An Analysis of Students' Mathematical Thinking Through Task Sequence on Division in Classroom Using Open Approach

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Abstract
The purpose of this research aimed to analyze students' mathematical thinking that occurred in task sequence on division. The participants were 9 students in grade 4. The research methodology was qualitative method based on Lesson Study processes: collaboratively plan 5 research lessons, collaboratively do with include 4 steps of Open Approach: 1) posing open-ended problems, 2) students' self-learning, 3) whole-class discussion and comparison, and 4) summarizing by connecting students' mathematical ideas. Finally process was collaboratively reflection after lesson. Data collected from students' worksheet, textbook, and field note. The results showed that in mathematics classrooms using the Open Approach, students' mathematical thinking occurred in order of task sequence on division from all 4 characteristics which were 1) The specification of the tasks allowed students to discover the rules of division by seeing the relations of the symbolic sentences generated from the tasks, 2) The generalization of tasks prompted students to generalize the relations of division rules discovered in tasks with specificity, 3) The extension of the tasks allowed students to apply division rules to expand learning to solve division tasks of tens and hundreds, and 4) The integration allowed students to integrate the rules of division and also the division of tens and hundreds in order to solve the division of large numbers.

Keywords: Mathematical Thinking, Task Sequence, Open Approach

Introduction

The purpose of mathematics education extends beyond the acquisition of numerical proficiency. It is a vital tool for fostering essential human qualities and nurturing a creative and competent citizenry equipped to enhance societal harmony (Mangao, Ahmad, and Isoda, 2017). The basic principle of learning mathematics is that student should learn by or for themselves, and in every class we teach the methods of developing mathematics, mathematical ideas, and its values for student's further learning. By teaching mathematical thinking consistently, we can prepare student to think by or for themselves (Isoda & Katagiri, 2012).

Teaching by Open Approach aims that all students can learn mathematics in response to their own mathematical power, accompanying with certain degree of self-determination of their learning, and can elaborate the quality of their process and products toward mathematics (Nohda, 2000). Inprasitha (2022) stated that he incorporated the Open Approach into the Do step of Lesson Study to generate students' independent learning and transform teachers' traditional teaching approach. A classroom using the Lesson Study and Open Approach is a mathematics classroom where teachers use the Open Approach as a teaching approach and use the Lesson Study as a way to improve the teaching approach (Inprasitha, 2015). The Lesson Study process consists of 3 steps: Step 1 collaboratively design lesson (Plan), Step 2 collaboratively observe lesson (Do) which includes 4 steps of the Open Approach: 1) posing open-ended problems, 2) students' self-learning, 3) whole-class discussion and comparison, and 4) summarizing by connecting students' mathematical ideas, and Step 3 collaboratively reflect on teaching practice (See) (Inprasitha, 2011, 2022).

The Center for Research in Mathematics Education at Khon Kaen University translated Japanese mathematics textbooks for use in schools implementing Lesson Study and Open Approach as innovations in teaching mathematics (Inprasitha, 2006). In the textbook, tasks and problems are arranged according to the curriculum sequence. A task with various possible solutions is presented to help student distinguish between what they have already learned and the aspects that are yet to be learned, rather than focusing solely on the answer to the specific task. To solve these tasks, student need to make the unknown understandable. The textbook employs a sequence of extension based on what the student have learned previously and teaches them how to extend mathematical ideas using this extension sequence (Isoda & Katagiri, 2012). This approach is supported by Isoda & Olfos (2021), who highlight the outstanding feature of the textbook-well-configured task sequences for extension and integration. The opportunity for extension and integration is a chance to reorganize their mathematics by comparing what they already knew and their developed mathematical ideas. At the moment of extension and integration on the task sequence, students are able to establish the significant meaning.

Mathematical ideas serve as the basis of content knowledge related to promoting and developing mathematical thinking. (Mangao, Ahmad, and Isoda, 2017). The most important ability that needs to be cultivated in order to instill in student the ability to think and make decisions independently is mathematical thinking (Isoda & Katagiri, 2012). The ability to think mathematically and to use mathematical thinking to solv eproblems is an important goal of schooling. In this respect, mathematical thinking will support science, technology, economic life and development in an economy. Such mathematical thinking is foundational in preparing student to navigate the complexities of the world with analytical and reflective minds. If teachers are to encourage mathematical thinking in students, then they need to engage in mathematical thinking throughout the lesson themselves (Stacey, 2007).

Purpose

The purpose of this research was to analyze students' mathematical thinking that occurred in task sequence on division.

Methodology

Context of Study

The researcher has been taught mathematics with the Lesson Study and Open Approach during the pre-service within teaching practicum school. Currently, the researcher is an in-service teacher in Thailand and has been implementing the Open Approach as a teaching approach to teach mathematics at the present school which is a small school.

The target group comprised 9 students from a total of 9 students in grade 4. These students were enrolled in mathematics classrooms that implemented the Open Approach for 1 semester and were taught by the researcher.

The research content on division for grade 4 students is derived from a mathematics textbook produced in collaboration with the Center for Research on International Cooperation in Educational Development (CRICED), University of Tsukuba, Japan, and the Center for Research in Mathematics Education (CRME), Khon Kaen University, Thailand. In this book, tasks on division, specifically focusing on division rules, are arranged in the following sequence:

1. The rules governing the relationship between divisors and quotients.
2. The rules governing the relationship between dividends and quotients.
3. The rules governing the relationship between dividends and divisors.
4. Apply division rules to solve problems involving the division of tens and hundreds.

Research Procedure

The research methodology was a qualitative method based on Lesson Study and Open Approach (Inprasitha, 2011, 2022):

Step 1: Collaboratively Plan: The Lesson Study group gathers to discuss and design a lesson, taking into consideration students' mathematical thinking, and determines open-ended mathematics problems from the task sequence on division in the grade 4 textbook.

Step 2: Collaboratively Do: The Do step is implementing the planned lesson. The Lesson Study group observes the learning activities with the aim of focusing on students' mathematical thinking. The Do step is incorporated with 4 phases of the Open Approach: 1) Posing open-ended problems is the process in which the teacher presents a problem situation as a task to the student making problem posing, 2) Students' self-learning is the step where students take action to solve their problems, 3) Whole-class discussion and comparison is the stage where students present their ideas to the class, engage in collective discussions, and compare different perspectives. This provides students with the opportunity to learn from their classmates' ideas, and 4) Summarizing by connecting students' mathematical ideas is connecting various student ideas that arise in class. The teacher and students summarize the lesson together based on students' mathematical ideas.

Step 3: Collaboratively See: The Lesson Study group would meet to collectively reflect on the learning activities they have observed.

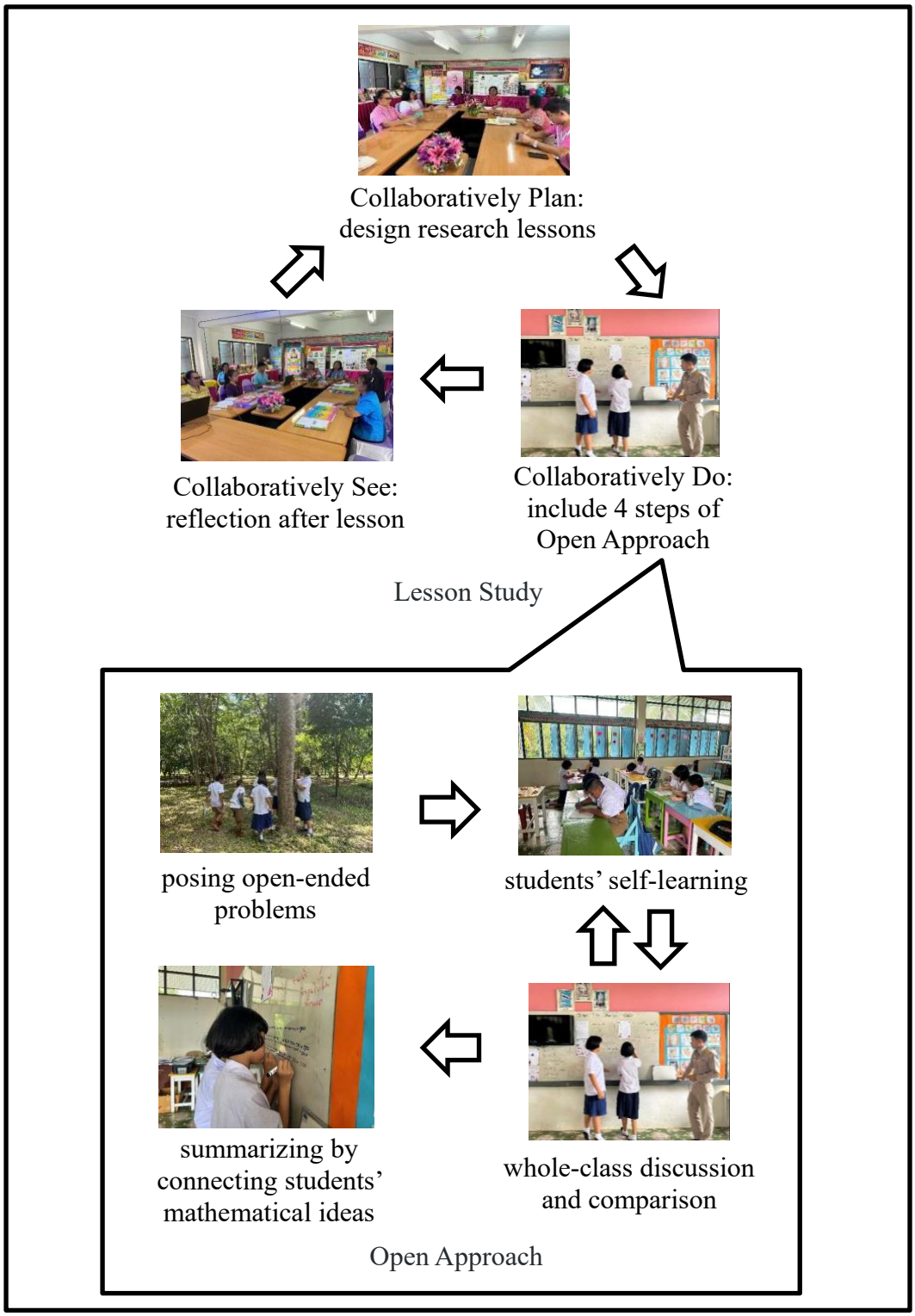

Figure 1: Research Procedure based on Lesson Study and Open Approach (Inprasitha, 2011, 2022).

Data Collection and Data Analysis

Data were collected from students' worksheets, textbooks, and field notes. The data were analyzed using the conceptual framework of Mangao, Ahmad, and Isoda (2017), which comprises the following components: 1) Specification, 2) Generalization, 3) Extension, and 4) Integration.

Results

The research results, derived from empirical evidence collected in grade 4 mathematics classrooms using the Open Approach and implementing the task sequence on division from the grade 4 textbook, posing open-ended problems for students, students take action in solving their problems, students present their ideas to the class, engaging in collective discussions, comparing different perspectives, and summarizing by connecting students' mathematical ideas, demonstrate that students' mathematical thinking occurred in the order of the task sequence on division. These findings will be presented following the conceptual framework of Mangao, Ahmad, and Isoda (2017), encompassing all four characteristics, which are:

1. Specification: Students discover the rules of division by seeing the relations of the symbolic sentences generated from the tasks in Figure 2.

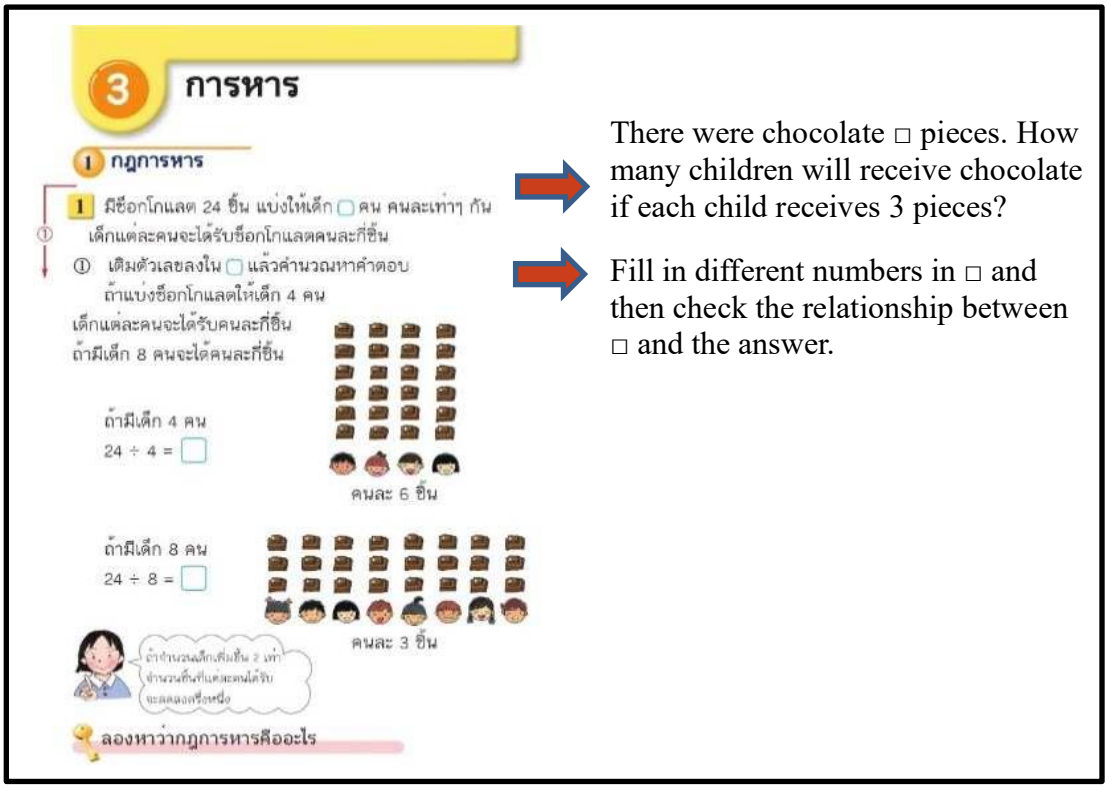

Figure 2: Tasks in textbook on Specification (Inprasitha et al., 2010).

In this task, the mathematical thinking of the students will be illustrated in Figure 3 and Figure 4.

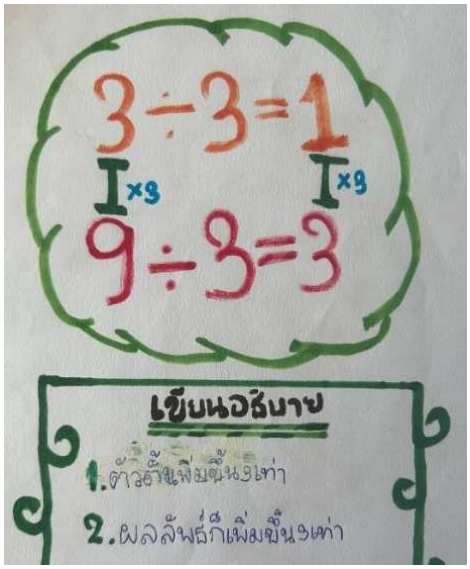

Figure 3: Students' worksheet on Specification (1)

In Figure 3, Students discover the rules of division by examining the relationships in the symbolic sentences: 3 ÷ 3 = 1 and 9 ÷ 3 = 3. These symbolic sentences illustrate the rule of division that if the dividend is tripled, the quotient will triple as well.

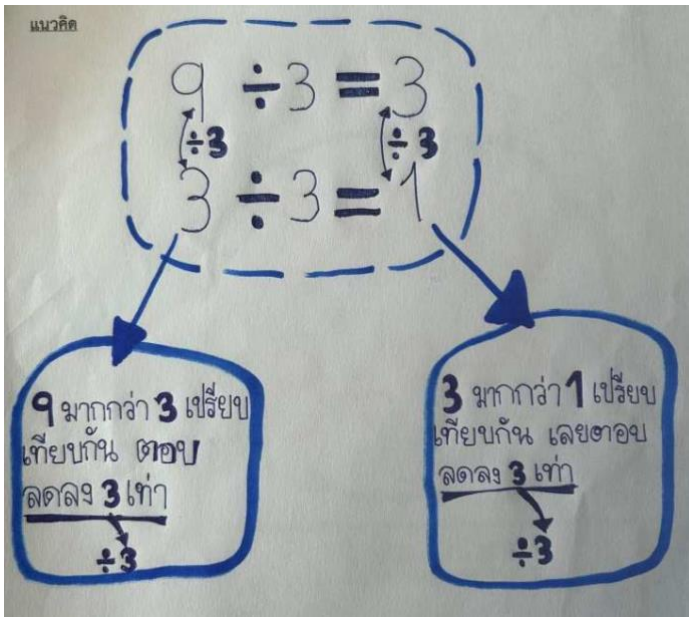

Figure 4: Students' worksheet on Specification (2).

In Figure 4, Students discover the rules of division by examining the relationships in the symbolic sentences: 9 ÷ 3 = 3 and 3 ÷ 3 = 1. These symbolic sentences illustrate the rule of division that if the dividend is divided by 3, the quotient will also be divisible by 3.

2. Generalization: Students generalize the relations of division rules discovered in tasks with specificity in Figure 5.

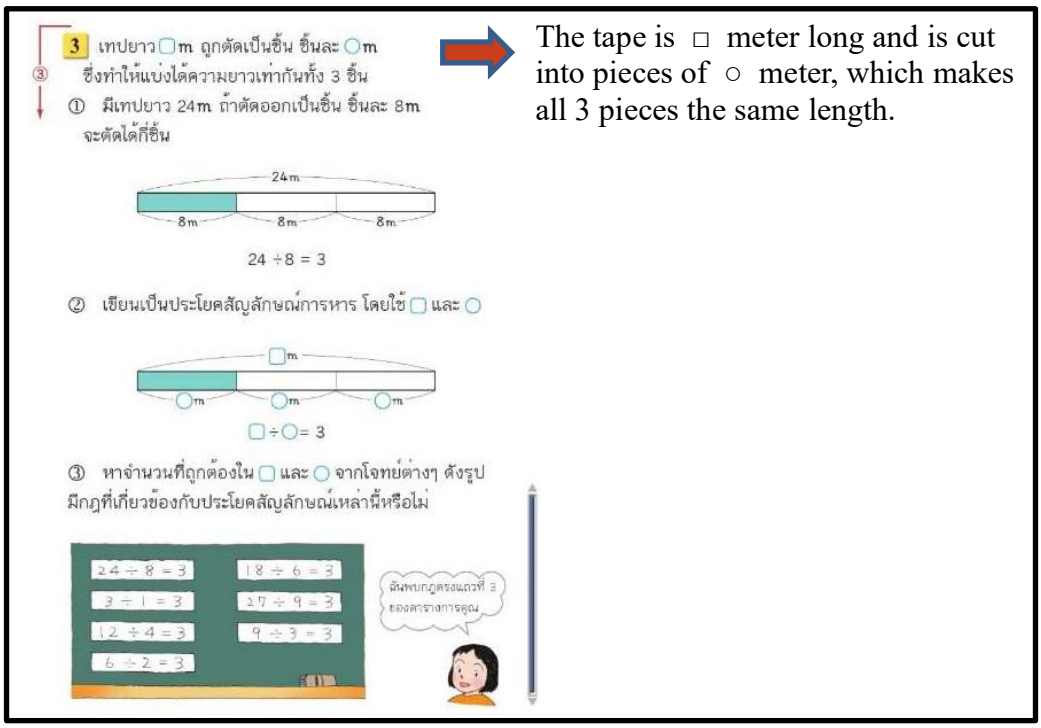

Figure 5: Tasks in textbook on Generalization (Inprasitha et al., 2010).

In this task, the mathematical thinking of the students will be illustrated in Figure 6 and Figure 7.

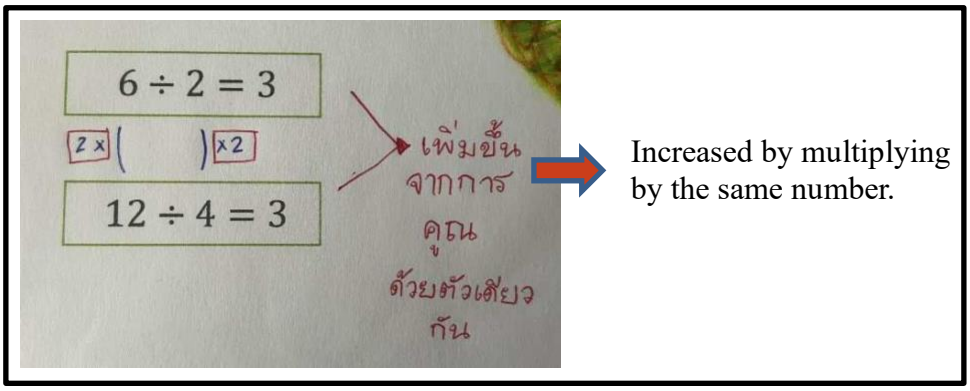

Figure 6: Students' worksheet on Generalization (1).

In Figure 6, Students can generalize by changing from multiplying the dividend by 2, and from multiplying the divisor by 2 to multiplying by the same number. This will result in the same quotient.

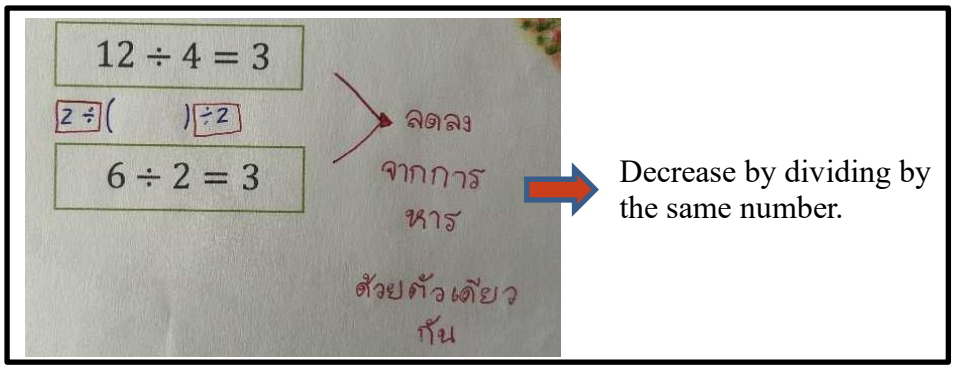

Figure 7: Students' worksheet on Generalization (2).

In Figure 7, Students can generalize by changing from dividing the dividend by 2, and from dividing the divisor by 2 to dividing by the same number. This will result in the same quotient.

3. Extension: Students apply division rules to expand their learning to solve division tasks of tens and hundreds in Figure 8.

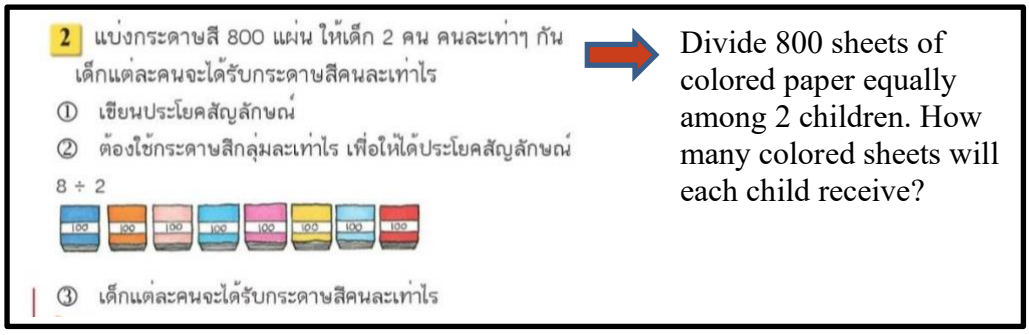

Figure 8: Tasks in textbook on Extension (Inprasitha et al., 2010).

In this task, the mathematical thinking of the students will be illustrated in Figure 9.

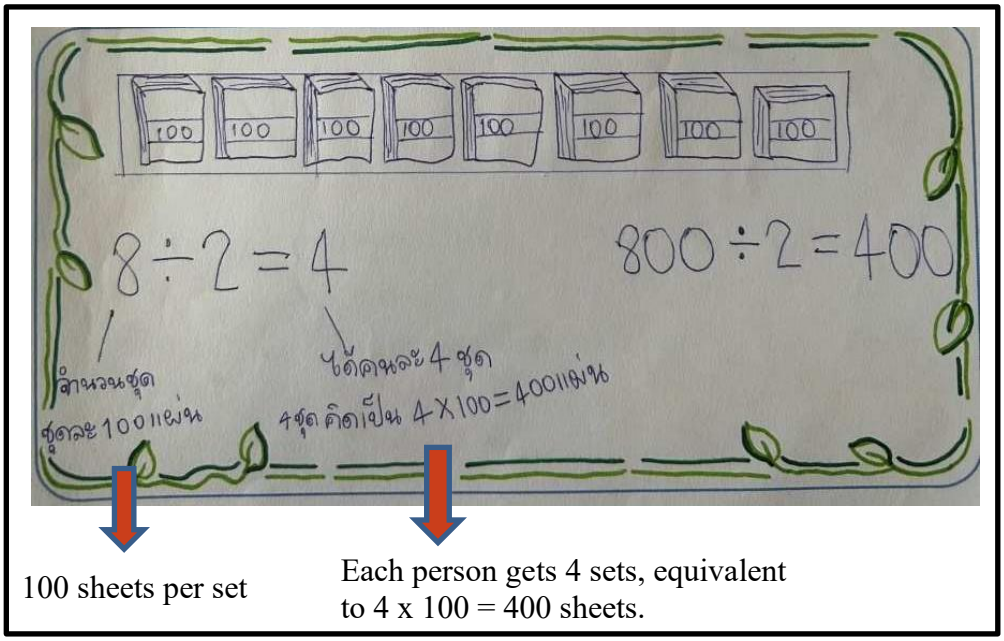

Figure 9: Students' worksheet on Extension.

In Figure 9, Students apply division rules governing the relationship between dividends and quotients to expand learning to solve division tasks of hundreds.

4. Integration: Students integrate the rules of division and also the division of tens and hundreds in order to solve the division of large numbers in Figure 10.

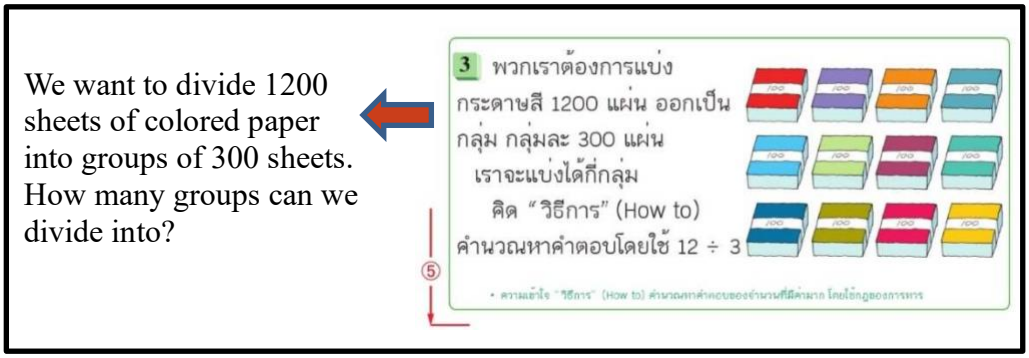

Figure 10: Tasks in textbook on Integration (Inprasitha et al., 2010).

In this task, the mathematical thinking of the students will be illustrated in Figure 11.

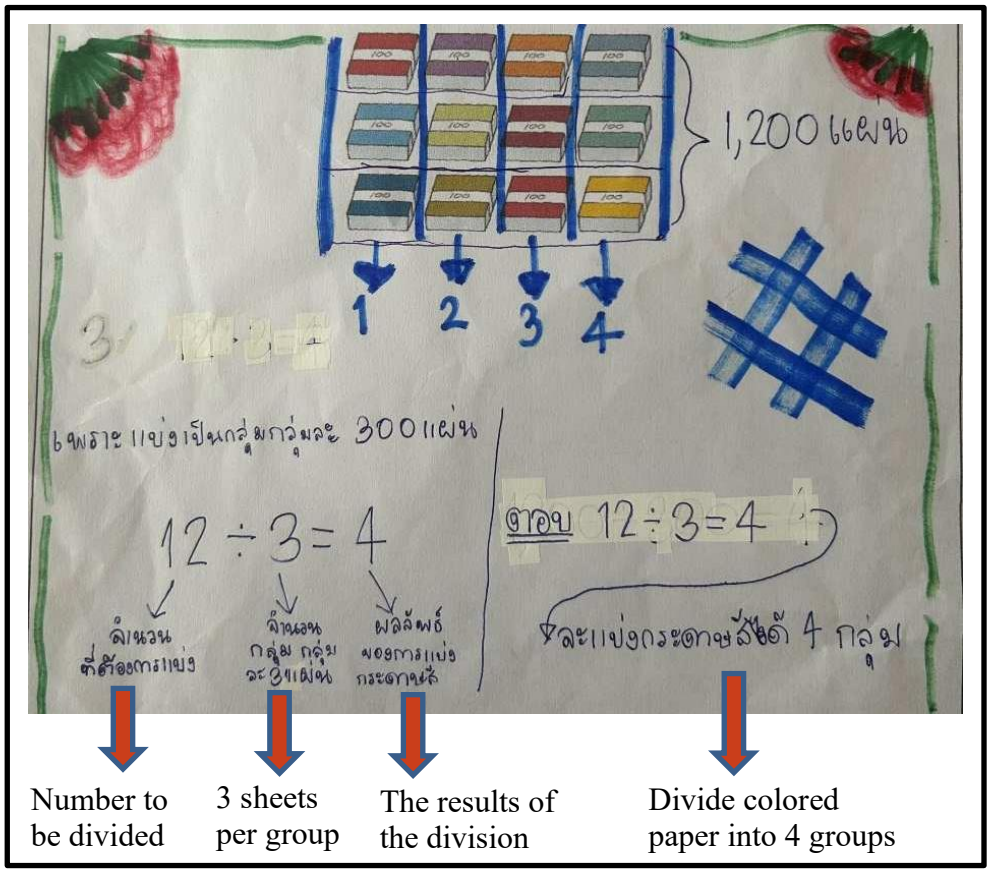

Figure 11: Students' worksheet on Integration.

In Figure 11, Students integrate the rules governing the relationship between dividends and divisors and also the division of hundreds in order to solve the division of large numbers.

Conclusion

The results showed that students' mathematical thinking occurred in order of task sequence on division from all 4 characteristics which were:

1. The specification of the tasks allowed students to discover the rules of division by seeing the relations of the symbolic sentences generated from the tasks.
2. The generalization of tasks prompted students to generalize the relations of division rules discovered in tasks with specificity.
3. The extension of the tasks allowed students to apply division rules to expand learning to solve division tasks of tens and hundreds.
4. The integration allowed students to integrate the rules of division and also the division of tens and hundreds in order to solve the division of large numbers.

Acknowledgments

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