The Performance of Factor and Multiple Problem-Solving for the Fifth Grade with Mathematics Underachievers

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Abstract

The research method adopted was a survey study in quantitative research. The purpose was to explore the performance of factor and multiple problem-solving in fifth-grade underachievers, including performance on the tests, patterns of error and causes of error. The participants came from a public elementary school in Tainan City, a total of 53 mathematic underachievers from 14 classes. The data were collected using quantitative and qualitative methods. The "learning achievement test" designed by the researcher was used as quantitative analysis and 12 students were interviewed as qualitative analysis. The findings were as follow: (a) Regarding factor and multiple problem-solving, the students showed poor performance on the tests; (b) Six patterns of error were identified, including "misunderstanding of prior knowledge," "missing concept of factor and multiple," "fuzzy concept of factor and multiple," "careless calculation and a slip of pen," "problem-solving only by the use of keywords," and "the lack of semantic understanding abilities;" and (c) Six causes of error were identified, including "the lack of prior knowledge led to the misconceptions," "the lack of operational abilities suppressed concept construction," "insufficient life experience hindered conceptual understanding," "semantic comprehension error interfered with problem-solving activity," "the lack of operational concept resulted in problem-solving difficulties," and "insufficient integration capacity affected the effectiveness of problem-solving."

Keywords: mathematics underachievers, factor, multiple, problem-solving performance

Introduction

In recent years, improving the mathematics literacy of students has become the primary educational goal of mathematics education groups around the world. Based on "*The Grade 1-12 Curriculum Guidelines for Mathematics*" in 2018, our country focuses on the ability of students to solve problems and to appreciate and cultivate a positive attitude toward mathematics. The ultimate goal is to achieve the vision of "talent development—a fair chance for all." In 2014, the National Council of Teachers of Mathematics (NCTM) published the Common Core State Standards (CCSS) for Mathematics. It was pointed out in 2010 that the mathematics curriculum standards should include the principle, "Students shall use mathematical reasoning from multiple sources and apply mathematics to solve problems in the real world (CCSS, 6-8)."

According to research data included in the 2015 "*Trends in International Mathematics and Science Study"* (TIMSS) and the "*Program for International Student Assessment"* (PISA), there are great disparities between the performances of high-scoring and low-scoring children in Taiwan. The most worrying of these is the excessive number of mathematics underachievers. Furthermore, today's inclusive education emphasizes that teachers should provide each student with learning and interaction with his or her peers in a natural and integrated environment. However, it is imperative to establish the underachievers in ordinary classes to eliminate fear of learning and to cultivate confidence in solving problems with mathematics.

In terms of the researcher's teaching practice, the application of factor and multiple problems is one of the most difficult skills for elementary school students. The primary reason for this is that the concept of factoring is an extremely abstract mathematical term and quite removed from the life experience of fifth-grade students. Modern cognitive psychologists regard learning as the process of learners actively constructing knowledge, especially in the arena of education. The psychologists pay more attention to process-oriented and error-patterns analysis. The results of incorrect reaction processes can provide many rich messages for instructors and serve as a reference for remedial teaching (Chyn, 1995).

In examining the domestic discussion of the factor and multiple problem-solving process, the research object is aimed mainly at junior high school students and students with learning disabilities. However, few studies have been conducted on the problem-solving performance and the patterns of error in mathematics of underachievers. Therefore, the researcher hopes to understand the learning difficulties by exploring the factor and multiple problem-solving processes of underachievers and, to this end, analyzes the types and causes of errors that may arise. The main purpose of this research is to provide teachers with references in classroom teaching, curriculum design and teaching evaluation in the future. Teachers can implement differentiated instruction according to the different ability levels of students to help underachievers to develop their learning potential.

Purpose of the Study

The study is mainly aimed at the mathematical problem-solving performance of fifth-grade underachievers in a public primary school and discusses the rate of correct results, the patterns of error and the causes of error in the concepts of factor and multiple problems.

Research Questions

Based on the goals list above, the research questions are as follows:

1.What is the rate of correct results in the concept of factor and multiple problem-solving for the fifth-grade underachievers?
2.What are the patterns of error in the concept of factor and multiple problem-solving for the fifth-grade underachievers?
3.What are the causes of error in the concept of factor and multiple problem-solving for the fifth-grade underachievers?

Literature Review

The Meaning of Mathematical Problem-Solving

In mathematics culture, specific activities are generated to solve problems. Santos-Trigo (2014) asserts that "Mathematical problem solving is mainly to foster an inquisitive approach to develop and comprehend students' mathematical knowledge. These activities involve making sense of concepts or problem statements; looking for different ways to represent, explore, and solve the tasks; extending the tasks' initial domain; and developing a proper language to communicate and discuss results (496)." As can be seen from the above, problem-solving can be used as a method of reconstructing knowledge content and testing learning effectiveness. Developing students' problem-solving skills is the primary goal of curriculum and teaching.

The Process of Mathematical Problem-Solving

This study combines the problem-solving theory of two scholars, Schoenfeld (1985) and Mayer (1992). The researcher believes that Mayer explored the problem-solving process from the perspective of cognitive psychology, focusing on the psychological process analysis between the problem-solver and the problem. Schoenfeld's problem-solving process described the problem-solving behavior at each stage, and focused on the behavior analysis and verification process, and evaluated the problem-solver's confidence. Therefore, this study integrates the problem-solving process proposed by the two scholars and divides it into five stages: reading (R), analysis (A), planning (P), execution (I), and verification (V).

The Meaning and Type of Misconceptions Analysis

The misconception also includes the preconception. Fujii (2014) pointed out that misconceptions or alternative conceptions are concepts that are considered reasonable and viable conceptions based on the problem-solver's experiences in different contexts or in their daily life activities from a child's perspective, including students' mental models, children's arithmetic, preconceptions, native theories, conceptual primitives, private concepts, alternative frameworks, and critical barriers (453). In 1985, Mayer divided problem-solving mistakes into three patterns: (a) omission error, (b) specification error, and (c) conversion error. The most serious reason for the conversion error is that many students do not recall the relation-representational model and lack linguistic knowledge.

Methods

The purpose of this study was to understand the performance of factor and multiple problem-solving of fifth-grade mathematics underachievers in public elementary school. Therefore, the survey research in the quantitative analysis was used.

Participants

Participant in the Paper-and-Pencil Test

The pretest phase was to select the fifth grade of the southern A school using the H mathematical version. A total of 56 students in the two regular classes were used for the group test. In the formal stage, a convenience sampling method was adopted to select the required research participants. Therefore, underachievers in the fifth-grade general class of the southern B school were selected, and a total of 53 students in 14 classes were used for the test.

Participants in Individual Interviews

In order to gain a deeper understanding of the patterns and causes of errors in the performance of factor and multiple problem-solving, the researcher analyzed the test contents and answers of the students after the test was completed. The 12 students who chose various problem-solving problems were selected as representatives, and the semi-structured interview was conducted according to the syllabus of the interview.

Materials

Factor and Multiple Achievement Test

This study was based on the relevant literature, the Grade 1-9 mathematics curriculum guidelines, and the contents of the three commonly used versions of the current fifth-grade textbook.

The achievement test questions were divided into three cognitive dimensions, namely "conceptual understanding," "procedure implementation", and "problem-solving." Each dimension included five formats: "factor," "common factor and greatest common factor," "multiple," "common multiple and least common multiple" and "multiple of 2, 3, 5, 10."

In the matter of difficulty and the discrimination index of the test questions, the P value of the "conceptual understanding" dimension was between 0.3 and 0.6 and the D value was between 0.3 and 1.0. The P value of the "procedure implementation" dimension was between 0.2 and 0.7 and the D value was between 0.3 and 0.6. The P value of the "problem-solving" dimension was between 0.2 and 0.4 and the value of D was between 0.2 and 0.4. All three were in line with the P and D values of the ideal questions proposed by the experts.

Procedure

Implementation and Score

In this study, the group test method was first determined, and a total of 56 students in the fifth grade of the southern A school were selected as the pretest participants. The test time was 40 minutes. In succession, a total of 53 underachievers for the fifth grade of the south B school were selected for the group test.

In terms of scoring, this test had a total of 14 questions, and the scoring approach was one point for each question. The highest score that could be achieved was 14, and the lowest score was 0. If the scoring principle was to write the correct answer for each question, 1 point would be awarded. If the question had clerical error, calculation error, solution process error, incorrect answers, or unanswered situation, the question was not given.

On the subject of validity, the researcher, who prepared the test papers, invited mathematics educational experts, senior teachers, and resource teachers to discuss, amend and review all aspects of the test questions. Finally, the researcher also invited three underachievers in the fifth grade to take the test in order to ascertain the speed of the students' answers and the quality of the questions.

In order to determine the feasibility of the test questions and the required test time, the pretest would be conducted by the fifth-grade students of the A school. The Cronbach's α value for this pretest was .75. For the purpose of understanding the patterns and causes of learning errors, the fifth-grade underachievers of the B school were selected for formal testing. The Cronbach's α value for this test was .80, which meant that the reliability of this test was quite good.

Analysis

Paper-and-Pencil Test

This study would summarize the results of the participants' test results and calculate the rate of correct answers for each question in the achievement test, then calculate the correct rate, average correct rate, and total correct rate of each type according to the type of problems, in order to ascertain the problem-solving performance of the fifth-grade students.

Interview Data

The researcher conducted qualitative interviews according to the syllabus of the interview. During the interview, the researcher used the triangulation method of different data sources such as problem-solving performance, on-site recording, and recording, hoping in this way to analyze the problem-solving performance of students objectively and improve the validity of data analysis.

Results and Discussion

Student's Problem-Solving in the Concepts of Factor and Multiple

After the collation and summary of the test papers, the researcher calculated the rate of correct answers for each type of problem, the conceptual average correct rate, and the overall correct rate for factor and multiple problems, as shown in Table 3. It can be seen from Table 3 that the overall correct rate of the "Factor and Multiple Achievement Tests" for students was 47.0%, which showed that the effect of learning factors and multiple urgently needed to be strengthened. It was worth further discussion.

Table 3

Students' Problem-Solving Performance in the Factor and Multiple Achievement Test

Concept	Type of problem	Question number	Correct rate for each type of problem (%)	Conceptual average correct rate (%)	Overall correct rate (%)
Factor	Factor judgment	1	71.7%	56.6%	47.0%
	Factor algorithm	6	67.9%		
	Factor application	11	49.1%		
Common factor、Greatest common factor	Judgment of common factor and greatest common factor	2	32.1%	47.2%	
	Algorithm of Common factor and greatest common factor	7	67.9%		
	Application of Common factor and greatest common factor	12	41.5%		
Multiple	Multiple judgment	3	71.7%	49.1%	
	Multiple algorithm	8	52.8%		
	Multiple application	13	22.6%		
Common multiple、Least common multiple	Judgment of common multiple and least common multiple	4	60.4%	34.6%	
	Algorithm of common multiple and least common multiple	9	22.6%		
	Application of common multiple and least common multiple	14	20.8%		
Multiple of 2, 3, 5, 10	2, 3, 5, 10 multiple judgment	5	66.0%	48.1%	
	2, 3, 5, 10 multiple algorithm	10	30.2%		

Notes: 1. Correct rate for each question type = (number of correct answers to the question ÷ total number of questions) ×100%

2. Overall correct rate = (total number of correct answers ÷ total number of questions) ×100%

Students in the Patterns of Error in Factor and Multiple Problem-Solving

The researchers compared the problem-solving performance of the 53 underachievers in the test according to the three concepts of "conceptual understanding," "procedure execution," and "problem-solving." The statistics were classified into six possible patterns:

Conceptual Understanding

In the students' performance in conceptual understanding, the total error rate was 39.6%. Moreover, incorrect answers were more serious in the concept of common factor and common multiple problem-solving than in other types of questions. These errors mainly followed two patterns: "misunderstanding of the prior knowledge" and "missing concept of factor and multiple."

Procedure Implementation

In the students' performance in procedure implementation, the total error rate was 51.7%. Moreover, incorrect answers were more serious in the concept of common multiple and least common multiple than in other types of questions. These errors mainly followed two patterns: "fuzzy concept of factor and multiple" and "careless calculation and a slip of pen."

Problem-Solving

In students' performance in problem-solving, the total error rate was as high as 66.5%. Many of the students were left blank spaces on the test papers, showing that they had given up trying to solve the problems. These errors followed two patterns: "problem-solving only by the use of keywords" and "the lack of semantic understanding abilities."

Causes of Error of Factor and Multiple Problem-Solving

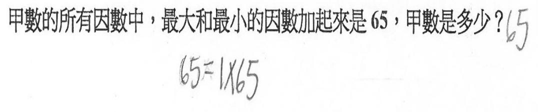
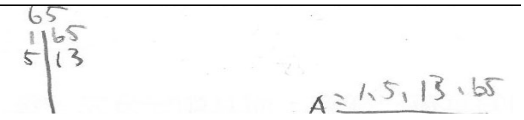
After classifying the factor and the multiple patterns of error, the researcher further analyzed the causes of the students' errors through qualitative interviews.

The Lack of Prior Knowledge Led to the Misconceptions

一、40÷8=5，Is 8 the factor of 40? Is 8 a multiple of 40?	三、12×6=72，Is 72 a factor of 12？Is 72 a multiple of 12?
SB01	SG07
40÷8＝5，請問 8 是 40 的因數？還是倍數? A：倍數	12×6=72，請問 72 是 12 的因數？還是倍數? A：因數
Problem-Solving Analysis	
⦿ **Analysis** T: What is this question asking about？ SB01: Eight is the factor of 40, or 8 is a multiple of 40. ⦿ **Planning** T: Can you tell me what this question is being asked about? How do you judge? SB01: Eight is a multiple. Because 8 multiplied by 5 is 40, when multiplication is used, the number becomes larger, so it is a multiple.	⦿ **Analysis** T: What is this question asking about？ SG07: Seventy-two is a factor of 12, or 72 is a multiple of 12. ⦿ **Planning** T: Can you tell me what this question is being asked about? How do you judge? SG07: Seventy-two is a factor, because 12 multiplied by 6 is 72, so 12 is a multiple and 72 is the factor.

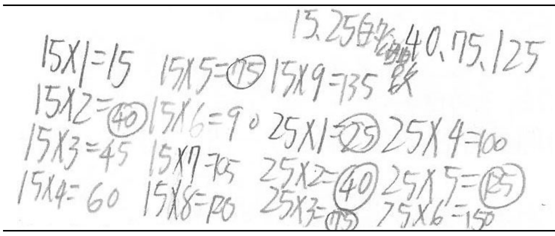
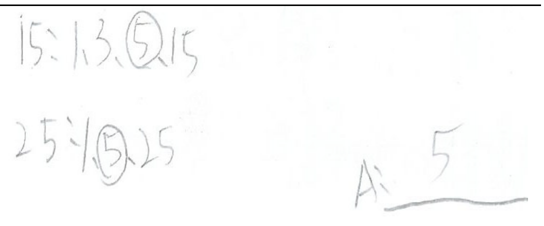
It was found in the interviews that the students mistakenly believed that the multiplication was greater and the division was more and less. Therefore, as long as the title appeared to be multiplied, it was recognized as a multiple, and if it appeared to be divided, it was recognized as a factor. This kind of erroneous prior knowledge led to many misconceptions in learning factor and multiple problem-solving, and indirectly affected learning outcomes. This comports with Graeber and Campbell's (1993) argument. Most students have the misconception that "multiplication will become larger and division will become smaller" when they solve multiplication or division word problems.

The Lack of Operational Abilities Suppress Concept Construction

二、Among all the factors of the number 甲, the maximum factor plus minimum factor is 65. What is the number of 甲?	
SB02	SB03
	
Problem-Solving Analysis	
⦿ **Planning** T: What do you think is the number of 甲? What is the maximum factor? What is the minimum factor? SB02: The number of 甲 is 65, the maximum factor is 65 (. . . Hesitate about 20 seconds) the minimum factor…… I don't know. T: Why do you write the formula 1✕65 at that time? SB02: (. . . Hesitate about 15 seconds) Because 65 is equal to 1 x 65.	⦿ **Planning** T: What do you think is the number of 甲? What is the maximum factor? What is the minimum factor? SB03:The maximum factor is the largest number in it, the minimum factor is the smallest number in it, so the number of 甲 is 65. T: Could you talk about why this formula is listed? SB03: I list all the factors of 65, with possible answers of 1, 5, 13, and 65.

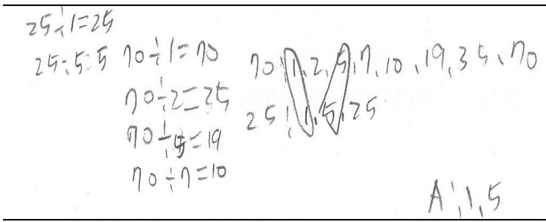
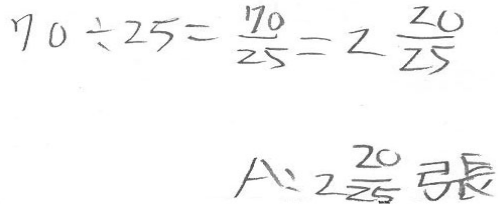
It was discovered from the interviews that students had difficulty in making meaningful connections between factor and common factor. If the structure of the factor was used, the student needed to have the measurement operations ability to understand the concept of the factor. The so-called "measurement operations" meant that the student could grasp the two parts of the part – the whole relationship. From the interviews, it was found that the students' understanding of factor and multiple was often lacking in a tightly connected network, so incorrect interpretations of rules were often used to define or interpret the relationship between the two nouns, which led to failure in problem-solving.

Insufficient Life Experience Hindered Conceptual Understanding

七、Please write all the common factors in 15 and 25.	
SB04	SB05
	
Problem-Solving Analysis	
⦿ **Analysis** T: What is this question asking about? SB04: Seek common factor。 T: What do you think common factor is? SB04: The common factor is the same number when calculating. ⦿ **Planning** T: Can you talk about why this formula is listed? SB04: I will write the factors of 15 and 25 first, then circle the same number, which is their common factor.	⦿ **Analysis** T: What is this question asking about? SB05: Find common factor. T: What do you think common factor is? SB05: The common factor is the same number with both factors. ⦿ **Planning** T: Can you talk about why this formula is listed? SB05: I first find out the factors of 15 and 25, and then find two common factors, . . . Wow! I don't circle this number 1.

In the context of the life experience of students, because the concepts of factor and multiple were more abstract these terms were rarely discussed in daily life, and because students did not understand these specific terms they were confused about the concepts of factor and multiple. Finally, it was found in the interviews that it was difficult for students to develop an understanding of the meaning of factor and multiple through specific activities. In terms of the lack of the relationship between factor and multiple, the ability to link and classify was often affected by their confusion about the concepts of both, and indirectly led to a high error rate.

Semantic Comprehension Error Interfered with Problem-Solving Activity

十三、Wei has more than 70 game cards, which are divided equally among 25 people. How many game cards does Wei have?	
SB06	SG08
	
Problem-Solving Analysis	
⦿ **Analysis** T: What is this question asking about? SB06: How many game cards has Wei ? T: Do you think that 70 game cards are more than 70 cards or exactly 70? SB06: (Mm. . .) Because I don't know how many more than 70 game cards there are, I use 70 game cards to calculate.	⦿ **Analysis** T: What is this question asking about? SG08: How many game cards has Wei ? T: Do you think that 70 game cards are more than 70 cards or exactly 70? SG08: More than 70 game cards mean 70 game cards.
⦿ **Planning** T: Can you talk about why this formula is listed? SB06: Because it is divided equally, so I find the factors of 70 and 25.	⦿ **Planning** T: Can you talk about why this formula is listed? SG08: Because it is a bisector, the number will be smaller. So 70 divide by 25.

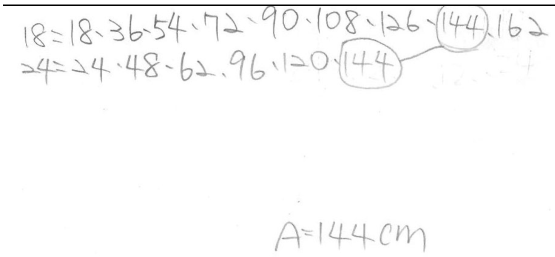
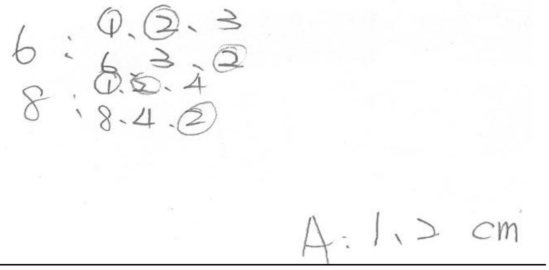
Reading and understanding the meaning of the word problem was the primary task of the problem-solving. When students understood the problem situation, they could determine the best problem-solving strategy. Moreover, through the students' interview data, it was found that many students face problem-solving difficulties because they did not think deeply enough about the problem. Students judged the surface meaning of the keyword in the title, and then calculated or decided how to solve the problem. For example, if the word "divide" appeared in the title, "division" was used. This decision ignored the understanding of the overall problem situation, which leads to an incorrect solution.

The Lack of Operational Concept Resulted in Problem-Solving Difficulties

十一、Hao harvests 56 strawberries and distributes them to the friends. Everyone will get as many strawberries as they can. If the strawberries are just divided equally, how many strawberries can they get? (Please write all possible answers.)	
SG09	SG10
56÷ 1、2、4 56、28、4 A: 1、2、4、28、56顆	56÷2=18 56÷8=7 56÷4=14 56÷9=6 56÷6=9 56÷7=8 A: 2.4.6.7.8.9
Problem-Solving Analysis	
⦿ **Planning** T: Can you talk about why this formula is listed？ SG09: Because this question is divided, the equal division is the meaning of division. So it is the factor. I find the factor of 56. ⦿ **Execution** T: How do you find the answer to this question? SG09: Mm. . . (After talking to myself for 10 seconds, write the process on paper). I use the division method, because 56 divided by 1, 2, . . . and until it is contained.	⦿ **Planning** T: Can you talk about why this formula is listed？ SG10: Because the question is said to be divided equally to friends. If you give it to two friends, you can get 18. If you give it to four friends, you can get 14. . . . T: Do you think that you can send all 56 strawberries to one friend? SG10: Mm. . . .(Hesitate about 9 seconds) I think it shall be ok? T: Why don't you think of sending all the strawberries to one friend at that time? SG10: Because the question say to be sent to the friends, I think it must be sent to at least 2 friends.

It could be seen that both “factor” and “multiple” had the characteristics of an operation concept. The learning of operation concepts was subject to complex information processing. Learners had to undergo identification, analysis, reasoning, and other operational activities to internalize and master this concept. Through interviews with students, it was found that many students were often unable to follow this process when searching for factors or that they did not develop in order to enumerate and used the “trial and error” method. Because the way to find the factor and multiple was not correct, it was easy to find more, mistaken, or time-consuming situations, indirectly causing the failure to exhaust all the numbers when looking for a factor or multiple of a certain number, so that the problem-solving activities failed.

Insufficient Integration Capacity Affected the Effectiveness of Problem Solving

十二、There is a rectangular cardboard with a length of 18 cm and a width of 24 cm. The cardboard is cut into several squares of equal size. The length of the square is an integer centimeter. What is the length of the side of this square?	十四、Yun uses a number of rectangular cards that are 6 cm long and 8 cm wide to form a square. What is the minimum length of the side of this square?
SG11	SB12
	
Problem-Solving Analysis	
⦿ **Planning** T: Can you talk about why this formula is listed？ SG11: Because the item ask the length of square, and the length multiplied by the width equal the area. So use multiples, and then find two identical numbers. ⦿ **Execution** T: What do you think the answer you calculate is? SG11: It refer to the length of the square. ⦿ **Verification** T: Will you check your calculated answer? Why? SG11: No, because the teacher don't teach it. I use the check answers only when I is calculating. T: Are you confident in the answer you have calculated? SG11: (Laugh. . . .) It is fine to write. Anyway, at least the success probability will be.	⦿ **Planning** T: Can you talk about why this formula is listed？ SB12: The question is what the minimum side length of the square is it? So I have to find the factors of 6 and 8, and then figure out that the common factors are 1 and 2. ⦿ **Execution** T: What do you think the answer you calculate is? SB12: The side length of the square is 1 cm or 2 cm at least. ⦿ **Verification** T: Will you check your calculated answer? Why? SB12: No, because the teacher don't teach it. T: Are you confident in the answer you have calculated？ SB12: Not necessarily. Sometimes I will have confidence when I see it very simple, but I will have no confidence in the word problem.

From the problem-solving process and interview data of the students, it was found that in the analyzing stage, the students are confused and had a poor understanding of the basic concepts of factor and multiple, and they constructed false or incorrect representations of the knowledge model. In the planning stage, because of the lack of basic mathematical concepts, it was impossible to balance the correctness of the problem-solving strategy in writing, which made it impossible to assess whether the calculated answer was reasonable. During the execution phase, it was more likely that the students would complete the problem-solving task in an inefficient manner because they could not accurately grasp the information presented on the topic, or their understanding was affected by the mental set or the functional fixedness. In the verification phase, the students lacked the ability and method to test answers, so they did not know how to check the calculated answers, which affected the correct rate of problem-solving.

Conclusions

This study mainly focused on the results of the factor and multiple achievement test and interviews and discussed the performance, error patterns, and causes of underachievement in the fifth grade. The total rate of correct student answers in the multiple achievement test was 47%, and the rate of correct answers in the factor, common factor, and greatest common factor test was about 50%. The rate of correct answers in the multiple, common multiple, and least common multiple test was about 50%. The results showed that the students were not well versed in overall problem-solving of the factor and multiple concepts.

Six patterns of error were identified, including “misunderstanding of prior knowledge,” “missing concept of factor and multiple,” “fuzzy concept of factor and multiple,” “careless calculation and a slip of pen,” “problem-solving only by the use of keywords,” and “the lack of semantic understanding abilities.”

Six causes of error were identified, including “the lack of prior knowledge led to the misconceptions,” “the lack of operational abilities suppressed concept construction,” “insufficient life experience hindered conceptual understanding,” “semantic comprehension error interfered with problem-solving activity,” “the lack of operational concept resulted in problem-solving difficulties,” and “insufficient integration capacity affected the effectiveness of problem-solving.”

In response to the conclusions, the researcher put forward five suggestions for teachers’ reference in the teaching of factor and multiple materials, including “providing students with factor and multiple prior knowledge and experience,” “introducing the concept of factor and multiple through life situations,” “promoting students’ reading comprehension abilities regarding factors and multiple,” “paying attention to the development of the concept of factor and multiple operation,” and “improving students’ problem-solving beliefs in mathematic.”

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