Van Hiele's Instructional Model In Teaching Of Tangents And Secants Of Circles

Melchor Del Rosario, Don Jesus Gonzales High School, The Philippines

Abstract
The research study aimed to identify the pupils' skill in proving mathematical concepts about tangents and secants of circles using Van Hiele's instructional model. Quasi – experimental method of research was used in this study. Quantitative, since descriptive analysis was employed on the scores of the participants and qualitative since the research dealt with the description of the different performance of group of students in terms of learning, for the participants belonging to low performing, average performing or high performing. The study was conducted during the first semester of S.Y.2017 – 2018 using fifteen students from Grade 10 as the participants of the study. Participants were given a pretest prior to the discussion about tangents and secants of circles and posttest was administered after the instruction using the Van Hiele's Instructional Model's five levels of learning. The results were then statistically evaluated, analyzed, and interpreted. Percentage, mean and t-test were utilized to analyze and interpret the data. Qualitative analysis was drawn from the descriptive rating of the selected 15 participants of the experimental group. This study found out that the participants with the use of Van Hiele Instructional Model showed poor and satisfactory performances on the pretest and very satisfactory and excellent on the posttest. Moreover, the participants showed improved scores in all levels in their mathematics performance. On the pre-test and posttest mean scores of each student, high performing participants improved their skills in Mathematics from Level 1 to level 5 instructional model. Average performing participants and low performing participants also showed improvement from their pre-test mean scores to their posttest mean scores. Van Hiele Instructional Model is effective.

Keywords: Van Hiele's Instructional Model, Tangents, Secants

Introduction

Mathematics is indeed a difficult and of little interest to certain people. Ganal and Guiab (2014) added, "Mathematics has importance over and above the application of basic numeracy skills. It is also the primary tool for developing student's logical thinking and higher-order thinking skills. Mathematics also plays a major role in some other scientific fields, such as engineering and statistics. With this, a positive attitude towards learning mathematics among students is an important goal of mathematics educators in the education system."

Paja (2001) said that students learn hardly anything that is being taught but if they learn through hands-on and minds on approach, with the law of readiness, and if what they learn will be incorporated into everyday activities and in subjects like languages, arts, sciences, social studies, music, physical education, life skills and performing arts, it will just be easier for them. Since students in today's generation are practical work learners, investigational learners, and sociable learner.

Thus, ln 2001, the Department of Education emphasizes that teachers must assist the Filipino child to discover his/her full potential in a child-centered and value-driven teaching-learning environment and thereby, enable him/her to create his/her destiny in the global community. Learning, therefore, must be focused in nurturing a child through a child-centered instruction.

Student-centered learning, according to Jones (2007), also known as learner-centered education, broadly encompasses methods of teaching that shift the focus of instruction from the teacher to the student. In original usage, student-centered learning aims to develop learner autonomy and independence by putting responsibility for the learning path in the hands of students. Young and Patterson (2007) added that student-centered instruction focuses on skills and practices that enable lifelong learning and independent problem-solving. The constructivist learning theory of Jean Piaget is the basis of student-centered learning theory and practice which emphasizes the learner's critical role in constructing meaning from new ideas, information and prior experience.

The Mathematics teachers' approach to geometric instruction as cited by Dongwi (2014), determines a large extent in the mathematical thinking strategies and dispositions of learners. Cabahug (2012) as cited by Mendoza (2016) mentioned that Mathematics is an essential subject in the curriculum. But in teaching mathematics, one of the common problems of teachers is how to teach symbols and abstract concepts in such a way that the students would easily understand the topic and make sense of them. One way is the use of the different teaching aids which are reflected in Bruner's theory. In his theory, Bruner (1967) says that conceptual understanding can be enhanced if students will be exposed to different representations of concepts. In particular, he identifies three main types of representations: concrete, iconic, and symbolic.

Studies such as Ismael (1998) pointed out that geometry is one of the hardest branches of Math facing the students and this difficulty refers to the lack of geometric concepts acquisition.

Van Hiele's (1986) solution to overcoming these problems is for teachers to take responsibility for their teaching and to make appropriate choices. For example, if the curriculum is not suitable for learners, design your own and if the textbook is inappropriate for teaching and learning in your classroom, restructure it to suit the environment. The difficulties that learners experience with geometric conceptualization arise from various factors, but their inability to reason at a higher level of geometric thinking does not lie solely within their learning ability or motivation.

The Van Hiele theory identifies a sequence of five hierarchical levels of geometric thinking. These thinking levels are recognition, analysis, ordering, deduction, and rigor. According to the Van Hiele's theory, "students move sequentially from one level of thinking to the next [level] as their capability increased" (Gutierrez, Jaime & Fortuny, 1991).

The levels of thinking comprise a hierarchical nature. They are logically structured to suggest that learners move from lower to higher levels of thinking in geometry. The current is a prerequisite for the next level. For example, "the recognition of a figure at Level 1 is an essential prerequisite for Level 2. The consideration of properties at Level 2 will eventually lead to Level 3 understanding where students see relationships between them, i.e., how one or two properties lead to a third" (Pegg, 1992). The fourth level leads to conceptual understanding of geometrical proof and development and of theorems and postulates.

Difficulties in teaching geometry persisted in the Van Hieles' years of teaching, despite their change of geometric instruction over the years. Van Hiele (1986) chronicled; "in the years that followed, he changed explanation many times, but the difficulties remained. It seemed as though he was speaking a different language". They then developed a framework of teaching phases that helped teachers to move their learners from one level to the next. Van Hiele-Geldof (1958, as cited in Fuys, et al., 1984) stresses that learners cannot progress through the levels of thinking without proper instruction. Hence, it is important that the teachers' instruction is pegged at the appropriate Van Hiele level to enable learners to attain the highest possible level in their learning environments according to Dongwi (2014).

One of the goals of teaching mathematics subject is to improve the students' geometric thinking levels. Geometric thinking is valuable in many scientific, technological and professional subjects as stated by Olkun, Sinoplu, & Deryakulu, (2005). Some studies have pointed to the significant role of the teaching method in developing geometric thinking skills (Ghneim, 2012). As per the Philippine Mathematics Curriculum is concerned, Cruz (2015) in one of his articles in Philippine Star Global cited that the enormous change is the spiral approach of the content. Today, students learn all the areas of mathematics starting in Kindergarten to twelfth grade. The learning is enhanced little by little and continuously as the students go up and promoted in the ladder of basic education. Learning Geometry as early as Kindergarten will enable students to master different skills in order to describe geometric concepts like shapes, analyze their characteristics, and make a comparison between the geometric systems.

One of the theories that help greatly and efficiently in teaching geometry is Van Hiele which attracted educationists' attention because it helps effectively in teaching

geometry to the students through the different school stages. The Van Hiele model from this theory includes five phases which are: information, directed orientation, explication, free orientation where students are involved in tasks relying on them, and integration where the students summarize the concepts learned from activities (Mistretta, 2000).

The Van Hiele model is considered one of the most important models in teaching the subject geometry and the geometric concepts and thoughts will be developed through five phases within an educational system. These phases represent the development of the thinking process in geometry in addition to the acquisition of geometric knowledge. The progress in the student's thinking enables him to summarize what he learned and employ it in his daily life activities (Tall & Pegg, 2005).

Theoretical and Conceptual Framework

When Van Hiele suggested a geometrical learning model, he realized that, to master the intended learning levels as prescribed in the model, an appropriate pedagogical approach would be required. She, therefore, suggested a series of instructional disciplines that could facilitate the learning of geometry. Van de Walle (2004) notes that Van Hiele clarified the specific activities expected of the teacher while teaching geometry.

The instructional steps were composed of five steps which were to guarantee that students move from one Van Hiele learning level to a higher one in their geometric thinking: "In the Van Hiele model an instructional plan, which is made up of five steps, was formed to provide a transition from one level to another in students' geometric thinking." (Crowley, 1987; Erdoğan, Durmuş & Bekci, 2007).

Interview (research): The first step is the step in which the geometric thinking levels of students are determined. In this step, the students' geometric thinking levels will be determined through communication between the teacher and the student.

Direct Orientation: In this step, the teacher gives instructions and assignments related to the studies which is done in the light of the answers he gets from the students. The purpose of the teacher giving tasks is to make students explore the structures about the topic through research.

Making clear (explanation): Teacher introduces the topic to students in this step and students combine their experiences with the words they used related to the discussion. In this level, it is of value for the teacher to arouse students' interests.

Free Performance (activities): Students work on different solutions to multiphase problems in this step. They discover the relationships/the effect of the Van Hiele Model-based to the various objects of the structure in the topic they work. The teacher should guide students in their thinking about different solutions.

Integration: This step is the step in which students summarize and gather what they acquired. Students internalize what they learned as a new thinking structure." The instructional model was visually structured by John A. Van de Walle (2004), to summarize the instructional models stated above. See figure 1 below.
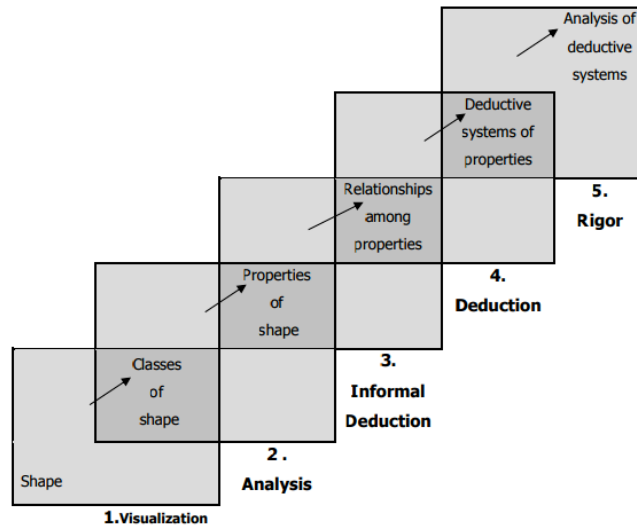
Figure 1

This study applied the Van Hiele's Instructional Model. The researcher taught Tangents and Secants of Circles applying the Van Hiele's Instructional Model in the whole class. He selected 15 students as experimental groups. All groups were required to take the pretest and posttest to evaluate their understanding of theorems of tangents and secants of circles. The test results of each group was analyzed to determine their skills in proving mathematical concepts and to identify their readiness in moving to the next stage of the 5-Levels of Thinking of Van Hiele's Instructional Model.

Paradigm of the Study
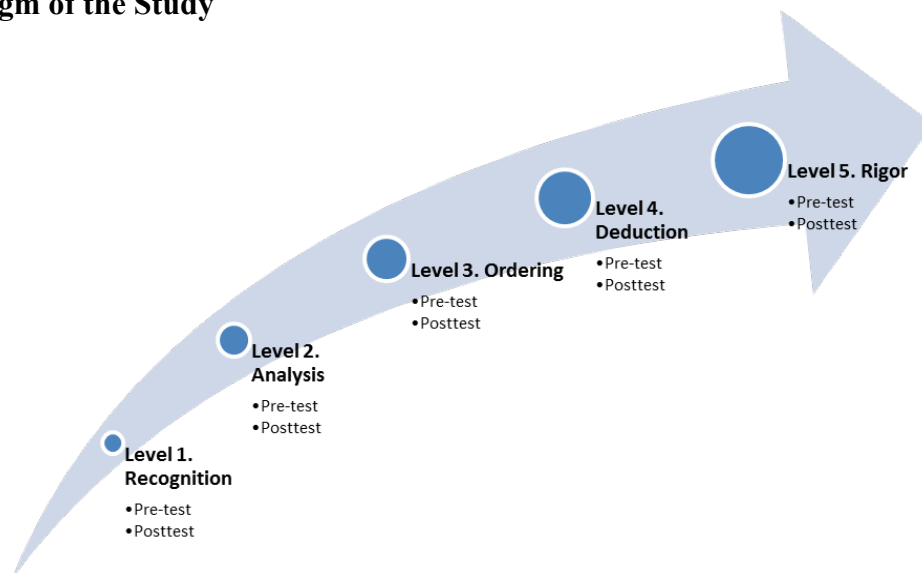
Research Problems

The research study aimed to identify the pupils' skill in proving mathematical concepts about tangents and secants of circles using Van Hiele's instructional model. The researcher sought answers to the following questions: (1.) how may the performance of the participants be described in terms of the following levels of the

Van Hiele's Instructional Model?; (2.) how may the performance of the following participants be described during the experiments? 2.1) low? 2.2) average? and 2.3) high?; and (3.) does Van Hiele's Instructional Model significantly affect the performance of the students?

Hypothesis

It was hypothesized in this study that Van Hiele's Instructional Model does not significantly affect the performance of the students.

Methods

The study was both quantitative and qualitative research. Quantitative, since descriptive analysis was employed on the scores of the participants and qualitative since the research dealt with the description of the different performance of group of students in terms of learning, for the participants belonging to Low Performing, Average Performing or High Performing.

Participants

The participants of this study were randomly selected from one section of Don Jesus Gonzales High School, Mexico, Pampanga during the school year 2017 - 2018. Participants were composed of males and females regardless of their age, culture and socio – economic status. Grade 10 - Patience were chosen as experimental group of the experiment. Qualitative part of the study focused on the 15 participants from the experimental group who were analyzed based on their methods and means of solutions in solving measurement of angles of tangents and secants of circles. These 15 participants were chosen through the diagnostic test administered to them. There were 5 high – performing, 5 average – performing and 5 low – performing participants identified after the diagnostic test. Participants 1 to 5 were the high – performing participants, participants 6 – 10 were the average – performing participants while participants 11 – 15 were the low – performing participants.

Instrument

The researcher utilized 2 sets of instrument per level of the Van Hiele's Instructional Model, 5 sets of problems involving tangents and secants of circles for pretest and 5 sets of problems for posttest were given to the participants.

The problem sets were composed of tangents and secants problems as per the delivery of the Van Hiele's Instructional Model. The first pretest was administered to identify the fifteen selected participants: 5 high – performing, 5 average - performing and 5 low – performing participants. These selected fifteen participants were the subject of qualitative discussion of the study to analyze the effect of Van Hiele's Instructional Model to their skills in solving problems involving tangents and secants of circle.

Each participant was given a test paper. During the pretest, problems were solved using any method that was taught to the participants in their previous grade level or the method known to them. In the posttest, the experimental group applied what they had learned in the Van Hiele's Instructional Model in solving the given problems. The

researcher used all or nothing point system for the study. The researcher used 20-points parallel questions for the pretest and posttest. The researcher divided the scores into two and got the scale in order to descriptively rate the performances. The following scale was utilized to rate the score of the participants.

Scale for the Pretest/Posttest
8.01 – 10.00	Excellent
6.50 – 8.00	Very Satisfactory
5.00 – 6.49	Satisfactory
5.00 below	Poor

Procedures

The researcher wrote a letter to Don Jesus Gonzales High School to inform its office that the researcher will have an initial visit to the locale for this research. After an initial visit, the researcher prepared the instruments and instructional materials to conduct the study. The researcher conducted and applied the Van Hiele's instructional model to his classes. However, only one participating class was chosen and the results of their outputs, both pretest and posttests were recorded.

A pretest was administered to the experimental group to identify the five high – performing pupils, five average performing pupils and five low – performing pupils. These 15 pupils were the subjects of the study.

Before every discussion, a pretest was administered by the researcher. And this time, only the scores of the 15 participants were recorded. Then, discussion of the lessons in the Tangents and Secants of Circles was done. The experimental group was given two sessions of discussion per level of the Van Hiele's Instructional Model. A total of ten sessions was utilized in discussing the Tangents and Secants of the Circle.

After every discussion, the researcher administered the posttest to evaluate the understanding of the participants. Lastly, interventions were done for each level depending on the learner's grasp of the mathematical skill. The proposed intervention materials, in line with the Van Hiele's Instructional Model, was administered per level before proceeding to the next level to develop mastery of learning.

Levels of the Van Hiel's Instructional model were determined during the discussion. In the Van Hiele model an instructional plan, which is composed of five steps, was formed in order to provide a transition from one level to another in students' geometric thinking." (Crowley, 1987; Erdoğan, Durmuş & Bekci, 2007).

Level 1 focused on the interview (research) in which the geometric thinking levels of students were determined. In this step, the students' geometric thinking levels were analyzed through indirect communication between the teacher and the student. The students were given a pre-test to check what each student already knew. The teacher then taught the students by giving different tasks and activities. To check if they improved, a posttest was administered.

Level 2 was Direct Orientation. In this step, a pre-test was also given before other activities. The teacher gave instructions and assignments related to the studies which was done in the light of the answers he got from the students. The purpose of the

teacher giving tasks was to make students explore the structures about the topic through research. To check if the students improved, a posttest was also administered. In Level 3, the teacher focused in making clear explanation. Prior to this step, a posttest was given to determine prior knowledge about the lesson. The teacher introduced the topic to students while combining their experiences with the words they used related to the discussion. In this level, it was of value for the teacher to arouse students' interests. The teacher taught and gave different activities to improve the skills of the students. A posttest was also given afterwards.

Free Performance was the center theme of Level 4. It was focused on activities that deepened the critical thinking skills of the students. A pre-test was administered before any other activities. They were able to work on different solutions to multiphased problems in this step. They discovered the relationships/the effect of the Van Hiele Model-based to the various objects of the structure in the topic they work. The teacher guided students in their thinking about different solutions and geometric proofs.

Table 1. Performance of the participants in Van Hiele's Instructional Model

			Excellent	Very Satisfactory	Satisfactory	Poor
Level 1	Pre-test	f	5	5	0	5
		%	33	33	0	33
	Post test	f	9	5	1	0
		%	60	33	7	0
Level 2	Pre-test	f	0	0	12	3
		%	0	0	80	20
	Post test	f	6	5	4	0
		%	40	33	27	0
Level 3	Pre-test	f	0	0	0	15
		%	0	0	0	100
	Post test	f	8	4	2	1
		%	53	27	13	7
Level 4	Pre-test	f	0	0	0	15
		%	0	0	0	100
	Post test	f	4	1	2	8
		%	27	7	13	53
Level 5	Pre-test	f	0	0	5	10
		%	0	0	33	67%
	Post test	f	6	3	5	1
		%	40	20	33	7%

In Level 5 or Integration, a pre-test was also administered. This step was the step in which students summarized and gathered what they acquired. Finally, a posttest was administered to check the improvement on the skills of students.

Van Hiele's Instructional Model is an instructional model where students followed the five levels namely: a) Visualization; b) Analysis; c) Informal Deductions; d) Deductions; and e) Rigor.

Percentage score was utilized to determine the performance of the participants using the instructional model. Mean and Standard deviation were also used to analyze the scores of the participants and t-test was used to determine if the Van Hiele's Instructional Model significantly affect the performance of the students.

Qualitative analysis came from the participant's descriptive rating of the selected 15 students of the experimental group.

Results and Discussion

Table 1 reveals the performance of the participants in Van Hiele's Instructional Model. In Level 1, the pretest revealed that there are five participants who get a descriptive rating of excellent which is equivalent to 33% while the other 5 participants (33%), were recorded very satisfactory and the other 5 (33%) were rated poor. On the other hand, after the discussion, Posttest revealed that 9 out of 15 participants (60%) were rated excellent, 5 (33%) were very satisfactory and 1 (7%) was satisfactory. In the pre-test of Level 2, 12 participants (80%) were recorded as satisfactory and 3 (20%) were noted as poor while the post test revealed that 6 participants (40%) were rated as excellent, 5 (33%) were very satisfactory and 4 (27%) were satisfactory. In level 3, 15 participants or 100% were rated poor on the pretest while on the posttest, 8 out of 15 (53%) were recorded as excellent, 4 (27%) were very satisfactory, 2 (14%) were satisfactory and there was only 1 (7%) rated poor. The pre-test of Level 4 was also rated as poor since 100% among 15 participants got zero. Some improvements were done after the discussion because 4 participants (27%) were rated excellent, 1 (7%) were very satisfactory, 2 (13%) were satisfactory and 8 (53%) were still categorized as poor. In Level 5, the pretest revealed that there are 5 participants who get a descriptive rating of satisfactory which is equivalent to 33% while the other 10 participants (67%) were recorded poor. Hence, after the discussion, posttest revealed that 6 (40%) got excellent rating, 3 (23%) got very satisfactory, and 5 (60%) got poor rating.

Thus, the table implied that the participants with the use of Van Hiele Instructional Model showed mostly poor and satisfactory performances on the pretest and mostly very satisfactory and excellent on the posttest, however, participants showed improving scores in all levels in their mathematics performance

Graph 1. Pre-test and posttest mean scores of the participants
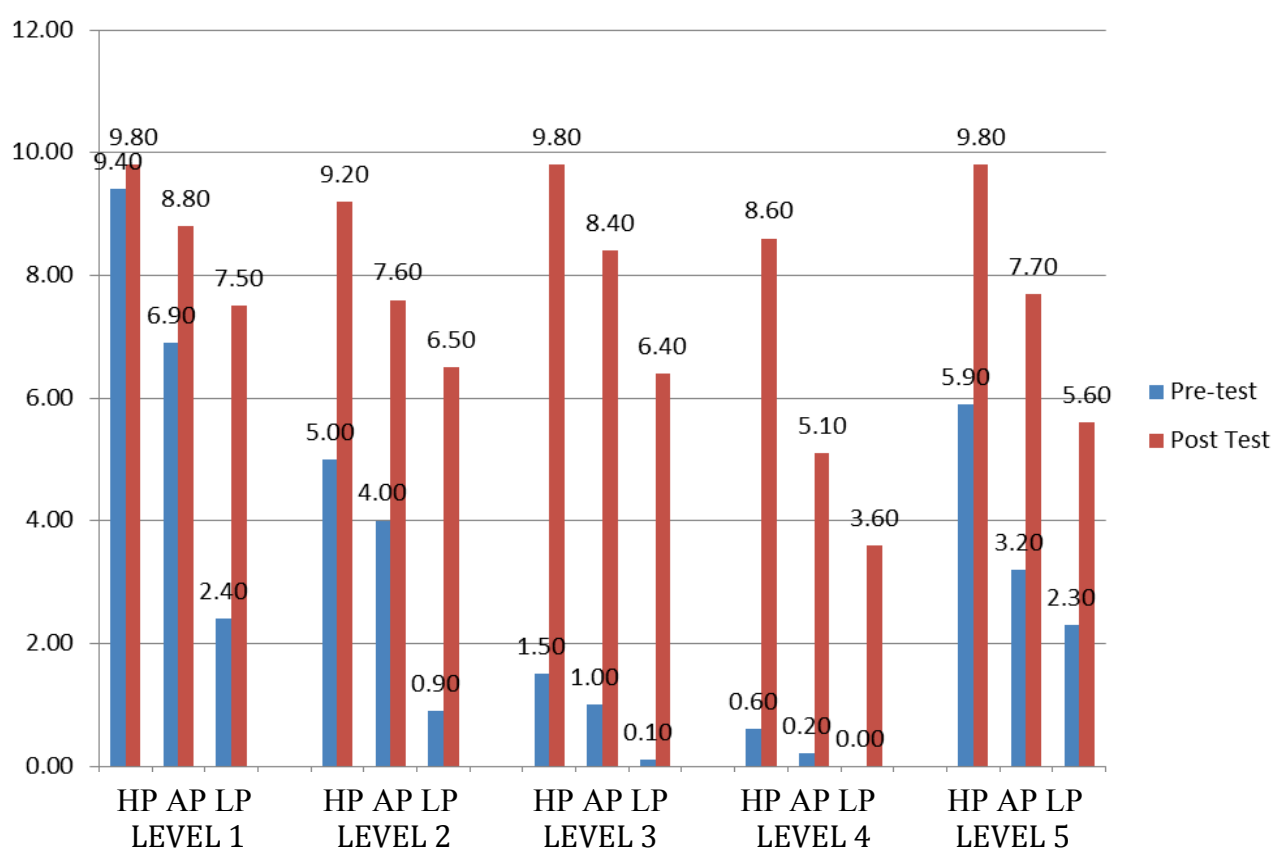
Graph 1 showed the pre-test and posttest mean scores of the participants. It will be described on the graph that most students improved greatly from their pre-test scores to their posttest scores. Level 1 showed that high-performing participants improved 0.40 from their pre-test mean scores to their posttest mean scores. Average performing participants improved from 6.90 to 8.80 and low performing participants improved the most by 5.10 mean difference of the posttest to their pre-test.

Level 2 also showed improving skills in Mathematics. High performing participants' posttest improved to 4.20 from its pretest mean score of 5.00. Average performing participants also improved from 4.00 to 7.60. There was also an improvement among the low performing participants from the pre-test mean score of 0.90 to posttest mean score of 6.50.

On Level 3, students showed a great height of improvement. High performing participants had a mean difference of 8.30. Average performing participants upgraded from pre-test mean score of 1.00 to posttest mean score of 8.40 while low performing participants mean score was increased by 6.30.

There was a visible upgrade shown on Level 4. Participants from the high performing group increased by 8.00 from the pre-test mean scores and posttest mean scores. Average performing participants had a mean of 0.20 from the pre-test to 5.10 on the posttest. All students from the low performing group did not scored on the pretest but there's an improvement on the posttest with a mean score of 3.60.

On level 5, all participants showed improving skills in Mathematics. High performing participants' posttest improved to 3.90 from its pretest mean score of 5.90. Average performing participants also improved from 3.20 to 7.70. There was also an improvement among the low performing participants from the pre-test mean score of 2.30 to posttest mean score of 5.60.

Based on the pre-test and posttest mean scores of each students, high performing participants improved their skills in mathematics from Level 1 to level 5 of the Van Hiele's instructional model. Average performing participants and low performing participants also showed improvement from their pre-test mean scores to their posttest mean scores.

Table 2. Difference between the posttest scores of the participants

Levels	Mean Differences	p-value Sig. (2-tailed)	Remarks
Level 1	2.47	0.001	Significant at 0.01
Level 2	4.47	0.000	Significant at 0.01
Level 3	7.33	0.000	Significant at 0.01
Level 4	5.50	0.000	Significant at 0.01
Level 5	3.90	0.000	Significant at 0.01

Table 2 reveals that there was a significant difference at 1% between the pretest and posttest of the Van Hiele's Instructional Model with a computed p – value of 0.001 for the Level 1, 0.000 for the Level 2, 0.000 for the Level 3, 0.000 for the Level 4 and 0.000 for the Level 5. The mean difference of pre-test and posttest per level was also computed. The mean difference of Level 1 is 2.47, 4.47 for Level 2, 7.33 for Level 3, 5.50 for Level 4 and 3.90 for Level 5. This indicates that participants improved their performance after the discussion of the lesson. This proves that the Van Hiele's Instructional Model showed significant improvement in their skill in mathematics with regards to tangents and secants of circles.

Conclusion

Based on the findings, the researcher derived at the following conclusions: 1) The participants used Van Hiele Instructional Model showed mostly poor and satisfactory performances on the pretest and mostly very satisfactory and excellent on the posttest however participants showed improving scores in all Levels in their mathematics performance; 2) on the pre-test and posttest mean scores of participants, high performing participants improved their skills in Mathematics from Level 1 to level 5 of the Van Hiele's instructional model. Average performing participants and low performing participants also showed improvement from their pre-test mean scores to their posttest mean scores; and 3) the Van Hiele Instructional Model is effective. Considering the aforementioned findings and conclusions, the following recommendations are hereby suggested: 1) Teachers are encouraged to test the prior knowledge of their students before discussing their lessons about tangents and secants of circles to determine pupils' strengths and weaknesses; 2) Teachers may adopt the use of Van Hiele's Instructional Model in teaching Mathematics from primary level to junior high school to enhance students' performance in Mathematics and to strengthen the foundation of knowledge; 3) Curriculum planners and textbooks

writers must take into consideration the difficulties of students in order to come up with easier methods to be used in teaching to produce better students' outputs. Moreover, researchers on the different techniques on how to address students' learning barriers in Mathematics may be conducted; cultural background and techniques could be observed before applying a foreign method.

References

Bruner, J. (1966). Toward a Theory of Instruction. Cambridge, MA: Harvard University Press. Retrieved August 19, 2017, from http://www.instructionaldesign.org/theories/constructivist.html

Cruz, I. (2015) The Math Curriculum Mini Critique (The Philippine Star) | Updated August 13, 2015 12:00am

Crowley, M. L. (1987) cited by Erdoğan, Durmuş & Bekci, (2007) The Van Hiele model of the development of geometric thought. In M.M. Lindguist (Ed.), Learning and teaching geometry (K–12 (pp. 1-16). Reston, VA: NCTM.

Dongwi, B. L. (2014) Using the Van Hiele phases of Instruction to Design and Implement a Circle Geometry Teaching Programme in a Secondary School in Oshikoto Region: A Namibian Case Study.

Ganal, N. N. & Guiab, M. R. (2014) Problems and Difficulties Encountered by Students towards Mastering Learning Competencies.

Genz, R. (2006). Determining high school geometry students' geometric understanding using Van Hiele levels: Is there a difference between standards-based curriculum studentsand non standards-based curriculum students? Retrieved August 19, 2017, from http://contentdm.lib.byu.edu/ETD/image/etd1373.pdf

Gneim, M. (2012). Effect of teaching geometry using Van Hiele Model in the ninth grade students' achievement and developing critical thinking skills in Jordan (Unpublished thesis). University of Jordan.

Gutiérrez A., Jaime, A. & Fortuny, J. M. (1991). An alternative paradigm to assess the acquisition of the Van Hiele levels. Journal for Research in Mathematics Education, 22(3), 237–251.

Ismael, M. R. (1998).The effect of using mathematics laboratory in teaching geometry for fourth grade students on their achievement and performance of scientific thinking skills and engineering. Journal of research in education and science, University of Almniaa. 11(4).

Jones, L. (2007). The Student-Centered Classroom. Cambridge University Press. Mendoza, A. R. (2016) Singapore Bar Model Drawing Method And Traditional Method

Mistretta, R. M. (2000). Enhancing geometric reasoning. Adolescence, 35(138), 365-379.

NCTM. (2000). Principles and Standards for School Mathematics. National Council of Teachers Mathematics Reston.

Olkun, S., Sinoplu, N. B., & Deryakulu, D. (2005). Geometric explorations with dynamic geometry applications based on van Hiele levels. International Journal for Mathematics Teaching and Learning.

Paja (2001). Problem – Based Learning in Teaching and Learning High School Geometry: Its Effects on Students' Attitude and Performance

Pegg, J. (1992). Students' understanding of geometry: theoretical perspectives. Retrieved August 21, 2017 from http://www.merga.net.au/documents/Keynote_Pegg 1992.pdf
Piaget, Jean. (1950). The Psychology of Intelligence. New York: Routledge.

Sadiki, M/ W. R. (2016). The Effect Of Using Van Hiele's Instructional Model In The Teaching Of Congruent Triangles In Grade 10 In Gauteng High Schools.

Tall, D., & Pegg, J. (2005). The fundamentals cycle of concept construction underlying various theoretical frameworks. International reviews on mathematics Education, 37, 468-475.

The Vision & Mission of the Department of Education. (2001) Retrieved from http://www.deped.gov.ph/mandate

Van de Walle, J. A. (2004). Elementary and Middle School Mathematics-Teaching Developmentally.5th ed. Boston: Pearson Education.

Van Hiele, P. M. (1959). The child's thought and geometry. In D. Fuys, D. Geddes & R. Welchman Tishchler (Eds.), (1984). English translations of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele (pp. 1-259). ERIC/SMEAC.

Van Hiele, P. M. (1986). Structure and insight: A theory of mathematics education. Netherlands: Academic Press.

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. Teaching Children Mathematics, 5(6), 310–316.

Young, L.E. & Patterson, B.L. (2007). Teaching nursing: Developing a student-centered learning environment. Philadelphia, PA: Lippincott Williams & Wilkins.

Contact email: melchormdr@gmail.com