A Comparison of Visual Representations of Integer Operations in Middle School Mathematics Textbooks in the Turkish and United States

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Abstract
Textbooks are the principal teaching material in mathematics, as in other subjects, and therefore it is one of the main reasons for analysing mathematics textbooks. Despite the increasing interest of teachers and students in the visuals in textbooks, further information about the representations is needed. The visualisation of mathematical concepts has always been a critical issue in teaching and learning processes due to their abstract nature. Hence, analysing visual representations brings with it the necessity to examine the learning and teaching opportunities that mathematics textbooks offer to both students and teachers. Visual representations are widely used in mathematics textbooks to facilitate students' understanding of integer operations, which they have difficulties with. The study comparatively analysed the visual representations of integers operations in Turkish and United States mathematics textbooks through content analysis using the visual representation analysis scheme in mathematics textbooks, which was formed from three categories considering the related literature. Findings of the study revealed that there were no statistically significant differences in the visual representations of the two countries' textbooks in integer operations. Diagrams are more prominent in Turkish textbooks, whereas pictures and manipulatives are more prominent in US textbooks. However, the visual representations in the textbooks of the two countries are generally used for informative and problem solving. The findings are discussed in terms of mathematics curriculum developers, teachers and researchers to improve the effectiveness of textbooks on teaching and learning.

Keywords: Visual Representations, Mathematics Textbook, İnteger Operations

Introduction

In the mathematics, where students often have difficulty, materials such as textbooks have always been important. As a signifier of the teachind and learning process, the textbook is one of the most fundamental tools in the process (Van den Ham & Heinze, 2018). Sievert and colleagues (2019) state that the textbooks influence the learning opportunities and experiences offered to students. Textbooks are the most important reference source for both teachers and students. The fact that textbooks are so vital brings along debates on their content and quality. Despite having such a wide scope and importance, studies on textbooks have increased especially in recent years (Fan et al., 2013). In order to contribute to the gap in the literature on the comparison of visual representations (VR_s) in textbooks, this study compares Turkish and US mathematics textbooks.

Visual Representations in Mathematics Textbooks

The abstract nature of mathematical concepts requires different ways of teaching or learning mathematics. Textbook content needs to support the presentation of these concepts in the most effective ways. Among these ways, VR_s stand out as one of the most important aids for students in clarifying and understanding mathematical concepts (Presmeg, 1986). The purpose of the visuals in textbooks and their connections with the content should be organized in accordance with the principles of cognitive learning (Smith et al., 2021). Diezmann and English (2001) report that the development of VR_s skills consists of three stages: understanding VR_s, creating appropriate representations, and reasoning with representations. Emphasizing the importance of textbook representations for students, Vinisha & Ramadas (2013) report that the quality and use of VR_S in mathematics textbooks have a direct impact on the learning environment.

Integer operations are among the topics that students have the most difficulties in school mathematics (Turan & Ipek, 2022). Stephan and Akyuz (2012) point out the difficulties in making sense of and modelling the concept of integers and operations with the numbers, which is focusing of students' first reasoning towards the concept of negative numbers. Mathematics textbooks and content contribute to the creation or elimination of difficulties in the subject.

The Cognitive Load Theory

Cognitive load theory was developed by focusing on how information is processed in relation to the capacities of long-term and working memory in general. Sweller (1988) defines cognitive load as the pressure on the learner's cognitive system during the learning process. The structure of the material presented to the learner has an impact on short-term memory and the content of this material creates a cognitive load as it is processed by this type of memory. If the load exceeds the limited capacity of short-term memory, learning cannot take place; however, if this cognitive load is reduced, learning is possible (Paas & Ayres, 2014). When learning materials are not well designed, the cognitive resource requires longer processing time and this leads to the serious problem of learning disabilities. According to a theory developed to control overloading of working memory by building strong schemas in long-term memory, each cognitive process in limited memory generates different cognitive loads.

A single mental representation can be constructed in the brain from verbal or visual information (Mayer, 2009). According to Paivio's (1991) dual coding theory, representation theories consist of two functions: verbal and visual cognitive processing systems. Paivio (1991) also argues that verbal stimuli such as speech and non-verbal stimuli such as touch, taste or sight are processed differently. The difference affects the speed at which information is processed in long-term memory and meaningful learning processes. In view of their importance in school mathematics, the visuals in textbooks should be examined in more detail in order to reduce the cognitive load of students.

The Conceptual Framework and Significance of the Study

Cellucci (2019) described visual representations in mathematics as a means of discovering and understanding mathematical knowledge. Due to the nature of mathematical concepts, it is inevitable to use the representations to understand and solve problems (Presmeg, 2006). In the respect, the use of VRs in mathematics textbooks is a topic that needs to be investigated. In the study, the visual representations used in Turkish and US middle school textbooks on operations with integers were compared. For the purpose, the factors towards the comparison of VRs in a mathematics textbook are listed. The research questions that frame this study are:

1. What is the general distribution of the VRs used in the textbooks of both countries on operations with integers?
2. What are the similarities or differences between the types of VRs (pictures, diagrams, tables, materials) of operations with integers in the textbooks of the two countries?
3. What are the similarities or differences between the roles of VRs (decorative, informative, interpretive) in the two countries' textbooks?
4. What are the similarities or differences between the purposes of VRs in problem solving (understanding, solve, self-regulate) in the textbooks of the two countries?

Method

Textbook Selection and Sampling

The Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) mathematics performances of students in different countries reveal that US students perform better in mathematics than Turkish students, but the difference is not very significant (Table 1 and Table 2). The scores reveal that the two countries are in a *'stand still'* in mathematics. The datas in Table 1 and Table 2 show that both countries are far from their intended levels.

Table 1: TIMSS Mathematics Performances of Turkiye and USA in Grade 4th and 8th

Grade	4			8		
Years	2011	2015	2019	2011	2015	2019
Turkiye	469	483	523	452	458	496
USA	541	539	535	509	518	515
Difference	72	56	12	57	60	19

Levels: Low (400-475); Intermediate (475-550); High (550-625); Advanced (625>)

Table 2: PISA Performance in Mathematics of Turkiye and USA

Years	2012	2015	2018	2022
Turkiye	448	420	454	453
USA	481	470	478	465
Difference	33	50	24	12

Levels: Level1(<420); Level2 (420-482); Level3(482-545);(Level4 (545-607);Level5 (607-609); Level6 (669>)

Textbooks in Turkiye are determined by the central government, while in the US they are determined by local governments and school districts. It is possible to argue that the selection of only one textbook from each country for analysis limits the representativeness of the textbooks available for comparison. However, concepts deal with integers are one of the core mathematical topics in middle school mathematics curriculum in both countries. The selection of middle school textbooks from Turkish and the US was based on the similarity of their content and their accessibility to students (Table 3).

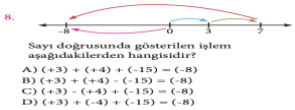
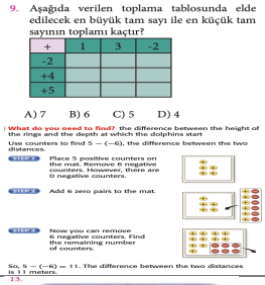
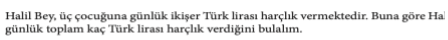
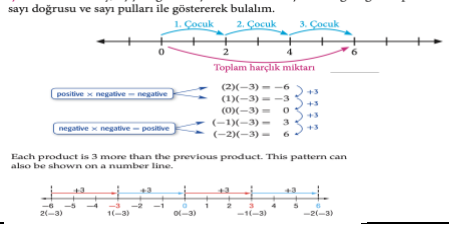
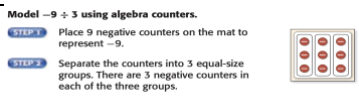
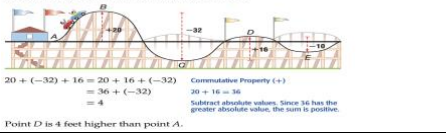
Table 3: Textbooks and Units Selected for Analysis

Country	Publisher Textbook title (Year)	Unit number and title	Pages
Turkiye	Ministry of Education Ortaokul matematik ders kitabı Grade 7 (2023)	1.Tamsayılarla işlemler	13-38
USA	McGraw-Hill Tennessee Math Connect Grade 3 (2012)	2. Add and subtract integers	86-99
		3. Multiply and divide integers	100-114

Data Analysis

As the VR_s in the textbooks contain different shapes and structures, a three-stage coding scheme was developed for the analysis of these representations by using the relevant literature (Carney & Lewin, 2002; Van Garderen, et al., 2021). The categories were revised in accordance with the content of operations with integers. In this context, a total of 111 visual representations were analyzed, 53 in the Turkish textbook and 58 in the US textbook. The categories and related subcategories used to code the representations are presented in Table 4. Also, a Chi-square test was applied to examine whether there were significant statistical differences between the variances of the VR_S in the textbooks of both countries.

Table 4: Framework for the Analysis of Visual Images

Category	Subcategory	VRs examples in math textbooks
Type of VRs	Picture	
	Diagram	
	Table	
	Manipulatives	
Role of VRs	Decorative	
	Informative	
	Interpretive	
Purposes of VRs	Understanding	
	Solve	
	Self-regulate	

Results

A total of 111 VRswere analyzed, 53 in the Turkish textbook and 58 in the US textbook. The findings on the distribution of the visuals used in the textbooks are summarized in Table 5.

Table 5: Summary of VRs Used in Math Select Textbooks

Country	Number of pages sampled	Total number of VRs	Average number of representations per page	% of pages with at least 1 representation
Türkiye	27	53	1.96	78
USA	28	58	2.07	97

Hence, the number of VRs per page in the textbooks of the two countries is quite close to each other. However, the distribution of representations in the US textbook is more balanced than in the Turkish textbook (Table 5). This is because almost 97% of the pages in the US textbook have at least one visual representation, compared to 78% in the Turkish textbook. The distribution of VRs types (pictures, diagrams, tables and manipulatives) is presented in Table 6. The frequencies and percentages of the representation types were analysed and the chi-square test was used to determine whether there was a statistically significant difference.

Table 6: Types of VRs in the Mathematics Textbooks

Country	Picture	Diagram	Table	Manipulative	Total
Türkiye	12 (%22.64)	23 (%43.40)	10 (%18.87)	8 (%15.09)	53 (%100)
USA	17 (%29.31)	15 (%25.86)	10(%17.24)	16 (%27.59)	58 (%100)

**(X^2(3, 111)=685.57, p=.1719 (The result is not significant at p<.05)*

Table 6 shows that diagrams are used more in Turkish textbooks while pictures are used more in US textbooks. As the use of pictures and tables are close to each other in the textbooks, there are relative differences between the use of diagrams and manipulatives. In addition, a Chi-square test was conducted to determine whether there was a statistically significant difference between the types of VRs in Turkish and US textbooks. The results of the test (X^2(3,111)=685.57, p>0.05) reveal that the types of VRs do not differ significantly between the textbooks of the two countries. The distribution between the roles of the VRs (decorative, informative and interpretive) is presented in Table 7.

Table 7: The Roles of VRs in the Two Mathematics Textbooks

Country	Decorative	Informative	Interpretive	Total
Türkiye	12 (%22.64)	39 (%73.59)	2 (% 3.77)	53
USA	15 (%25.86)	39 (% 67.24)	4 (% 6.90)	58

**X^2(2, N=111)=.7764, p=.6782 (The result is not significant at p<.05)*

From Table 7 it is shown that there is no significant difference between the roles of the types of VRs used in the textbooks of each country. In the textbooks of the two countries, VRs were mostly used as informative. More than 2 out of 3 representations used were directed in the direction. However, the rates and numbers of interpretive representations are very low. Only 2 (3.77%) visual representation in the Turkish textbook and only 4 (6.9%) in the US textbook are used in the role. Also, the results of the Chi-square test (X^2(2, 111)=.7764, p>0.05) reveal that the roles of VRs use do not differ significantly between the textbooks of the two countries.

The distribution between the purposes of visual representations (understanding, solve and self-regulate) is presented in Table 8. The frequencies and percentages of each of the purposes of using these representations were calculated, and the chi-square test was used to see if there was a statistically significant difference.

Table 8: Purposes of VRs in the Two Mathematics Textbooks

Country	Understanding	Solve	Self-regulate	Total
Türkiye	10 (18.87)	39 (73.59)	4 (7.54)	53
USA	17 (29.31)	36 (62.07)	5 (8.62)	58

$*X^2(2, N=111 = 1. 8244, p=.4016$ (The result is not significant at $p<.05$)

Table 8 shows that VRs in both countries' textbooks are mostly used for problem solving. As 36 visuals (62.07%) of the representations in the USA textbook were used for problem solving, this rate was as high as 39 visuals (73.59%) in the Turkish textbooks. The use of self-regulation remained at very low levels in both textbooks. Also, a Chi-square test was conducted to determine whether there was a statistically significant difference between the purposes of visual representations in Turkish and US textbooks. The results of this test ($X^2(2,111)=1.8244$, $p>0.05$) reveal that the purposes of using VRs do not differ significantly between the textbooks of the two countries.

Conclusion

The multimodal nature of mathematical concepts is one of the main reasons for using representations. VRs are one of the most preferred types of representations. The results of the study reveal that there are some similarities and differences between the use of VRs in Turkish and US mathematics textbooks. In this context, the representations in the textbooks were analyzed in terms of presentation, type, role, and purpose in terms of operations with integers.

Firstly, it should be emphasized that both countries' textbooks use a large number of VRs in the content of operations with integers. The fact that the use of visuals is inevitable in some situations, as Jitendra and Woodward (2019) point out, is even more true for the topic of operations with integers, which is a very difficult topic for students and teachers. The results of the study show that there is no significant difference in the number of visual representations per page between the textbooks of the two countries. In other words, both textbooks have an average of 2 visual representations per page. However, while 97% of the pages in the US textbook had at least 1 visual representation, this rate was 78% in the Turkish textbook. This difference may be due to the different design logic of the school textbooks of the two countries, confirming Fan and colleagues (2013). It is important to use a variety of VRs in textbooks for understanding operations with integers because the representations contribute to meaningful learning when integrated into the text (Mayer & Moreno, 2003). VRs refer to any type of visual displaysuch as pictures, drawings, maps, diagrams, charts, tables and graphs (Eitel & Scheiter, 2015; Guo et al., 2018) and are presented with or separate from verbal representation (Roberts and Brugar, 2017). In the case of integers, the representations include pictures, tables and diagrams. When multiple representations are used in textbooks, students can more easily construct knowledge through associations between different types of representations. The data show that there is no significant difference between the two textbooks in the types of VRs used in operations with integers. However, the Turkish textbook used diagrams more extensively, whereas the US textbook preferred a more balanced use. Ainswort (1999) emphasizes the complementary nature of this type of representation, stating that diagrams are important because they reveal the relationships between the components that make up the content. While a diagram reveals all possible connections of a concept (Ge et al., 2018), other types of representations such as pictures can be easily misinterpreted if not clarified by text (Coleman et al., 2011). Cook (2006) points out that using diagrams requires less working memory and contributes to meaningful learning by

reducing cognitive load. At this point, focusing on one or more of the representation types may be related to the content or the mentality of the book authors. However, it is clear that a more balanced distribution of representation types in textbooks would contribute more to individual learning and equality of opportunity in learning. In the data obtained in the category of roles of VRs, it is noteworthy that the representations in both textbooks mostly have an informative role. The category implies that the representation is mainly used as an aid. Approximately 1 out of 4 images used in both textbooks is decorative. Chen (2017) considers the overuse of this type of representation, which does not have much effect beyond attracting the student's attention, as a common mistake for textbooks. This is because over-embellishing the content with such illustrations may distract the student from the main focus of the textbook. Interpretive roles, which have an important role in the problem solving process, were found to be given very little space in both textbooks. The situation reveals the need to improve the textbooks in the direction.

Finally, it is found that VRs were mostly used for problem solving in both countries' textbooks. Both 39 (73%) representations in the Turkish textbook and 39 (62.07%) representations in the US textbook were used for this purpose. Van Gardener (2021) draws attention to the activation of the solving angle in situations of justify, solve or explain thinking in the problem solving process. 10 VRs (18.87%) in the Turkish textbook and 10 (29.31%) in the US textbook are focused on organizing or making sense of information. However, VRs of self-monitoring or self-control were very low in both textbooks. In this case, VRs in problem solving processes in textbooks are considered as one of the areas that need to be improved.

The study examined how VRs of integer operations are presented in Turkish and US middle school mathematics textbooks. Since the study analyzed one textbook on operations with integers in each country, there is a limit to the generalizability of the results. The results of the study suggest a number of reasons for a more balanced distribution of VRs in textbooks, especially in the interpretive and self-regulated dimensions. Of course, studies on a broader scale (country, grade level, subject, etc.) need to be conducted. Researches on the relationship between visual representations in textbooks and students' and/or teachers' perspectives can also make a significant contribution to the literature in the context.

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References

Ainsworth, S. (1999). The functions of multiple representations. *Computers &Education, 33* (2-3), 131-152.

Carney, R. N., & Levin, J. R. (2002). Pictorial illustrations still improve students' learning from text. *Educational Psychology Review, 14*, 5-26

Cellucci, C. (2019). Diagrams in Mathematics. Foundations of Science, *24*(3), 583–604. http://doi.org/10.1007/s10699-019-09583-x

Coleman, J. M., McTigue, E. M., & Smolkin, L. B. (2011). Elementary teachers' use of graphical representations in science teaching. *Journal of Science Teacher Education, 22*(7), 613–643. https:// doi.org/10.1007/s10972-010- 9204-1

Cook, M. (2006). Visual representations in science education: The influence of prior knowledge and cognitive load theory on instructional design principles. *Science Education*, *90*(6), 1073– 1091.https://doi.org/10.1002/sce.20164

Diezmann, C. M., & English, L. D. (2001). Promoting the Use of Diagrams as Tools for Thinking. *The Roles of Representation in School Mathematics: 2001 Yearbook*, 77-89, Virginia, USA: National Council of Teachers of Mathematics.

Eitel, A., & Scheiter, K. (2015). Picture or text first? Explaining sequence effects when learning with pictures and text. *Educational psychology review, 27*, 153- 180.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. *ZDM, 45*(5), 633– 646. https://doi.org/10.1007/s11858-013-0539-x

Guo, D., Wright, K. L., & McTigue, E. M. (2018). A content analysis of visuals in elementary school textbooks. *The Elementary School Journal, 119*(2), 244– 269. https://doi.org/10.1086/700266

Jitendra, A. K., & Woodward, J. (2019). The role of visual representations in mathematical word problems. *Cognitive Foundations for Improving Mathematical Learning*, 269–294.

Mayer, R. E. (2009). *Multimedia learning*. 2nd Ed. University Press.

Mayer, R. E., & Moreno, R. (2003). Nine ways to reduce cognitive load in multimedia learning. *Educational psychologist, 38*(1), 43-52.

Paivio, A. (1991). Dual coding theory: Retrospect and current status. *Canadian Journal of Psychology/Revue canadienne de psychologie, 45*(3), 255.

Paas, F., Tuovinen, J. E., Tabbers, H., Van Gerven, P. WM. (2003). Cognitive load measurement as a means to advance cognitiveload theory. *Educational psychologist, 38*(1), 63-71, Lawrence Erlbaum Associates, Inc.

Presmeg, N. (2006). A semiotic view of the role of imagery and inscriptions in mathematics teaching and learning. *In Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, *1*, 19–34. Prague; Charles University in Prague Faculty of Education.

Presmeg, N. C. (1986). Visualisation in high school mathematics. *For the learning of mathematics*, *6*(3), 42-46.

Roberts, K. L., & Brugar, K. A. (2017). The view from here: Emergence of graphical literacy. *Reading Psychology*, *38*(8), 733-777.

Sievert, H., Van den Ham, A. K., Niedermeyer, I., & Heinze, A. (2019). Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic. *Learning and Individual Differences*, *74*, 101716.

Smith, R., Snow, P. Serry, T. & Hammond, L. (2021). The role of background knowledge in reading comprehension: A critical review. *Reading Psychology*, *21*(3), 214-240.

Stephan, M., & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal for research in mathematics education*, *43*(4), 428- 464.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive science*, *12*(2), 257-285.

Turan, S., & İpek, A. S. (2022). An Action Research on Developing Students' Connection Skills in Operations with Integers. *International Journal of Educational Studies in Mathematics*, *9*(2), 67-83.

Van den Ham, A.-K., & Heinze, A. (2018). Does the textbook matter? longitudinal effects of textbook choice on Primary School Students' achievement in Mathematics. *Studies in Educational Evaluation, 59*, 133– 140. https://doi.org/10.1016/j.stueduc.2018.07.005

Van Garderen, D., Scheuermann, A., Sadler, K., Hopkins, S., & Hirt, S. M. (2021). Preparing pre-service teachers to use visual representations as strategy to solve mathematics problems: What did they learn? *Teacher Education and Special Education*, *44*(4), 319-339.

Vinisha, K., & Ramadas, J. (2013). Visual representations of the water cycle in science textbooks. *Contemporary Education Dialogue*, *10*(1), 7-36.