COVID-19 pandemic, there were noticeable shifts in Singapore's national curriculum. As part of Singapore Edtech Masterplan 2030, Singapore has pushed for more integration of technology into educational practices. Students are encouraged to develop digital literacy and technology skills. In addition, a new examinable topic on infectious diseases was introduced in the Biology Syllabus (2023) meant for 16 year-olds.

Given the stated challenges in teaching epidemic models and the recent shifts in Singapore's national curriculum, we were motivated to design an interdisciplinary lesson on modelling an epidemic using the Susceptible-Infected-Recovered (SIR) model. Instead of directly introducing differential equations, students are to build the model from scratch using spreadsheet software such as Microsoft Excel. Through this process, we hope to help students grasp the mathematical reasoning behind a basic epidemic model while fostering basic data competencies in the use of spreadsheet software.

Susceptible-Infected-Recovered (SIR) Model

Susceptible-Infected-Recovered (SIR) model is one of the most foundational models in epidemic modelling. It divides a population of N individuals into three categories:

- **S**usceptible (S) – number of individuals who are vulnerable to a disease
- **I**nfectious (I) – number of individuals who are infected and can spread the disease
- **R**ecovered (R) – number of individuals who have recovered and acquired immunity, or died

The model assumes that the population remains constant and is homogenously well-mixed. All individuals in the population are susceptible at time t = 0 (Day 0), and immunity is conferred after a single infection. The relationship between the above categories can be summarised by a system of differential equations:

- **S(t)** - number of individuals who are vulnerable to a disease at time t
- **I(t)** - number of individuals who are infected and can spread the disease at time t
- **R(t)** - number of individuals who have recovered and acquired immunity, or died at time t

$$\frac{dS}{dt} = -\beta \frac{I}{N} S$$
$$\frac{dI}{dt} = \beta \frac{I}{N} S - \Upsilon I$$
$$\frac{dR}{dt} = \Upsilon I$$

β and ϒ are the transmission rate and recovery rate respectively. The former is influenced by two factors: the number of contacts per unit time (π) and the probability of infection upon contact (p). The transmission rate captures the average rate at which a susceptible individual will be infected upon contact with an infected.

The "Modelling an Epidemic" Lesson Design

The "Modelling an Epidemic" lesson was designed to be two-hours long for secondary and pre-tertiary students with little or no prior exposure in Calculus. This lesson was offered as a workshop package to local and international schools under the Science Demonstration Laboratory in the National University of Singapore as part of local educational outreach efforts.

For implementation of lesson, access to a laptop with spreadsheet software, such as Microsoft Excel or Google Sheets, is required. If access to computers is limited, students can work in pairs or small groups.

1. Lesson Introduction

As a brief introductory segment, the etymology of common epidemiological terms – specifically “outbreak”, “epidemic” and “pandemic” – is introduced to students in relation to their definitions and scale of impact using real world news articles.

2. A Simple Model: Susceptible – Infected

A study by Lofgren et al. (2016) highlighted the potential of using zombie epidemic as an effective teaching tool for modelling infectious diseases as students may already have an intuitive sense of how the disease spread or can gain familiarity over a film. In her study, she incorporated zombie epidemics into a workshop for a group of public health professionals. A three-category “SZR” model was employed, where the population is divided into susceptible (S), zombie (Z), or removed (R) groups before building up to more complicated models. The models use difference-equations to represent the relationships between the categories. These equations were then written into Python and R codes and provided to students as part of the hands-on implementation segment.

Drawing inspiration from popular media such as the 2016 Korean horror film “Train to Busan” (Yeon, S., 2016) and the study by Lofgren et al (2016), a fictitious scenario 1 was posed to the students (Figure 2).

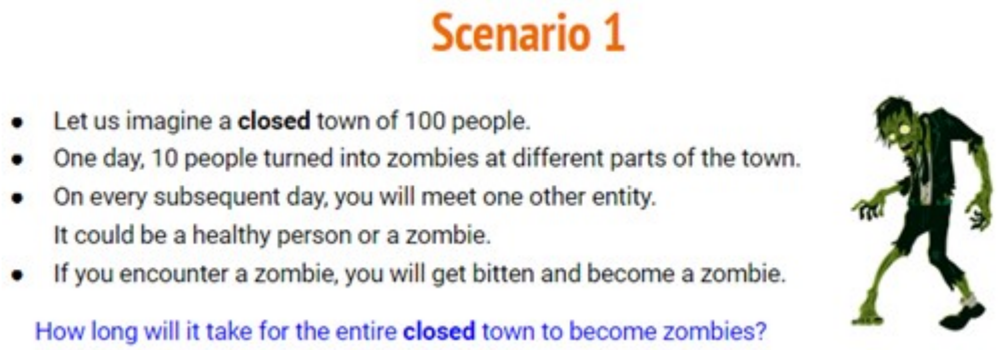


Figure 2: A Simple Zombie Scenario Was Posed

Instead of diving into and discussing a three-category “SZR” model as a start, a simpler Susceptible-Infected (SI) model was adopted as we found that it is easier to consider fewer categories of entities in teaching spreadsheet skills in the later segment of the lesson. In the SI model, students could naturally identify and describe the two primary entities that would exist: the susceptible (S) are healthy individuals who are yet to be bitten and infected but who can be, and the infected (I) are zombies who can bite others and spread the infection.

After understanding the problem, students were guided to simplify the problem by exploring possible model assumptions. A series of carefully framed questions, designed with deliberate word choices, were posed to guide students through the inquiry process (Table 1). These questions were helpful in facilitating them to reason and construct appropriate model assumptions.

Table 1: Some Examples of the Carefully Framed Questions Posed to Students

Examples of Questions	**Assumption**
Let us imagine a **closed** town of 100 individuals and 10 individuals turned into zombies. What does “closed” suggest to us about the population?	A constant population size.
If a susceptible meets a zombie, they will **definitely** be bitten and become a zombie. What does this suggest to us about the chances of being infected once we meet a zombie?	Chance of getting infected by a zombie upon contact is 1.

To support students in grasping the mathematical reasoning behind SI model, students were tasked to determine the number of days required for the entire town to become zombies through a pen-and-paper exercise. In the SI model, the pen-and-paper-exercise consists of five columns (Figure 3). The unit for time t is ‘Day’.

- Day – day of infection
- S(t) - number of healthy people who can be bitten and thus infected at time t
- I(t) - total number of zombies who can bite others and spread the disease by time t
- I_new(t) – a helper variable created to track the number of newly formed zombies at time t
- Total – total number of entities in the closed town

Students explore how the ratio of zombies to the total population affects the chances of a healthy person meeting a zombie each day, in turn affecting the number of newly formed zombies.

Day	S	I_new	I	Total
1	100 - 10 = 90	0	10	90 + 10 = 100
2	90 - 9 = 81	$\frac{10}{100} \times 90 = 9$	10 + 9 = 19	81 + 19 = 100

Figure 3: Students Complete the Pen-and-Paper Exercise Manually

The pen-and-paper exercise serves as a scaffolding technique to help students work through the mathematical reasoning behind the SI model by internalising how the categories change every subsequent day using simple mathematical terms (Figure 4). Instructors could guide students through working the first 3 days and leave computations for the rest of the days to the students.

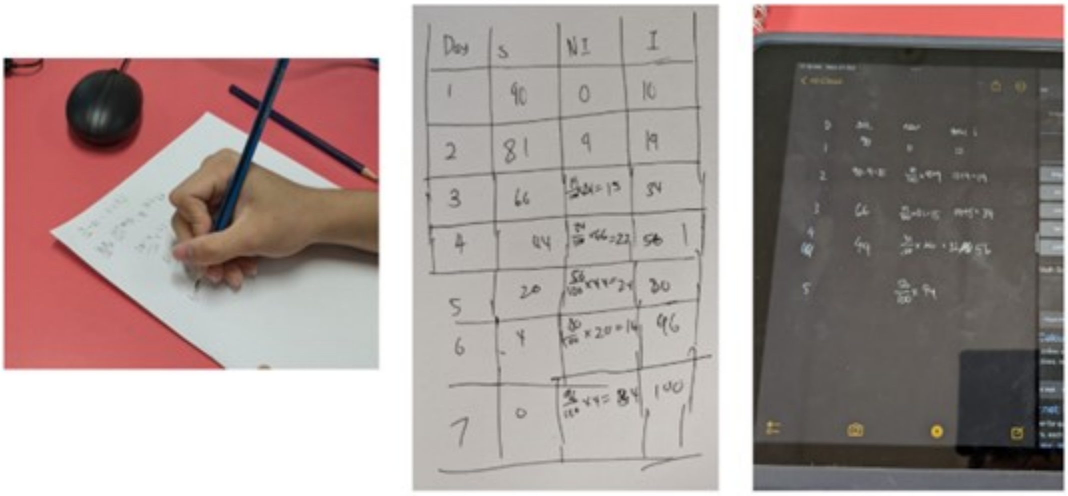


Figure 4: Some Examples of Students' Pen-and-Paper Exercise

After scenario 1, instructors can adjust the context such as exploring different population sizes or possibilities of a healthy person escaping from a zombie upon contact. The latter would influence the probability of infection upon contact with a zombie, in turn affecting transmission rate. Students are encouraged to explore how the change in context may affect the spread of disease. This is when a spreadsheet tool can feature as a more efficient apparatus to solve the problem. In Singapore, the majority of secondary and pre-tertiary students are not formally equipped with programming knowledge as computing is not a mandatory subject in the curriculum. Instead of introducing programming tools like Python or R (Lofgren et al., 2016), which presents an additional steep learning curve, students learn to appreciate the use of accessible spreadsheet tools like Microsoft Excel in building a model from scratch and implement recursions.

Microsoft Excel is introduced to handle tedious computations and model the pen-and-paper exercise. Two parameters are introduced for cell-referencing – 'N' which represents the population size and "b" which represents how likely a susceptible would get infected upon meeting a zombie. In Scenario 1, if a susceptible encounters a zombie, he will definitely get bitten and become a zombie as seen on Figure 2, hence "b" is given the value 1 (Figure 5). During this transition, students learn to input data into cells, use arithmetic operators (+, -, *, /) and automate calculations by cell-referencing ($).

	A	B	C	D	E	F	G	H
1	Day	S	I_new	I	Total		Population Size (N)	100
2	1	90	0	10	100		b	1
3	2	=D2/H1*B2*H2			100			
4	3							

Figure 5: Students Model the Pen-and-Paper Exercise on Spreadsheet Software

Students also explore basic data visualisation by plotting simple charts (Figure 6) to visualise and interpret the infection curve. Using a spreadsheet, students can imagine different contexts and evaluate how it affects the spread of disease in real-time.

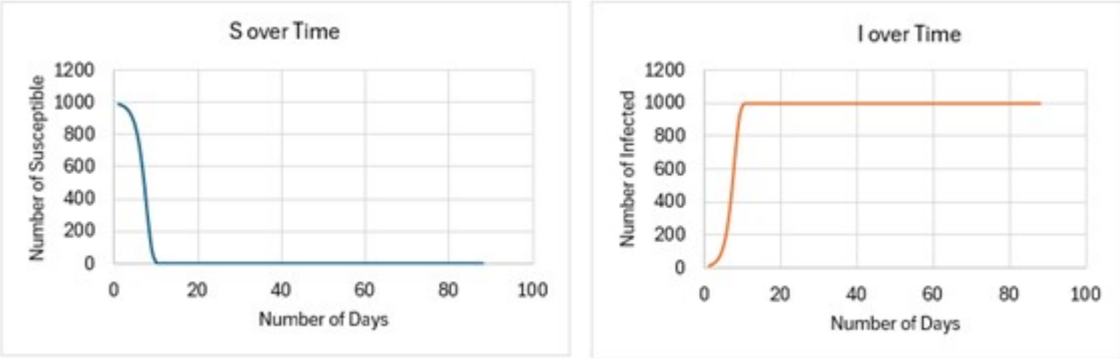


Figure 6: Examples of Simple Solid-Line Scatterplots Plotted to Visualise the Categories

3. A More Realistic Model: Susceptible – Infected – Recovered

After introducing the fictional scenario and SI model, the lesson transitions to a real-world problem with COVID-19 pandemic as a case study, reflecting a more realistic model with the inclusion of a third category – the recovered. The recovered (R) represents the number of people who has recovered or acquired immunity or died from the disease. With students having already gained a foundational understanding of navigating Microsoft Excel, this transition becomes less daunting and more intuitive as students extend their knowledge from the SI model.

The Susceptible-Infected-Recovered (SIR) model introduces a new parameter, k, representing the recovery rate, which depends on the number of days required for recovery. On average, if an infected takes ten days to recover from the disease, the recovery rate (k) would be 0.10. Instead of presenting the parameter through formal definition, we found that the provision of scenarios that allow students to visualise and internalise what the parameter represents helps them foster a deeper understanding of the problem. With this model, students iterate the modelling process by updating and contextualising the assumptions for SIR model, incorporating "Recovered" category and "R_new" helper variable into their spreadsheet, and revise their Excel formulas based on the mathematical reasoning behind how an infected (I) would transition to being recovered (R).

Lastly, the lesson concludes with discussing the significance of basic reproduction number (R_0). Students are encouraged to visualise the three categories and investigate how different public health interventions, such as enforcement of mask-wearing and lockdowns, may affect the peak of infection. By adjusting the values of "b" and "k", thereby changing the basic reproduction number, students are encouraged to discuss and share their insights on how interventions such as mask-wearing and lockdowns affect the disease dynamic (Figure 7).

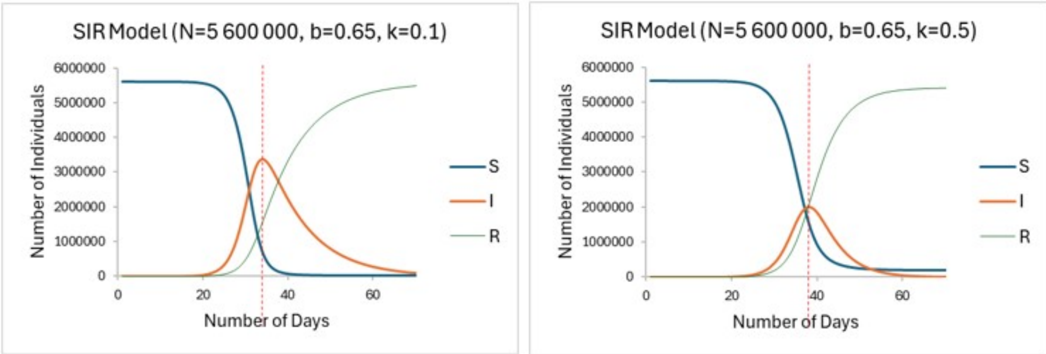


Figure 7: Examples of How the SIR Curves Change With Different Parameter Values of k

Students are also encouraged to compare their model with real world data from trusted government sources. For instance, students could discuss the implications of the infection peak exceeding the number of intensive care unit (ICU) beds available in their country's healthcare system. A mini discussion helps them to appreciate the significance of "flattening the curve", which is a common buzz phrase during the COVID-19 pandemic.

Data Collection and Outcomes

In 2024, 39 secondary 3 (Grade 9 equivalent) students and 20 pre-tertiary (Grade 11 equivalent) students attended the "Modelling an Epidemic" lesson on two separate occasions. Both groups of students are of mixed abilities and learnt to use Microsoft Excel.

A. Pre-Post Quiz Performance

Four learning outcomes were designed to evaluate if the students could grasp the etymology of common epidemiological terms and demonstrate some level of mathematical reasoning. To evaluate the effectiveness of the lesson in achieving these learning outcomes, a pre- and post-quiz was implemented. 37 Grade 9 and 20 Grade 11 responses were collected.

Learning Outcome 1: Distinguish the terms (i) outbreak, (ii) epidemic and (iii) pandemic

2 items were designed for students to identify if the real-world scenario posed should be categorised as an outbreak, epidemic, or pandemic. For both groups, the percentage of students getting a full score of 2 improved post-quiz (Figure 8).

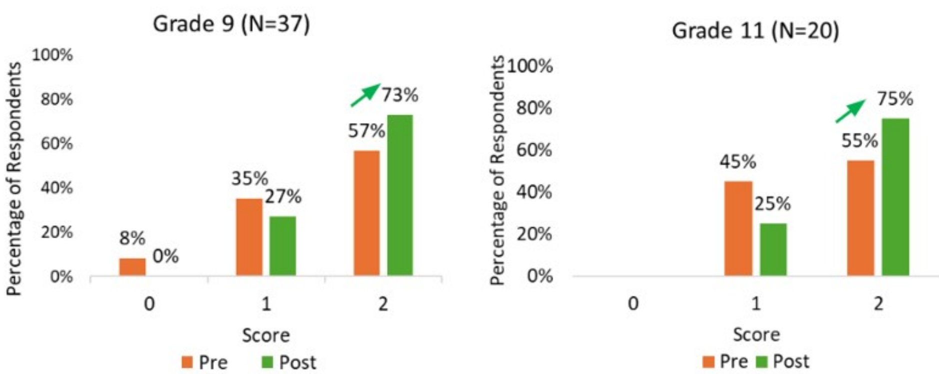


Figure 8: Positive Improvement Observed for Learning Outcome 1

Learning Outcome 2: Identify some assumptions of SI/SIR Model

When we transitioned from SI to SIR model, students were expected to be able to identify the foundational assumptions applicable to SIR model. For both groups, the percentage of students able to identify the assumptions of SIR model accurately improved post-quiz (Figure 9).

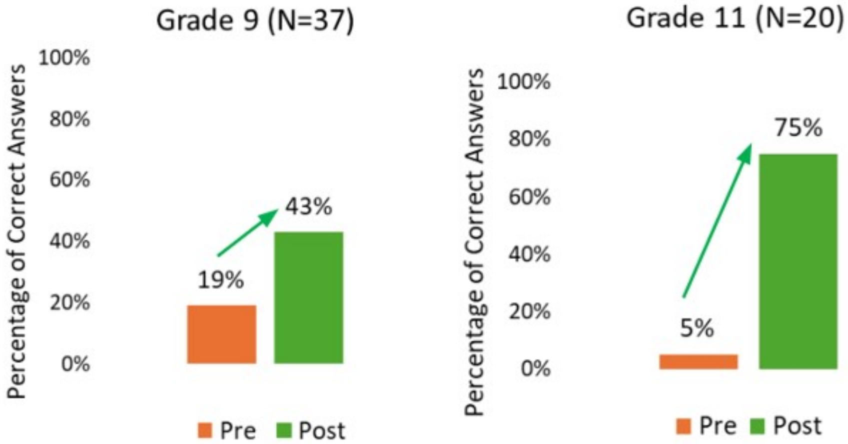


Figure 9: Positive Improvement Observed for Learning Outcome 2

Learning Outcome 3: Recognise and represent the relationships of categories by finding a mathematical expression for the nth day

With all variables defined on day 1, students were required to express the newly infected individuals on the next day (i.e. day 2) using a mathematical expression. Students were expected to be able to represent the relationships of SIR categories using a mathematical expression. For both groups, the percentage of students expressing a correct mathematical equation improved for post-quiz (Figure 10).

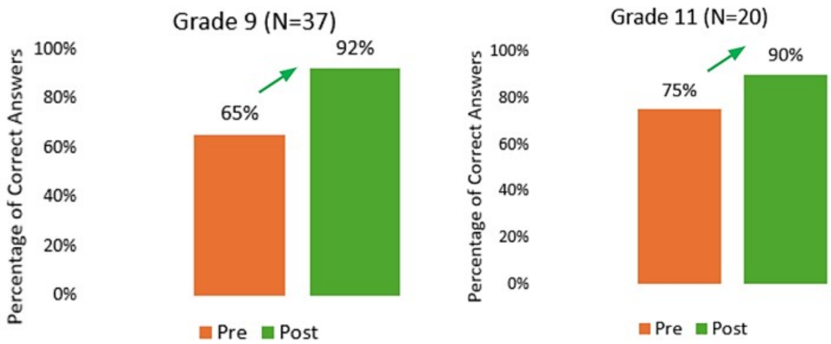


Figure 10: Positive Improvement Observed for Learning Outcome 3

Learning Outcome 4: Describe how parameters of SIR model would be affected based on real world interventions

Given a few real-world interventions, students were expected to predict how the parameters of SIR model (‘b’ and ‘k’) would be affected. For pre-quiz, no Grade 11 students were able to attempt the question correctly. After the lesson, 50% of the students attained a score of at least 2 out of 3 (Figure 11).

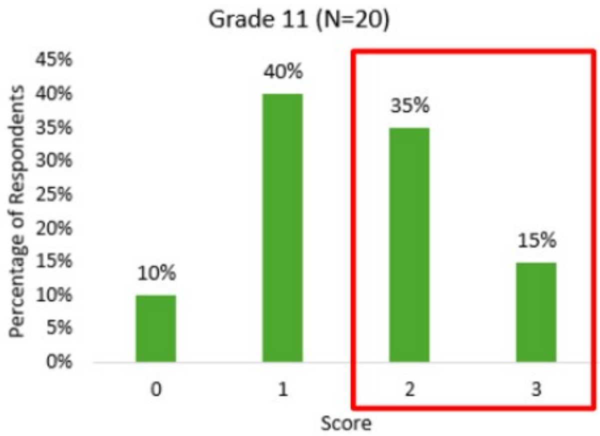


Figure 11: Positive Improvement Observed for Learning Outcome 4

B. Feedback on General Learning Experience

To understand the general learning experience of the students, a feedback form was designed and distributed via Googles Forms after the post-quiz was implemented. The attempt is optional. 19 Grade 9 and 20 Grade 11 responses were collected.

Pertaining to the clarity of the package, 98% of the students found the lesson well-organised and 95% of the students found the theoretical concepts presented in a clear manner. On the efficacy of the pen-and-paper exercise, 92% of the students found it helpful in understanding the mathematical reasoning behind the zombie model (Figure 12).

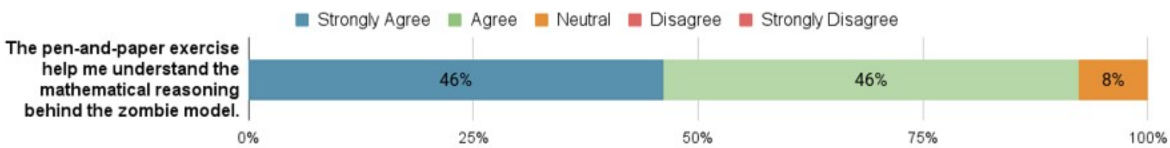


Figure 12: Students Could Understand the Mathematical Reasoning Behind the Zombie Model

On the usefulness of spreadsheet activities, 94% of the students found them helpful in understanding the theoretical concepts (Figure 13) and seeing relevance to the real world (Figure 14).

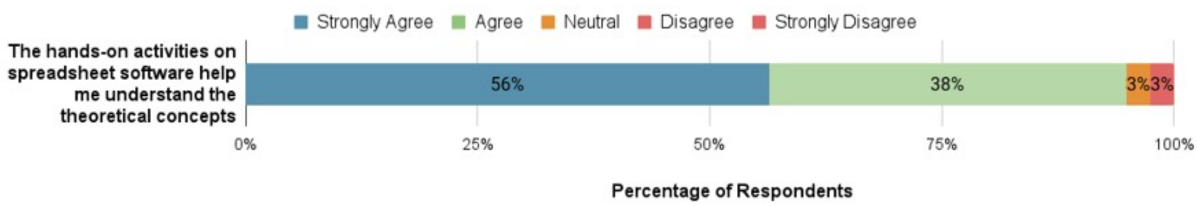


Figure 13: Students Could Understand the Theoretical Concepts

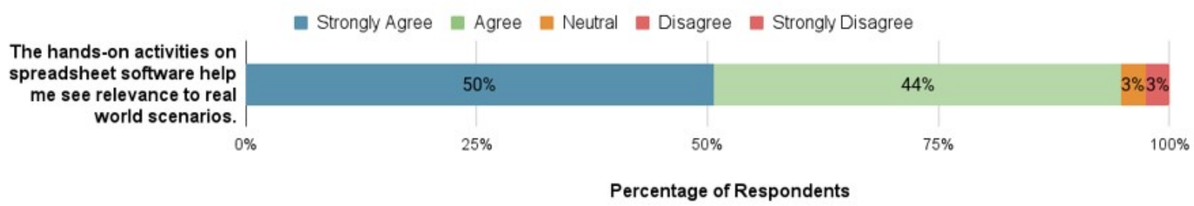


Figure 14: Students Could See Relevance to Real World Scenarios

Overall, in terms of general learning experience, 94% of the students felt engaged in the lesson and were satisfied with the learning experience (Figure 15).

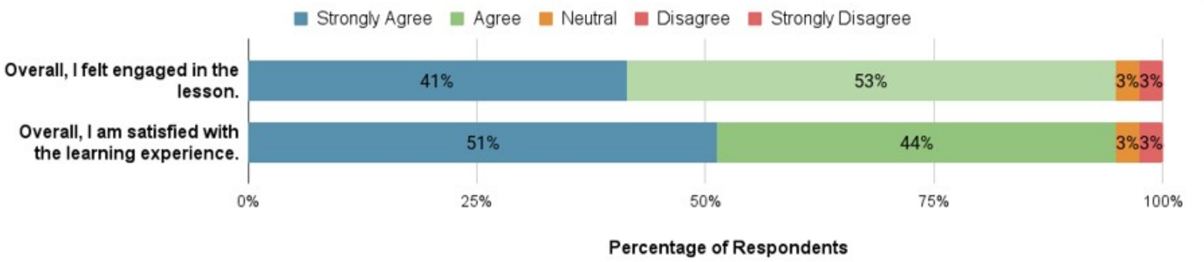
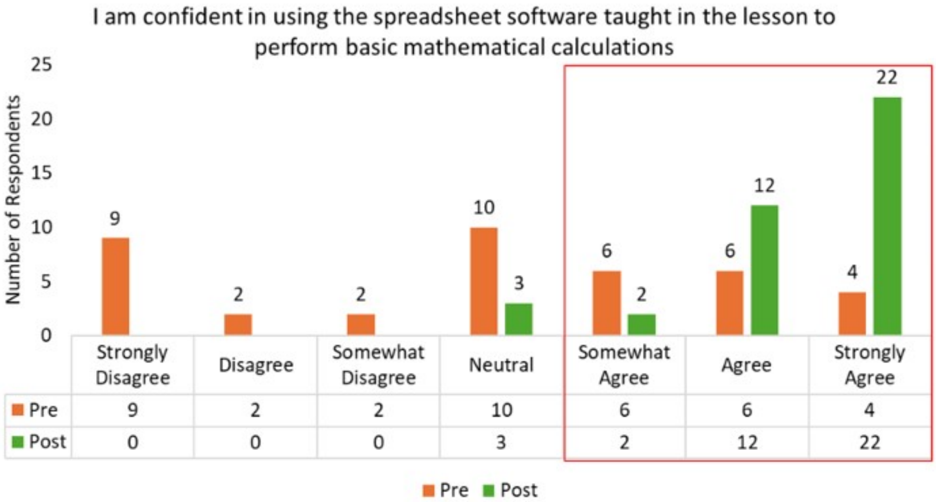


Figure 15: Students Have Positive Feedback on Their General Learning Experience

Lastly, the percentage of students confident in using spreadsheet software to perform basic mathematical calculations increased from 41% to 92% (Figure 16).



	Strongly Disagree	Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Agree	Strongly Agree
Pre	9	2	2	10	6	6	4
Post	0	0	0	3	2	12	22

Figure 16: Students Gained Confidence in Using Spreadsheet Software to Perform Basic Mathematical Calculations

Conclusion: Summary & Future Work

The lesson design demonstrated the process of introducing mathematical modelling in a more accessible and intuitive manner for students, in the context of epidemiology. To ensure proper scaffolding towards the Susceptible-Infected-Recovered (SIR) model, a simpler Susceptible-Infected (SI) model was first explored. Instead of diving into differential equations, we introduced parameters that matter and attempt to model the ground situation. We realised that it is helpful to introduce helper variables such as "I_new" and "R_new" to help students work through the mathematical reasoning behind the models.

Despite using a storyline without direct introduction of differential equations, the mathematical reasoning attained eventually maps back to coupled differential equations for SIR model (Figure 17).

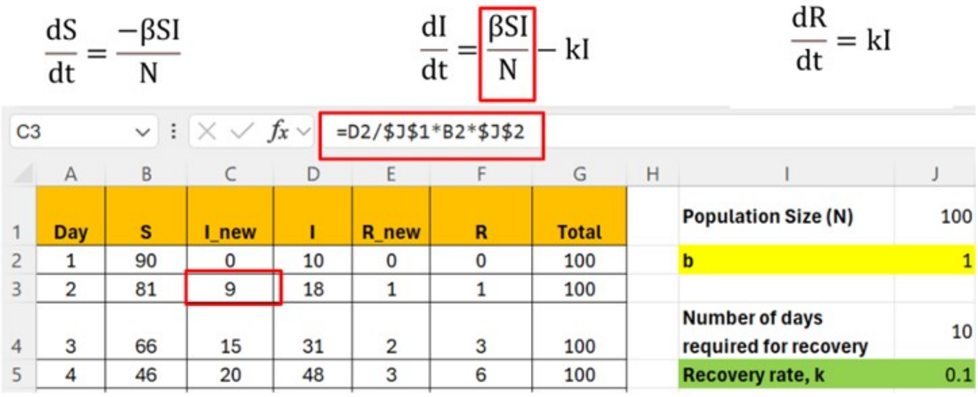


Figure 17: The Spreadsheet Formula Map Back to the Coupled Differential Equations for SIR Model

There is potential of using the same lesson design and narrative to introduce students to other epidemic models, such as the Susceptible-Exposed-Infected-Recovered (SEIR) by incorporating a scenario where a susceptible may be bitten but have a latent period before he turns into a zombie. This allows for a further extension of knowledge.

Given the interdisciplinary nature of this lesson, there is an initiative to adapt the lesson design into an e-learning package on Singapore Student Learning Space (SLS) (2024). This space acts as a national online learning platform that allows students to access curriculum-aligned resources, engaging lessons, and collaborative tools. With the lesson incorporated into this space, this will allow teachers to access and implement it as a teaching resource, while students can use it for self-directed learning.

Once the lesson package is made available, steps can be taken to collect feedback from a broader range of schools to evaluate its effectiveness as a lesson package. This feedback will be instrumental in refining the lesson design, ensuring that the concepts are more accessible and engaging for students. Also, it allows us to assess how can we promote transfer of learning through mathematical modelling.

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Contact emails: scilky@nus.edu.sg
ngmt@nus.edu.sg