Mastery Learning of Early Childhood Mathematics Through Adaptive Technologies

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Abstract
Childhood mathematics achievement has been associated with success in later schooling, and is predictive of later career success and adult socio-economic status. Despite the critical need for strong mathematics skills, not all learners are able to develop them. It is well-documented that children enter school with different levels of mathematics skills and understanding. These gaps in learners' mathematics foundations not only cause students to lag behind their peers, they also make it less likely that such students will find career success after formal schooling. A growing body of research exists documenting the impact of preschool and early elementary school mathematics achievement and its relationship to later mathematics achievement in school. It has proven challenging to help all young students achieve the mastery of important foundational concepts and skills in mathematics. There is promise, however, in using adaptive technologies designed to efficiently identify what the child already does or does not know, and what the child is ready to learn next, thus keeping students in the zone of proximal development. This paper discusses the design of a new adaptive platform that uses Bloom's Mastery Learning model to develop mastery of number sense foundations in a digital environment. Early results show that students are not only able to master important key concepts and skills in a very short amount of time, but that the system is able to efficiently monitor and move students through key concepts and skills using personalized learning trajectories optimized for maximum learning.

Keywords: mathematics education, early childhood, mastery learning, learning trajectories, achievement gap, number sense, digital learning, adaptive technology, mathematics achievement, personalized learning, game-based learning

Introduction

A primary goal of the U.S. educational system is to graduate students who are prepared for careers, college or both. Key to college and career readiness is a strong understanding of mathematics (NAEYC & NCTM, 2002; U.S. Department of Education, n.d.). According to the National Mathematics Advisory Panel (2008), "mathematics education has risen to the top of the national policy agenda as part of the need to improve the technical and scientific literacy of the American public. The new demands of international competition in the 21st century require a workforce that is competent in and comfortable with mathematics…" (p.1).

Strong academic skills, and mathematics skills in particular, are associated with positive professional outcomes in adult life (NCTM, 2000; NCTM, 2013; NMAP, 2008; Ritchie & Bates, 2013; Schweinhart et al., 2005). The relationship between children's mathematics achievement in early elementary school and later academic achievement in mathematics has been documented in many studies as well (Bailey et al., 2014; Claessens & Engel, 2013; Duncan et al., 2007; Entwisle & Alexander, 1998; NMAP, 2008; NRC, 2009; Siegler et al., 2012). Because of this, there has been a significant push by national governmental organizations, policy makers, and advisory groups to emphasize the importance of teaching mathematics throughout schooling and more recently, at the earliest levels of schooling: preschool and kindergarten (NCTM, 2000; NCTM, 2013; NMAP, 2008; NRC, 2009).

Despite the critical need for strong mathematics skills, not all learners are able to develop them, as has been well documented by both standardized measures of assessment, and ongoing research (Jordan, Huttenlocher, & Levine, 1992; Jordan & Levine, 2009; NAEP, 2015). Moreover, children enter school with different levels of mathematics skills and understanding (Bodovski & Farkas, 2007; Claessens & Engel, 2013; Clements & Sarama, 2011; Jordan, Huttenlocher, & Levine, 1992; Jordan & Levine, 2009; Siegler, 2009). Some of this is due to the naturally differentiated development of young children (NAEYC & NCTM, 2002), but other differences are due to factors such as socio-economic status (SES), ethnicity, mother's education level, etc. that have been shown to put children at an advantage or disadvantage for learning (Abedi et al., 2006; Entwisle & Alexander, 1990; Siegler, 2009).

Children from disadvantaged backgrounds, primarily those from low-income and minority families, lag behind their more advantaged peers on several measures of early mathematics skills at school entry (Claessens & Engel, 2013; Jordan, Huttenlocher, & Levine, 1992; Jordan & Levine, 2009; Siegler, 2009). As children progress through school, the disparities and differences do not disappear, even though students of different SES levels may be exposed to the same curriculum and teaching practices. The research consistently shows that students who begin school behind their peers are more likely to stay behind throughout the rest of their schooling (Claessens & Engel, 2013; Clements & Sarama, 2011; Seigler, 2009). These gaps in learners' mathematics foundations not only cause them to lag behind their peers, they also make it less likely that such students will find career success after formal schooling, thus reinforcing the cycle of poverty. Conversely, children that enter school with better mathematics understanding and skills will be better prepared to receive instruction and build upon that knowledge, even leading to placement in higher ability groups (Claessens & Engel, 2013). Such early advantages positively impact students as they move forward in their schooling.

The Importance of Early Mathematics Competencies

A growing body of research exists documenting the impact of preschool and early elementary school mathematics achievement and its relationship to later mathematics achievement in school (Claessens, Duncan, & Engel, 2009; Claessens & Engel 2013; Clements & Sarama, 2011; Duncan et al., 2007; Sarama & Clements 2004). While most studies have focused on general mathematics achievement in the early years, a few studies have focused on certain finer-grained mathematics skills and competencies that might have a particularly strong influence on later success in mathematics (Claessens & Engel, 2013; Clements & Sarama, 2011; Nguyen et al., 2016). The identification of key competency indicators in mathematics is critical, as it may provide assessment opportunities that allow for early identification of and intervention for students most likely to be at risk for lagging mathematics achievement. Even so, only a few studies have sought investigate the relationship between certain preschool mathematics competencies and later mathematics achievement, and whether or not specific competencies are *more* or *less* predictive of later mathematics achievement (Claessens & Engel, 2013; Nguyen et al., 2016).

The idea that certain early mathematics competencies might be more or less impactful on student success in the long term is relatively new, and the body of research is small. One study by Claessens and Engel (2013) focused on the relationships between school-entry mathematics skills and later achievement in both elementary and middle school across the subjects of math, reading, and science. The authors found that specific early math skills are important in predictors of later academic achievement. Number recognition, counting, shapes, and patterns, the ability to read all one-digit numerals, count beyond 10, recognize a sequence of patterns, and use nonstandard units of length to compare objects, were the "most consistent and important predictor[s] of later achievement test scores in both reading and math across elementary school" (Claessens & Engel, 2013, p.20). In a separate study by Nguyen and colleagues (2016), the authors found that advanced counting skills demonstrated by the end of kindergarten, including such competencies as being able to count beyond ten, count on one quantity to another, and to count forward and backward from any number within ten (e.g., start at four and count forward), were most predictive of later overall fifth grade mathematics achievement.

The findings of these studies and others are important for informing the creation of preschool curriculum and intervention programs, as well as parent education programs that build awareness around the kinds of math activities that are most likely to promote later success in mathematics. Knowing which mathematics competencies are most predictive of later achievement is important. However, that knowledge alone is not enough to help all students succeed. Even with targeted instruction, not all learners are able to improve (Lannin et al., 2013). The achievement gap between different subgroups of students is pervasive and has persisted for decades with only minor narrowing, despite the ongoing efforts of teachers and researchers to eliminate it (Barton, 2004; Goodman & Burton, 2012).

Bloom's Theory of Mastery Learning

Over the years, various theories have emerged to explain why some children seem to learn easily while others struggle. One such theory is Bloom's (1968) Mastery Learning theory. Bloom's Mastery Learning theory held that all students were capable of learning, albeit at different rates and in different ways. The more standardized the school curriculum became, the more difficult it became for all children to learn successfully. The standardized curriculum worked for some but did not work for many others. This has been true historically but may be even more so in today's diverse classrooms. A 'one size fits all' instructional approach does not work for all learners, which may explain the persistence of the achievement gap even today (Lannin et al., 2013).

Bloom developed his theory of Mastery Learning from studying both the interactions of one-on-one tutors with students and the behaviors of successful students. He compared these observations with his observations of traditional whole-group instruction in the classroom. Bloom observed that in the classroom, teachers working with groups of students provided the same instruction to all at the same pace, then followed that instruction with a test through which students were expected to demonstrate what they had learned. Some students performed well, while others did not. According to Bloom, "the test signifies the end of instruction on the unit and the end of the time they need to spend working on those concepts. It also represents their one and only chance to demonstrate what they learned. After the test is administered and scored, marks are recorded in a grade book, and instruction begins on the next unit, where the process is repeated" (Guskey, 1997, p.5). The problem remained one of competency; though some students had not demonstrated a level competency needed to move on to the next topic in the sequence, everyone still moved on. Across successive units, concepts that built upon prior concepts were understood less and less by students, until at last a large portion of the students were completely lost.

Conversely, Bloom observed a very different approach when studying the practices of effective one-to-one tutors with their students. Bloom considered the ideal learning situation to be when an "excellent tutor is paired with an individual student" (Guskey, 1997, p.6). In such situations, Bloom observed that the tutor was highly attuned to the needs of the student, provided ongoing feedback and corrective guidance, and only moved on once the student had demonstrated understanding of the concept or proficiency with the skill under study. With this in mind Bloom theorized that classroom instruction could be transformed based on these same principles. Beyond instruction, Bloom theorized that mastery learning should provide preassessment, feedback, correctives, enrichment, and alignment among instructional components, if it were to better replicate the ideal teaching conditions of one-to-one tutoring. Figure 1 below shows the relationships among key components of Bloom's Mastery Learning model.

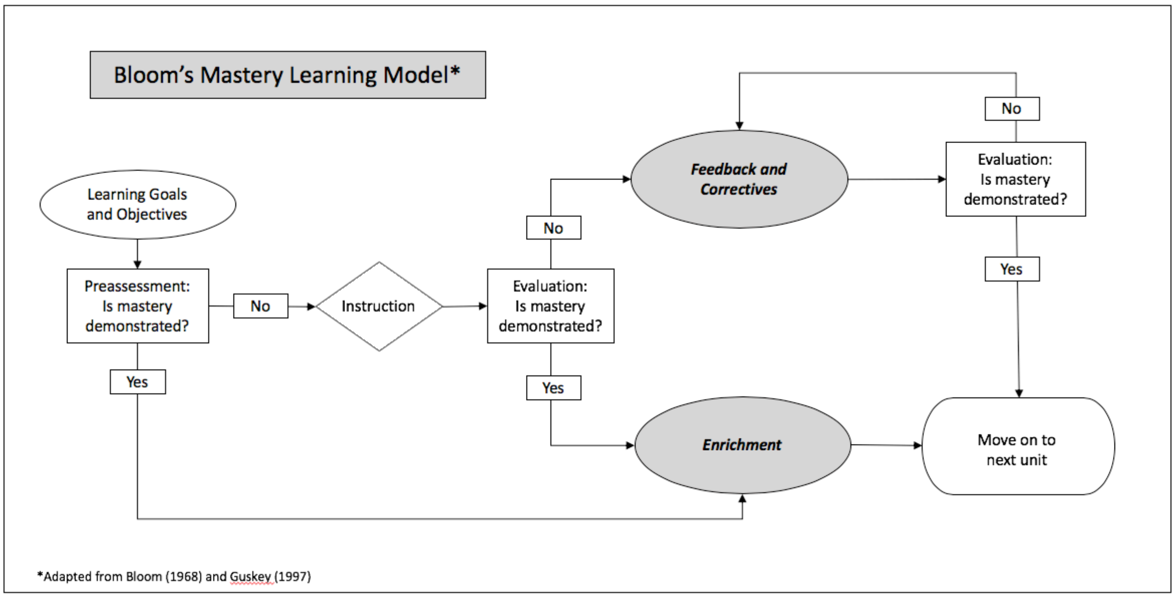

Figure 1: Bloom's Mastering Learning Model. Adapted from Bloom (1968) and Guskey (1997).

Preassessment. One of the critical components of the Bloom's Mastery Learning model is the step of preassessment. Before embarking on instruction, Bloom believed it was important to assess students to determine who, if any, already possesses mastery of the content to be studied. Any students who demonstrate mastery are summarily moved on to an enrichment group, where they can apply and extend their learning through independent projects. Those students who do not demonstrate mastery of the content receive targeted instruction from the teacher. In this manner, the teacher ensures that only students who need and will benefit from instruction receive it, while those students who do not need instruction are able to build on what they already know through enrichment.

Feedback & Correctives. In order to improve, students must receive feedback on their performance. Evaluation or summative feedback at the end of an instructional unit without the opportunity to improve is not beneficial for ongoing learning. Formative assessment that provides students with a better sense of what they do and do not understand, along with an opportunity to practice and improve, is more beneficial. Bloom asserted that feedback helps students to recognize what they have learned well, reinforces key concepts, and identifies the specific concepts upon which students need to spend more time. When such feedback is accompanied with interventions or targeted lessons ("correctives" as Bloom called them) designed to unpack and address any misunderstandings, students are able to effectively learn and move forward (Guskey, 1997).

Enrichment. A major challenge of Mastery Learning in the classroom context is promoting the ongoing learning of students who quickly and easily master the content. What is to be done with mastery students while the teacher is otherwise occupied delivering "correctives" to non-mastery students? In the Mastery Learning approach, Bloom asserts that students should be presented with independent learning opportunities that build on and extend their understanding of the concepts under study. In Bloom's view, these "enrichment" opportunities should be designed to stimulate intellectual curiosity and promote the independence of students, with the added benefit that no learning time is sacrificed while the teacher provides remediation to students who are not yet ready to move on.

Alignment. Perhaps one of the most important components of Mastery Learning is the idea that teaching and assessment should be aligned to learning objectives and outcomes. This has led many to criticize Mastery Learning as "teaching to the test," however that is not the case. It is more a matter of "testing what you teach." Bloom's Mastery Learning model holds that teaching should be based on targeted outcomes and objectives, while assessments should be aligned to and evaluate student performance on those same outcomes and objectives (Guskey, 1997).

Bloom's studies on Mastery Learning did show marked improvement in the individual performance of students participating in whole-group classroom instruction under Mastery Learning conditions. Though he was not able to replicate the achievement gains students made in one-to-one instruction with an excellent tutor, Mastery Learning did show great promise. In a landmark study, Bloom compared three groups of learners: (1) one-to-one instruction with an excellent tutor, (2) Mastery Learning whole-group instruction, and (3) traditional whole-group instruction. The results showed that the final achievement of the average student in the one-to-one tutoring condition was about two standard deviations above students in the control condition (traditional classroom teaching), while the final achievement of the average student in the Mastery Learning condition was about one standard deviation above those students in the control. Even more impressive was the result that "about 90% of the tutored students and 70% of the Mastery Learning students attained the level of summative achievement reached by only the highest 20% of the students under conventional instructional conditions" (Bloom, 1964, p. 40). This result was known ever after as the *2-sigma problem*.

Learning Trajectories, Bloom, and ABCmouse's Mastering Math™[1]

Academic achievement involves the accumulation and mastery of new skills, while improving and building upon already existing skills (Entwisle & Alexander, 1990). Much as Bloom and others had observed, competencies are built up and upon over time; doing well in one grade helps the child do better in subsequent grades (Entwisle, Alexander, & Olsen, 2005). Bloom, however, did not attempt to define what these competencies were, leaving such decisions up to the judgement and wisdom of the teacher (Guskey, 1997). Bloom did, however, assert that the curriculum must be based on clear learning goals, and divided up into successive, highly-focused units of instruction that could provide a hierarchy or trajectory of concepts and skills for learning (Bloom, 1968).

More recently, researchers and practitioners have built on the work of Bloom and others to develop the idea of learning trajectories. According to Simon (1995) a learning trajectory is defined as a framework that includes the "the learning goal, the

[1] Disclosure: the author of the present paper was one of the principle architects of the curriculum planning and design for Mastering Math, including the development of the knowledge map and learning trajectories, and remains involved through leadership of the team that plans and designs the Mastering Math system. Much of the information in this section comes from the author's personal experience as one of the architects of the Mastering Math system. Other sources of information are noted as warranted.

learning activities, and the thinking and learning in which the students might engage" (p.133). Another definition might describe a learning trajectory as the learner's pathway through a heirarchy of goals and activities where each successive objective and interaction is designed to build on the understanding and mastery of previous objectives (Clements & Sarama, 2004; Sarama & Clements, 2004). Accepting the premise that all learners are different, and learn in different ways and at different rates, it is assumed that learner trajectories are best individualized. Much like Mastery Learning, the Learning Trajectories approach depends on the learner's success with prior learning and uses that as a foundation for subsequent learning that is tailored to the needs of the individual student (Clements & Sarama, 2004; Sarama & Clements, 2004).

Nowhere are these learning trajectories of concepts, skills, and activities more critical than in mathematics, "with each step drawing upon skills laid down in the preceding steps" (Entwisle & Alexander, 1990, p.454). This is especially true for young learners whose early experiences with number are the very foundations of their future mathematical understanding. Indeed, some researchers have suggested that all subsequent learning may be driven in large part by what the child has learned by third grade (Bloom, 1964; Entwisle & Alexander, 1990).

Few early childhood mathematics programs make effective use of the learning trajectories approach. Two programs that do are *Number Worlds* (formerly called *Rightstart)*, and *Building Blocks.* Though both of these programs are currently distributed by McGraw Hill Education PreK-12, they originated as research-designed programs used to determine the efficacy of various pedagogical and curricular approaches to early mathematics instruction (Clements & Sarama, 2011). While *Number Worlds* and *Building Blocks* are both based on developmentally sequenced learning trajectories for each mathematical topic, neither program is explicitly grounded in Bloom's model of Mastery Learning. A more recent program, *ABCmouse's Mastering Math*™, is grounded in Bloom's Mastery Learning model and uses it to drive student progress through individualized learning trajectories. *Mastering Math* is an adaptive digital application that teaches mastery of number sense foundations to preschoolers through second grade.

Designed primarily for individual use by students in the home (without a teacher or mentor present), *Mastering Math* relies on a tightly structured sequence of learning objectives, and adaptive algorithms to determine what the student knows and has mastered, what the student does not know, and what the student is ready to learn next. This "smart" system places the student in structured game-based activities that most closely match the learner's anticipated zone of proximal development, and then adjusts in real time to the learner's individual needs based on ongoing interactions and game-play. As a result, each student's learning trajectory through the content is entirely unique, based on his or her prior knowledge, experience, learning ability, and agency within the game.

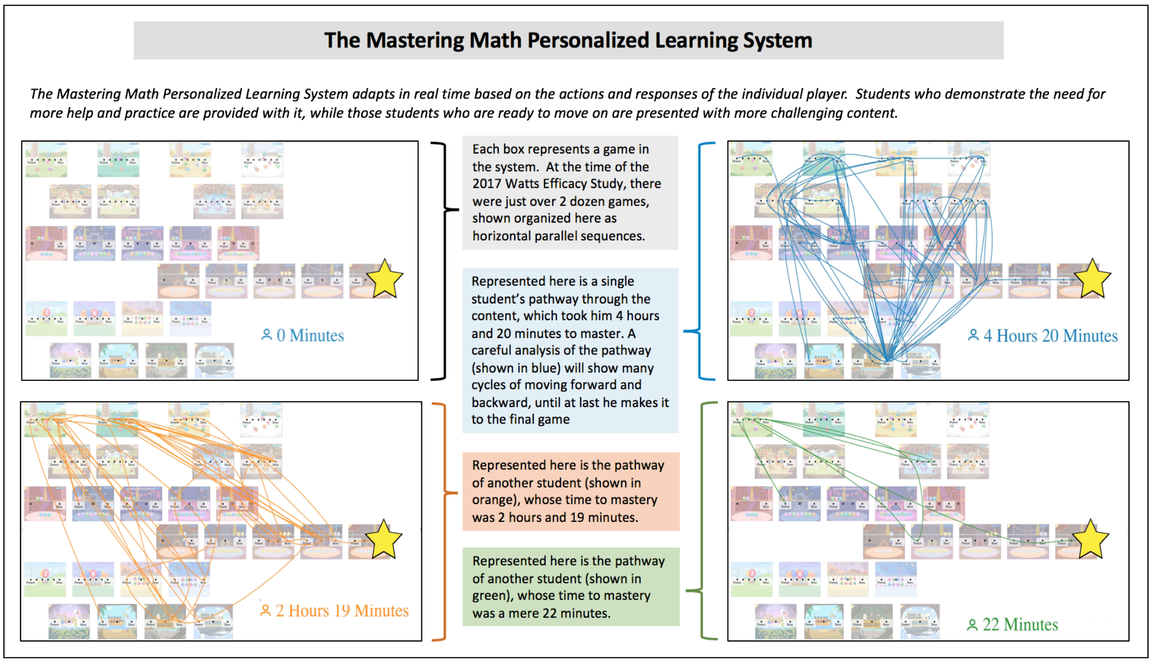

Figure 2: Examples of student learning trajectories in the Mastering Math system (Age of Learning, 2018)

Figure 2 above shows the diverse pathways of three different students through the same portion of the *Mastering Math* content. The lines on the images represent the different learning trajectories of each student, along with the time to completion listed at the bottom right of each student's trajectory map. As shown, one student was able to complete mastery of the content in 4 hours and 20 minutes; another student required 2 hours and 19 minutes, and a third student only needed 22 minutes. As the individual learning trajectories of these three students indicate, the need for personalization in a single classroom is great. It is possible that a teacher might be able to personalize instruction for these three students. It is less likely that a teacher might be able to personalize instruction for 30 students, each with their own unique learning trajectory.

In Bloom's model of Mastery Learning, the student who mastered the content in 22 minutes would have been moved to an enrichment group, while the other two students would have received instruction as well as feedback and correctives. However, in the Mastering Math system, students who demonstrate mastery are simply moved forward to the content they are ready to learn next, rather than into an enrichment group. The need for an enrichment group in Bloom's model is driven primarily by the demands of the classroom, where the teacher has a need to keep mastery students meaningfully occupied while the teacher delivers instruction, feedback, and correctives to non-mastery students. In the digital space, mastery students can be meaningfully occupied by moving on to the next topic, as there is no need to wait for the attention of a teacher.

The Mastering Math Personalized Learning Model

Key components of Mastering Math are aligned with the those of Bloom's Mastery Learning model, including preassessments, instruction, feedback and correctives, evaluation, and alignment to a hierarchy of learning goals and objectives. As mentioned previously, enrichment is not part of the Mastering Math model, since students who demonstrate mastery are able to move forward unhindered to the next topic they are ready to learn. Figure 3 below shares additional information regarding the ways in which the Mastering Math personalized learning model exemplifies key components of Bloom's Mastery Learning model.

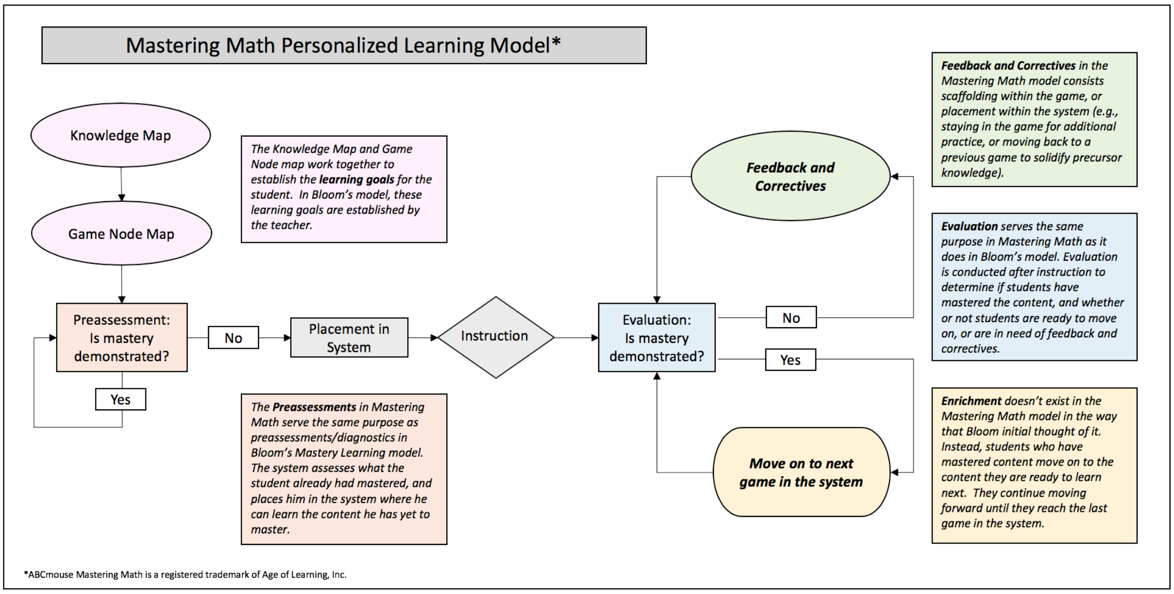

Figure 3: Mastering Math Personalized Learning Model

Early results for *Mastering Math* are encouraging. Students are not only able to master important key concepts and skills in a very short amount of time, but the system is able to effectively and efficiently monitor and move students through the architecture of concepts and skills using personalized learning trajectories optimized for maximum learning (Age of Learning, 2018). In an efficacy study conducted in the fall of 2017 (n = 460), learners in the treatment condition experienced a 36% greater gain in early number sense skills over the control group in during an average of only 5 hours of game play spread over 12 weeks. In addition, the schools participating in the study were 100% Title I schools with extremely low SES, little or no access to technology, and no existing math curriculum (teachers created their own). Moreover, through the use of *Mastering Math*, these students were able to demonstrate mastery of those very same advanced counting competencies identified by Nguyen and colleagues (2016) and Claessens and Engel (2013) as especially critical for later success in mathematics, including counting out (within 20), counting on (within 20), recognizing numerals (within 20), and counting forward and backward from any number (within 20) (Age of Learning, 2018). Though still in its infancy, *Mastering Math* has already shown strong potential for helping all students—including minority and low-SES students who are most at risk—learn the most critical mathematics competencies.

Conclusion

Early mathematics knowledge and skills are key for success in later schooling and beyond and are further critical for helping to eliminate the pervasive and persistent achievement gap that keeps low-SES and minority students from reaching their full potential. Yet, there are time-proven theories of learning and instructional methodologies available that can, if implemented effectively, help diminish or even eliminate the achievement gap, thus making it possible for all students to learn. Curricular approaches that employ both learning trajectories and key components of Bloom's Mastery Learning approach have tremendous potential for producing lasting gains. Moreover, the implementation of mastery learning and learning trajectories through digital mediums carry the promise of scalability and efficacy, as they are not dependent on the presence of a highly-qualified teacher, nor are they dependent on small group or class sizes. Carefully crafted, adaptive digital programs such as Mastering Math have the potential to closely replicate Bloom's ideal one-to-one condition with an "excellent tutor." With time and refinement, the team behind Mastering Math hopes to eventually solve, or come close to solving, the 2-sigma problem at scale.

Programs like *Mastering Math*, that attempt to integrate a wide body of research, theory, and practice, from a wide array of disciplines, are still in their infancy. More research is needed to determine just how impactful such programs will be. However, early signs indicate that the ability to effectively teach all young learners the mathematics competencies needed to ensure future success may finally be on the horizon.

References

Abedi, J., Courtney, M., Leon, S., Kao, J., & Azzam, T. (2006). *English language learners and math achievement: A study of Opportunity to Learn and language accommodation* [Technical Report 702]. National Center for Research on Evaluation, Standards, and Student Testing (CRESST). Retrieved from: https://files.eric.ed.gov/fulltext/ED495848.pdf

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, 17(5), 775-785.

Barton, P.E. (2004). Why does the gap persist? *Educational Leadership, 62*(3), 8-13.

Behrens, J.T., Mislevy, R.J., Bauer, M., Williamson, D.M, & Levy, R. (2004). Introduction to evidence centered design and lessons learned from its application in a global e-learning program. *International Journal of Testing, 4*(4), 295-301.

Age of Learning. (2018). *ABCmouse Mastering Math: An adaptive, game-based, personalized learning math program for PreK to 2nd Grade*. Unpublished manuscript.

Bloom, B. (1964). *Stability and change in human characteristics.* New York, NY: Wiley.

Bloom, B. (1968). Learning for mastery. In J.H. Block (Ed.), *Mastery learning: Theory and practice* (pp.47-63. New York, NY: Holt, Rinehart, & Winston.

Bloom, B. (1984). The 2-sigma problem: The search for methods of group instruction as effective as one-to-one tutoring. *Educational Researcher, 13*(6), 4-16.

Bodovski, K., & Farkas, g. (2007). Mathematics growth in early elementary school: The roles of beginning knowledge, student engagement, and instruction. *Elementary School Journal*, 108, 115-130.

Claessens, A., Duncan g., & Engel, M. (2009). Kindergarten skills and fifth grade achievement: Evidence from the ECLS-K. *Economics of Education Review*, 28, 415-427.

Claessens, A., & Engel, M. (2013). How important is where you start? Early mathematics
knowledge and later school success. *Teachers College Record*, 115(6), 1–29.

Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, 6, 81–89.

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science*,
333(6045), 968-970.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: the learning* *trajectories approach*. New York. NY: Routledge.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., &
Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428–1446.

Entwisle, D. R., & Alexander, K. L. (1990). Beginning school math competence: Minority and majority comparisons. *Child Development*, 61, 454–471.

Entwisle, D. R., & Alexander, K. L. (1998). Facilitating the transition to first grade: The nature of transition and research on the factors affecting it. *The Elementary School Journal*, 98(4), 351-374.

Goodman, R., & Burton, D. (2012). What is the nature of the achievement gap, why does it persist and are government goals sufficient to create social justice in the education system? *International Journal of Primary, Elementary and Early Years Education, 40*(5). 500-514.

Guskey, T. (1997). *Implementing mastery learning.* New York, NY: Wadsworth Publishing Company.

Jordan, N., Huttenlocher, J., & Levine, S. (1992). Differential calculation abilities in young
children from middle- and low-income families. *Developmental Psychology,* 28, 644-653.

Jordan, N.C., Huttenlocher, J., & Levin, S.C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle and low-income children. *Learning and Individual Differences, 6,* 413-432.

Jordan, n., & Levine, S. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews*, 15, 60-68.

Lannin, J., Van Garderen, D., Switzer, J.M., Buchheister, K., Hill, T., & Jackson, C. (2013). The mathematical development in number and operation of struggling first graders. *Investigations in Mathematics Learning, 6*(2), 19-47.

National Assessment of Educational Progress (NAEP). (2015). *The Nation's Report Card:*
Mathematics & reading assessments State comparisons. Retrieved from: https://www.nationsreportcard.gov/reading_math_2015/#reading/state/comparisons/NP?grade=4

National Association for the Education of Young Children and National Council of Teachers of Mathematics (NAEYC and NCTM). (2002). *Early childhood mathematics: Promoting good beginnings* (Position Statement). Retrieved from https://www.naeyc.org/sites/default/files/globally-shared/downloads/PDFs/resources/position-statements/psmath.pdf

National Council of Teachers of Mathematics (NCTM). (2000). (1st ed.). *Principles and standards for school mathematics* (Vol. 1) Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2013). *Why is mathematics important for early childhood learners?* [Position Statement]. Retrieved from https://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

National Research Council (NRC). (2009). *Mathematics learning in early childhood: Learning paths toward excellence and equity*. In C.T. Cross, T.A. Woods. H. Schweingruber (Eds.) Washington, DC: The National Academies Press.

Nguyen, T., Watts, T.W., Duncan, G.J., Clements, D.H., Sarama, J.S., Wolfe, C., & Spitler, M.E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly,* 36, 550-560.

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological science.* 24(7). 1301-1308.

Sarama, J., & Clements, D. (2004). *Building Blocks* for early childhood mathematics. *Early Childhood Research Quarterly, 19,* 181-189.

Schweinhart, L., Montie, J., Xiang, Z., Barnett, W., Belfield, C., & Nores, M. (2005). *Lifetime Effects: The HighScope Perry Preschool Study Through Age 40*. (Monographs of the HighScope Educational Research Foundation, 14). Ypsilanti, Mich.: HighScope Press.

Siegler, R. (2009). Improving the numerical understanding of children from low-income families. *Child Development Perspectives*, 3, 118-124.

Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M.,

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2). 114-145.

U.S. Department of Education. (n.d.). *College and career-ready students* [Policy Paper]. U.S. Department of Education. Retrieved from https://www2.ed.gov/policy/elsec/leg/blueprint/college-career-ready.pdf

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