*Analysis and Improvement of Distribution Routes and the Associated Vehicle Fleet - Application to a Real Case in Portugal*

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**Abstract**
Logistics management plays a key role in improving business efficiency, since it is the part of supply chain management that plans, implements and controls the flow of goods, services and information between the point of origin and the point of consumption. Customers are more demanding and want the right products, in the right place, at the right time and in the right quantity. Consequently, logistics plays a fundamental role because it allows adding value to the product, and is always seeking to minimize the total costs. In the field of supply chain management, the use of vehicles to deliver products to customers is one of the largest operations. However, before delivering the products it is necessary to optimize the routes of the vehicles, in order to provide an efficient and low cost service. This study intends to analyze and propose improvements in the distribution network of ABC, a Portuguese company dedicated to the commercialization and distribution of home decoration products. The main objective of this work is to provide a model that allows the company to optimize the distribution routes and the associated fleet, in order to minimize the total costs of the distribution, maintaining the level of service provided to the customer. Finally, a model was developed that allows the definition of optimal routes for the various delivery points, resulting in a reduction in the number of vehicles to be used, the total distance traveled and the time of the journeys, leading to a real reduction in logistics costs.

Keywords: Logistics, delivery points, distribution route optimization, total costs reduction.

## Introduction

Nowadays, business management has been forced to implement new strategies and to innovate to ensure competitive advantages and profitability. We can say that logistics plays a key role in improving the efficiency within companies, as it is the part of supply chain management that plans, implements and controls the flow of goods (in the direct and inverse sense), services and information between the point of origin and the point of consumption, in order to meet the customers' needs (Grant, Lambert, Stock & Ellram, 2006).

To help optimize the supply chain, shippers are becoming less concerned about the mode of transportation, and instead are opting for the most efficient means of moving products (Capgemini & Langley, Jr., 2016).

Customers are increasingly demanding and want the right products, in the right place, at the right time and in the right quantity. In this sense, logistics reveals a fundamental role, since it allows adding value to the product, creating time and site utility, always seeking to minimize total costs (Ho, Ho, Ji & Lau, 2008).

In the field of logistics management, the use of vehicles to deliver products from suppliers to customers is one of the largest operations. Before delivering the products it is necessary to optimize the routes of the vehicles in order to provide an efficient and low cost service (Ho et al., 2008).

Sometimes, in some products, transportation represents the largest share of the final price charged to the consumer. In this way, transportation is considered a priority in supply chain management, so optimization can increase the competitiveness of a company. The management of the distribution fleet is also fundamental for the correct allocation of resources to needs (Mester, Bräysy & Dullaert, 2007).

The present work intends to study the improvement of the distribution network of ABC, a Portuguese company dedicated to selling and distributing home decoration products. The main objective is to provide a mathematical model that allows the company to optimize the distribution routes and the associated fleet, in order to minimize the total costs of distribution, having into account the maintenance of the level of service provided to the customer.

In the context of the problem under study, the main objectives of this work are:

• Construction of a mathematical model that adapts to the problem, with the necessary simplifications to reduce the associated complexity;

• Study of the feasibility of implementing the company's routes, through a comparative analysis with the current solution;

• Provide solutions that enable a significant improvement in the company's distribution network.

**Literature review and research questions**

According to Eilon, Watson-Gandy and Christofides (1971), logistics can be defined as the provision of goods and services from the point of supply to the point of demand.

For Carvalho (2017), logistics can be divided into two types of activities: principal and secondary. The former include transportation, stock keeping and order processing, while the latter include warehousing, material handling, packaging, purchasing, product programming and information systems operations.

Strategic planning consists of determining the number and location of a set of facilities such as factories, warehouses, and distribution centers in order to minimize the costs of meeting a given demand, and maintaining a high level of service. According to Simchi-Levi, Chen and Bramel (2014), these decisions are a crucial determinant for materials and products to flow efficiently throughout the distribution system.

Vehicle Routing Problems (VRPs) are considered the central part of many decision support systems for actual situations of distribution problems. In VRP, a set of routes with the lowest total cost for a given number of resources (the fleet of vehicles) located at one or more points (distribution centers, warehouses) should be determined in order to efficiently serve a number of demand or supply points (Mester et al., 2007).

The study of VRP problems is considered very important as the efficient definition of vehicle routes becomes increasingly important as markets become more open, and if this efficiency is achieved, the economic benefits will be quite substantial. (Bianchi, Birattari, Chiarandini, Manfrin, Mastrolilli, Paquete & et al., 2006).

Given the importance of logistics operations for organizations, VRP has been extensively studied because of its great applicability in real situations. For example, Eksioglu, Vural and Reisman (2007), who studied VRP's whole domain, accomplish an all-encompassing taxonomy for the VRP literature, and delineate all of VRP's facets in a parsimonious and discriminating manner.

The models and algorithms proposed for the VRP solution can be used not only to solve delivery problems, but also to collect products (Toth & Vigo, 2002).

The most studied VRP is the Traveling Salesman Problem (TSP), in which the Traveling Salesman has to visit a set of cities and return to the city where he started, minimizing the total distance traveled (Larsen & Madsen, 2000).

The VRP is a generalization of the TSP (Figure 1), which consists of finding an optimal solution in the definition of goods' delivery routes from one or several distribution centers, to a certain number of clients dispersed geographically in a given region. All customer demands must be met, all operational constraints must be met, and overall transport costs minimized (Larsen & Madsen, 2000).

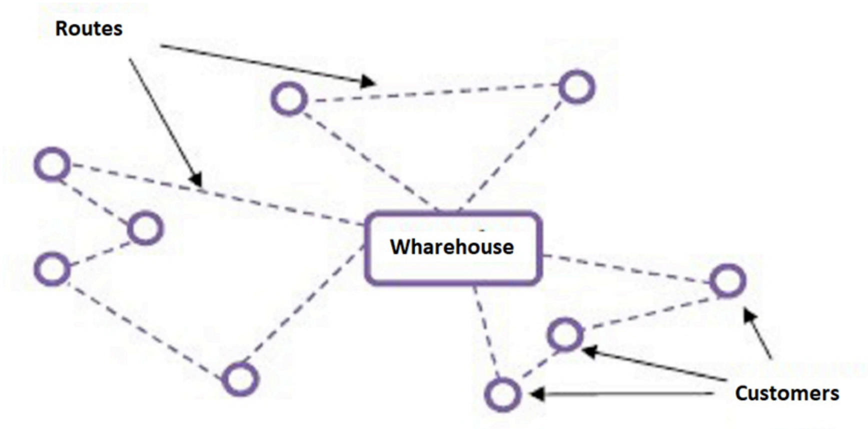


Figure 1: Graphic representation of a VRP
Source: Authors

Problems of route planning can be of different types depending on the characteristics of the problem to be solved. According to Berbeglia, Cordeau, Gribkovskaia and Laporte (2007), some of the most important problems identified in the literature are as follows:

- Capacitate Vehicle Routing Problem (CVRP);
- Vehicle Routing Problem Time Window (VRPTW);
- Vehicle Routing Problem with Pick-Up and Delivering (VRPPD);
- Dynamic Vehicle Routing Problem (DVRP);
- Split Delivery Vehicle Routing Problem (VRPSD);
- Dial-a-Ride Problem (DARP).

**Capacitate Vehicle Routing Problem (CVRP)**

The CVRP represent situations where the delivery charge is fixed, and all delivery vehicles have a uniform and limited capacity.

The CVRP was proposed in 1959 by Dantzig and Ramser (Dantzig & Ramser, 1959), and consists of finding a set of exactly K routes (one route for each vehicle available) with minimum cost, defined by the sum of the costs of each edge belonging to the routes such that:

- Each route begins and ends in the warehouse;
- Each client is visited by exactly one route;
- The sum of demands corresponding to the vertices visited by a route does not exceed the carrying capacity of the vehicle.

**Vehicle Routing Problem Time Window (VRPTW)**

VRPTW is another extension of the classic VRP, often used to decide the distribution of goods and services, with the additional constraint that each client has to be visited at a specific time interval ([ai, bi]), a so called time window.

The objective is to minimize the vehicle fleet, time spent on the journeys, and the waiting times required to serve all customers within their time requirements, without violating the capacity and travel time allowed to the vehicles, and the time windows established by customers (Kallehauge, 2007).

There are two types of formulations in a VRPTW. In the first one, with inflexible time windows, the vehicle has to wait if it reaches the customer before he is ready for service, and can never arrive after the required time. In the second, with flexible time window, temporal window violation is allowed, but has an additional cost associated (Kallehauge, 2007).

**Vehicle Routing Problem with Pick-Up and Delivering (VRPPD)**

The VRPPD is a VRP where the possibility of simultaneous deliveries and collects at the client are modeled. Thus, there are two quantities involved in the operation:

• Di, which represents the quantity that must be delivered to the customer;

• Qi, which represents the amount that must be collected at the customer.

In a problem of this kind it is therefore necessary to ensure that the goods (or persons) collected at the customer do not exceed the carrying capacity of the delivery vehicle. This constraint makes the planning problem more difficult, sometimes leading to poor utilization of transportation capacities, therefore increasing travel distances, or creating a greater need for available transportation vehicles (Cordeau, Laporte & Røpke, 2008).

For this reason, there is often only one point of departure and arrival of merchandise, with no exchange of goods between customers. In this way, all delivery requests depart from a warehouse, and all the collections are brought back to that same warehouse.

Finally, the problem can be further simplified by considering that each vehicle must make all deliveries before starting the collection. The objective is, in general, to minimize the fleet of vehicles and the sum of travel time, taking into account the restrictions concerning the transportation capacity of vehicles (Cordeau et al., 2008).

**Dynamic Vehicle Routing Problem (DVRP)**

DVRP is a problem of dynamic planning of vehicles' routes. Thus, unlike the static problems mentioned above, the relevant information for the planning process, such as the geographical location of customers, the time of service at the place of delivery/collection, the demand of each customer, or the travel times of vehicles between customers, is not fully known when the operation begins.

In most DVRP there are two types of requests:

• Anticipated request, which concerns static customers, whose orders are made before the operation is started;

• Immediate request, which refers to dynamic clients, whose orders occur in real-time during the operation (Larsen & Madsen, 2000).

Figure 2 shows a possible example of a DVRP, where there are 7 pre-orders and 2 immediate orders.

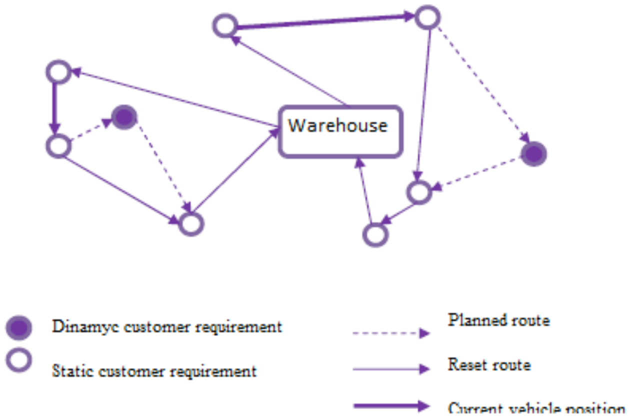


Figure 2: Representative scheme of a DVRP
Source: Authors

**Split Delivery Vehicle Routing Problem (VRPSD)**

In SDVRP, unlike the other problems described above, different vehicles can serve the same customer if this brings cost savings. The objective is to minimize the vehicle fleet and the total travel time required to serve all customers (Archetti, Speranza & Savelsbergh, 2008).

**Dial-a-Ride Problem**

The DARP is a generalization of the VRPPD (Pick-up and Delivery) and TSPTW (Traveling Salesman Problem with Time Windows) problems. This problem of route planning was introduced by Psaraftis (1980, 1988). In these first theoretical studies, an exact algorithm for the case of a single vehicle was developed in dynamic programming. Later, Desrosiers, Dumas and Soumis (1986) proposed an improvement to the previous algorithm, which was able to solve problems where there are more than 40 users.

Most algorithms known today, for multiple vehicle cases, are heuristics or metaheuristics, and the study by Jaw, Odoni, Psaraftis and Wilson (1986) is the first method capable of dealing with large-scale cases. Although this problem is common, it is relatively understudied due to its high complexity (Mauri & Lorena, 2009).

What makes the DARP different, and somewhat more difficult than most other route planning problems, is that the cost and travel time of the user have to be weighed against each other in the definition of a route (Lois & Ziliaskopoulos, 2017).

**Research questions**

This study aims to answer two research questions:

• Will the use of a mathematical model of route determination improve the distribution network of ABC?
• Which of the following ABC company variables (distance traveled, number of vehicles used, travel time, total cost) can be improved through the use of a mathematical model of route elaboration?

**Methodology**

Taking into account all the idiosyncrasies of the actual case under study, it was decided to apply a mathematical model of the type VRPPD with heterogeneous fleet to the problem under analysis. This model consists in a problem of route planning for a heterogeneous vehicle fleet, which begins and ends at the distribution center. Customers to visit may have associated a quantity of product to be delivered and another quantity to be collected. As they can only be visited once, delivery and collection takes place at a given time, and by a single vehicle.

The problem solving methodology chosen was to first divide the problem into smaller problems, then develop an optimization model through a mathematical programming language in the *Mathematics* software, and then apply it to each sub-problem obtained.

The ABC company has a fleet of commercial vehicles that are destined to the distribution of the requested orders, at any moment, being this an important requirement for its clients. Currently, there is a distribution network consisting of a warehouse, located in the industrial park of Seixal (near Lisbon), and customers are distributed throughout several areas of the country, according to Figure 3.


Figure 3: Location of the distribution center and customers
Source: Authors

The reception of the decoration material takes place at a weekly or biweekly pace, depending on the time of year. The decoration material comes from China, and the ordering is based on customer needs, Chinese production, and time of year. In order to understand the quantity of material for storage, please refer to Table 1, which refers to the number of pallets shipped per month for each region of the country in the year 20XX (for reasons of confidentiality, we cannot indicate the Year of the data collected).

| 20XX | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| North | 35 | 31 | 26 | 34 | 34 | 32 | 30 | 26 | 33 | 56 | 69 | 50 |
| Centre | 39 | 34 | 32 | 32 | 37 | 31 | 34 | 27 | 39 | 60 | 79 | 62 |
| South | 27 | 32 | 24 | 24 | 31 | 28 | 32 | 21 | 31 | 51 | 60 | 45 |
| Total | 66 | 66 | 56 | 56 | 68 | 59 | 66 | 48 | 70 | 111 | 139 | 107 |

Table 1: Quantity of pallets issued monthly
Source: Authors

The transportation to the clients is carried out with the company own fleet, that carries all the material to the North, Center and South of Portugal. Figure 4 presents a simplified scheme of the distribution structure.

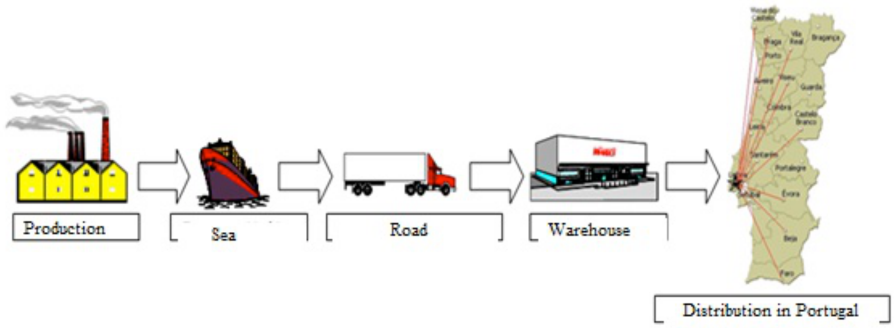


Figure 4: Scheme of the distribution structure
Source: Authors

Routes are organized on a daily basis by geographical area, and each route is associated with a specific driver and the vehicle assigned to him.

In order to define routes, three different groups of customers are established, depending on the frequency of orders placed (in S - when weekly orders are made, in Q - when they are made each 15 days, and in M - when they correspond to monthly orders). It is in these modes that orders are planned, checking the sales forecast for specific weeks with each salesperson.

**Distribution in Lisbon**

The distribution is governed by the opening hours of the distribution center, from 9:00 am to 6:00 pm, interrupting deliveries from 1:00 p.m. to 2:00 p.m., which corresponds to the lunchtime of the vehicle driver. The company's fleet is heterogeneous, since it includes vehicles with different transport capacities.

In the Seixal warehouse, the fleet consists of seventeen vehicles, four of which provide the Great Lisbon area. The products are very different, since their weight can vary from 5kgs to 35kgs, although on average it is considered that a box weighs between 10 and 25kg.

Another aspect to keep in mind is to avoid that the weight of the loads transported exceeds the weight accepted by law. Regardless the number of pallets transported by the vehicle, the maximum gross weight must be within the limits imposed by law.

The relationship between the type of vehicle operating in the Lisbon Area and the number of boxes it can carry is described in Table 2.

| Car model | Car Weight (kg) | Carrying capacity (kg) | Gross weight (kg) | Average number of boxes |
|---|---|---|---|---|
| Fiat Dobló | 1280 | 740 | 2020 | 35 |
| Fiat Dobló | 1280 | 740 | 2020 | 35 |
| Fiat Dobló | 1280 | 740 | 2020 | 35 |
| Citroen Jumper | 1624 | 1559 | 3183 | 90 |

Table 2: Characterization of the Lisbon fleet
Source: Authors

From the year 20X-2 to 20XX the number of customers has been growing. In Figure 5, we can see a graph illustrating this positive evolution.

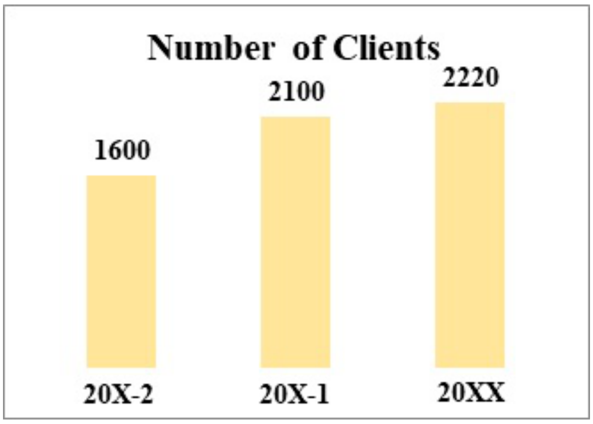


Figure 5: Evolution of the number of customers
Source: Authors

**Problem Restrictions**

The activity developed by the company presents some restrictions that have to be taken into account in order to guarantee the reality of the model and the proposed solution. The distribution constraints to be considered are:

• Demand must always be met, regardless the number of products involved. There are no minimum order quantities. Demand should be fulfilled, ideally, up to a maximum of 24 hours;

• It is necessary to respect the maximum load capacities imposed by law for each vehicle. When loading, each vehicle must have a number of boxes not exceeding the maximum load capacity;

• Each customer can only be visited once a day, and by a single vehicle;

• Usually, the distribution starts at 9:00 a.m., has a break from 1:00 p.m. to 2:00 p.m. for the driver's lunch, and ends at around 6:00 p.m., not existing a fixed timetable;

• Each route begins and ends at the distribution center;

• Each distributor must visit a number of customers that does not exceed his capacity to respond to orders. The sum of the quantities to be delivered and received from each customer cannot exceed the maximum capacity of the vehicle;

• Distribution vehicles only carry out distribution and collection tasks;

## Results analysis and discussion

After identifying the problem, an optimization model was developed for the establishment of the optimal set of routes, and the number of vehicles necessary to execute them.

### Fleet

It is necessary to characterize the type of vehicle (brand and model) used by the company, and its maximum capacity. These were already reported in Table 2.

### Clients

Although the sample focuses on 1200 customers, the amount of material ordered over a year varies greatly.

For this reason the month of November of 20XX was chosen, since it was during this period that a greater demand was observed, and also during which the number of delivery points varied daily.

Table 3 presents the number of customers served on each day of the distribution, and the respective aggregation in number of parishes (clusters) to visit.

| Day | Number of clients | Number of clusters | Boxes delivered | Boxes collected |
|---|---|---|---|---|
| 1 | 60 | 30 | 126 | 0 |
| 2 | 74 | 21 | 98 | 0 |
| 3 | 55 | 25 | 88 | 0 |
| 4 | 62 | 20 | 99 | 5 |
| 5 | 45 | 24 | 100 | 10 |
| 8 | 55 | 15 | 70 | 0 |
| 9 | 74 | 20 | 83 | 2 |
| 10 | 45 | 27 | 88 | 0 |
| 11 | 61 | 20 | 93 | 8 |
| 12 | 73 | 26 | 85 | 0 |
| 15 | 72 | 25 | 89 | 0 |
| 16 | 60 | 27 | 74 | 0 |
| 17 | 73 | 24 | 89 | 0 |
| 18 | 60 | 25 | 91 | 0 |
| 19 | 30 | 21 | 97 | 5 |
| 22 | 49 | 28 | 83 | 2 |
| 23 | 18 | 24 | 78 | 1 |
| 24 | 55 | 24 | 75 | 0 |
| 25 | 20 | 20 | 84 | 0 |
| 26 | 46 | 24 | 88 | 0 |
| 29 | 30 | 27 | 65 | 5 |
| 30 | 30 | 25 | 78 | 0 |

Table 3: Number of clients and clusters to visit, number of boxes delivered/collected
Source: Authors

The aggregation by parishes is related to the fact that customers are close to each other. In this way, clients were grouped in a single area, the center of this same parish. Although aggregated in parishes, the amount of data to be treated is still high, which leads to the application of the p-median model (which allows to divide the problem into smaller sub-problems), grouping the points of visit in a certain number of desired aggregates, with the objective of reducing the total distance to be traveled.

The application of this model groups the parishes logically, as it intends to minimize the distances to be traveled, thus reducing transportation costs.

Google Maps was used to determine the distance and the travel time between the various points of delivery, and from the warehouse to those points, by demonstrating the best routes between the two points, and the approximate values of the distance and travel times.

**Visiting times**

The time spent in each cluster to deliver the new material, and to collect the damaged material, is divided into two distinct components:

• tf - The fixed time needed to handle administrative issues on a daily basis, which is assumed to take approximately ten minutes;

• tv - The variable time in each cluster depends on the reason the visit was made. The time to unload and load at each delivery point depends on the number of boxes to deliver and defective products to collect.

**Costs**

The costs related to vehicle maintenance are as follows:

• Fiat Dobló ➔ cost of 365 € / month;

• Citroen Jumper ➔ Cost of 465 € / month.

The costs concerning fuel consumption for the period from 01/11/20XX to 30/11/20XX, depend on the type of vehicle. For the Citroen Jumper, the costs are summarized in Table 4, while the costs for Fiat Dobló are shown in Table 5.

| | Liters | Price (€/L) | Value (€) | Kilometers | Kms travelled |
|---|---|---|---|---|---|
| **Refuelling 1** | 68.04 | 1.18 | 80.29 | 62245 | |
| **Refuelling 2** | 70.01 | 1.18 | 82.61 | 62885 | 640 |
| **Refuelling 3** | 65.03 | 1.18 | 76.74 | 63485 | 600 |
| **Refuelling 4** | 68.02 | 1.18 | 80.26 | 64111 | 626 |
| **Total** | 271.1 | | 319.90 | | 1866 |

Table 4: Costs per km for the vehicle Citroen Jumper
Source: Authors

| | Liters | Price (€/L) | Value (€) | Kilometers | Kms travelled |
|---|---|---|---|---|---|
| **Refuelling 1** | 53.12 | 1.18 | 62.68 | 38489 | |
| **Refuelling 2** | 55.00 | 1.18 | 64.90 | 39169 | 730 |
| **Refuelling 3** | 51.67 | 1.18 | 60.97 | 39849 | 680 |
| | | | | | |
| **Total** | 159.79 | | 188.55 | | 1410 |

Table 5: Costs per km for the vehicle Fiat Dobló
Source: Authors

It is now possible to calculate the cost of running each Km (CdistCitroen and CdistFIAT), taking into account the fuel costs and the maintenance of the two vehicles:

- CdistCitroen = (319,90+465,00)/1866 Kms = 0,43€ / Km
- CdistFIAT = (188,55+365,00)/1410 Kms = 0,39€/Km

**Mathematical formulation of the problem**

We now present a mathematical definition of the simple VRP, based on Marinakis, Marinaki and Dounias (2010), and Secui (2015).

**Indexes**

i - Location visited

j - Place to visit

v - Vehicle

**Parameters**

• qi - Quantity of boxes to be delivered to location i.

• ri - Amount of defective material to be collected at location i.

• maqri - Maximum value between boxes to be delivered and defective material to be collected in each parish.

• capvv - Maximum capacity of vehicle v, measured in number of boxes it can carry.

• cdistv - Unit cost to cover one km for each vehicle, in €/Km.

• tvi - Variable delivery time of boxes and collection of defective material, in location i, in minutes.

• d_viagemij - Distance between location i and location j, in Km.

• t_viagemij - Travel time between points i and j in minutes.

• tf - Fixed time to handle administrative issues in a distribution operation, in minutes.

• total - Daily time available for the distribution operation, in minutes.

**Variables**

Decision Variables:

• xijv - It is a binary variable that takes the value 1 if vehicle v visits cluster i and then moves on to cluster j, otherwise it will take value 0.

• vfv - is a binary variable that takes the value 1 if the vehicle makes a route, and the value 0 otherwise.

Free Variable:

Z - Variable only present in the objective function, assumes the value of the minimum cost found.

Positive Variables:

• distotalv - This variable is required to store the value of the total distance traveled by each vehicle.

• tetotalv - It is a variable that saves the total time required to make the distribution plan calculated for each vehicle.

**Objective function**

$$MIN \sum_{v}^{m} \sum_{i}^{n} \sum_{j}^{n} x_{ijv} \,.\, d\_viagen_{ij} \,.\, cdist_v \tag{4.1}$$

**Restrictions**

Equation 4.2 ensures that all routes, if any, start at the distribution center.

$$\sum_{i>1} x_{jiv} = vf \text{ ,} j = Centro\ de\ Distribuição\ > \forall \text{V} \tag{4.2}$$

Equation 4.3 ensures that all routes, if any, end up in the distribution center.

$$\sum_{i>1} x_{ijv} = vf \text{ ,} j = Centro\ de\ Distribuição\ > \forall \text{V} \tag{4.3}$$

Equation 4.4 ensures that a single vehicle arrives to each customer.

$$\sum_{i \neq j} \sum_{v} x_{ijv} = 1 \text{ ,} \forall > 1 \tag{4.4}$$

Equation 4.5 ensures that a single vehicle departs from each customer. Thus, equations 4.4 and 4.5 define that for all routes, each customer will only be visited once.

$$\sum_{i \neq j} \sum_{v} x_{jiv} = 1 \text{ ,} \forall > 1 \tag{4.5}$$

Equation 4.6 ensures the movement of the vehicles within their route, that is, when a vehicle arrives at a point of delivery, must mandatorily leave this point and go to the next one.

$$\sum_{j} x_{jiv} = \sum_{j} ijv \ , \ \forall i > \forall \mathrm{v} \tag{4.6}$$

Equation 4.7 ensures that the vehicle making a certain route can handle all customers, and that no vehicle visits more customers than its capacity allows.

$$\sum_{i>1} \sum_{j\neq 1} maq_i \, .x_{ijv} \leq cap_v , \forall v \tag{4.7}$$

Equation 4.8 defines whether a vehicle is required for the distribution operation. If the vehicle leaves the warehouse, it can only make one route.

$$x_{ijv \leq} \leq vf_v, \forall i > \forall j < \forall v \tag{4.8}$$

Equation 4.9 ensures that the time in transit between the points i and j, given by the variable t_viagemij, and the time required for the entire distribution operation for each vehicle at location i, do not exceed the available time per day for the distribution operation for each driver, represented by the variable ttotal, which is 8 hours a day, that is 480 minutes.

$$\sum_{i>1} \sum_{j>1} (t\_viajem_{ij} + tv_i + tf) \leq ttotal, \forall v \tag{4.9}$$

Equation 4.10 eliminates the creation of sub-routes, being S a subset of clients.

$$\sum_{vi\in S} \sum_{vj \in S} x_{ijv} \leq |S| - 1 , \qquad \forall S \subseteq \mathrm{V} \setminus \{0\}; \mathrm{S} \neq \emptyset; \mathrm{v} = 1, \ldots, \mathrm{m} \tag{4.10}$$

Equation 4.11 allows calculating the total distance traveled by each vehicle.

$$distotal_v = \sum_{i} \sum_{j} (x_{ijv} . d\_viagem_{ij}), \qquad \forall v \tag{4.11}$$

Equation 4.12 calculates the total time required by each vehicle to make the distribution plan obtained with the model.

$$tetotal_v \sum_{i} \sum_{j} [\![ xi_{ijv} . (t\_viagem_{ij} + tv_i + tv), \qquad \forall v \tag{4.12}$$

After the mathematical formulation of the model was carried out, its implementation was done using the software *Mathematics*, which uses mixed integer optimization to find the optimal solution.

The model refers to the 22 days of November 20XX, which are grouped into 44 segregations, in total. This calculation was achieved by initially aggregating clients in their parishes and, in a second step, by using the p-median model.

Thus, we obtain groups of no more than 12 parishes, being possible to apply the model in a total of 44 times for each situation, equivalent to the number of groups obtained.

In this study the real model is compared with the VRPPD, during the month of November of 20XX.

**Real Scenario vs VRPPD**

Figure 6 presents the comparative analysis of the distance traveled between the Real Scenario and the VRPPD.

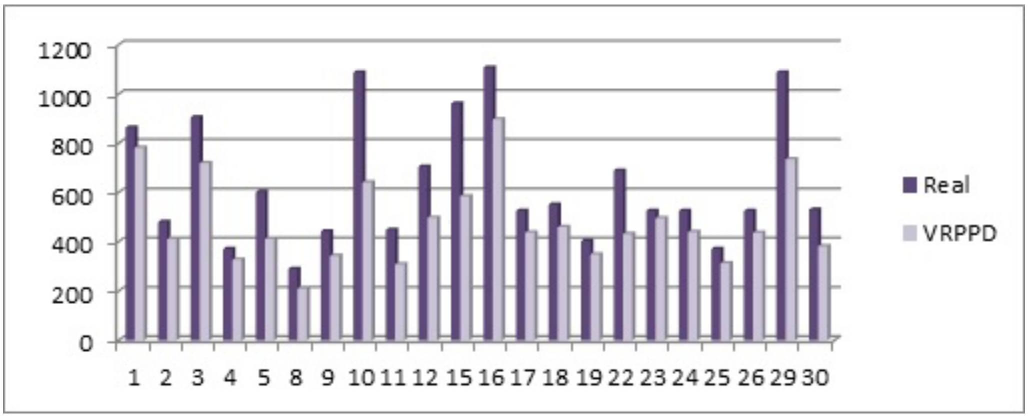


Figure 6: Comparative analysis of the distances traveled for the two scenarios
Source: Authors

Table 6 presents a summary table with the total distance and the average daily distance traveled in the month of November 20XX.

| | Actual distance traveled (Kms) | VRPPD distance traveled (Kms) |
|---|---|---|
| Daily average | 636.30 | 457.90 |
| Total | 14000 | 10470 |

Table 6: Summary of Actual Distance vs VRPPD Distance (Kms), November 20XX
Source: Authors

From the comparison we can verify that there is a significant monthly reduction of 3529 km, in average 160 km per day, which represents a very significant improvement of the results.

In Figure 7 we can observe the comparative analysis of the number of vehicles used between the Real Scenario and the VRPPD model.

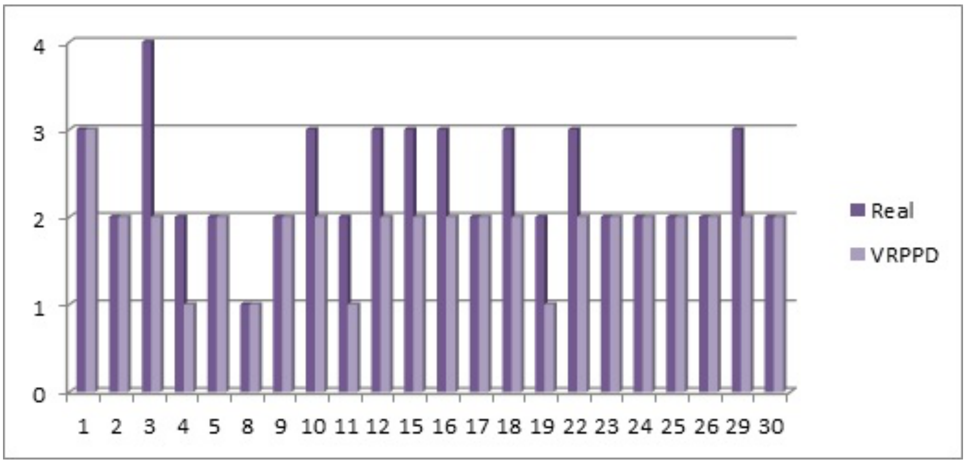


Figure 7: Comparative analysis of the number of vehicles used
Source: Authors

Table 7 presents a summary table with the number of vehicles used and the average daily number of vehicles for November of 20XX.

| | Actual number of vehicles used | VRPPD number of vehicles used |
|---|---|---|
| Daily average | 3 | 2 |
| Total | 53 | 41 |

Table 7: Summary of the number of vehicles used, November 20XX
Source: Authors

From the comparison, the Real Scenario shows the need for 53 vehicles to carry out the distribution operation for the entire month of November 20XX, while the VRPPD scenario only needs 41, which represents a decrease of 12 vehicles during that month.

Figure 8 shows the comparison between the Real Scenario and the result of the VRPPD scenario concerning the time required to perform daily operations.

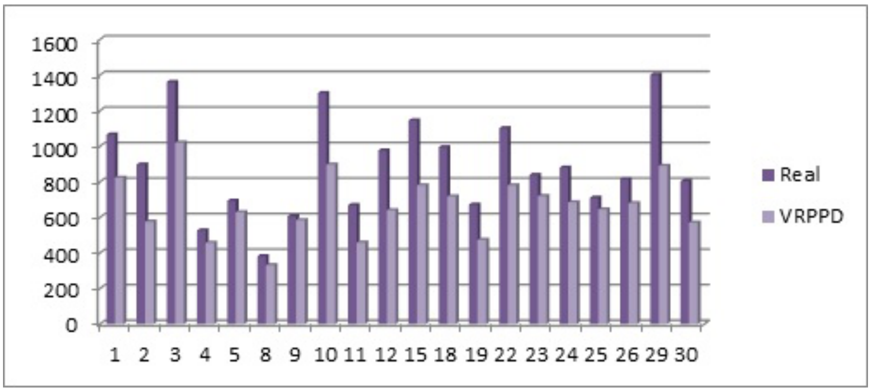


Figure 8: Comparative analysis of route duration
Source: Authors

Table 8 presents a summary table with the total and average daily duration of the routes, for the month of November 20XX.

| | Duration of the actual route (minutes) | Duration of the VRPPD route (minutes) |
|---|---|---|
| Average | 902 | 682 |
| Total | 19833 | 15004 |

Table 8: Summary of route duration, November 20XX
Source: Authors

Comparing the results obtained in the Real Scenario with the results of the VRPPD model, there is a considerable reduction (4829 minutes) in the total duration of the routes, approximately 80 hours in a month, representing a daily reduction of 220 minutes.

In Figure 9, the costs of the distribution operation can be compared between the Real Scenario and the VRPPD scenario.

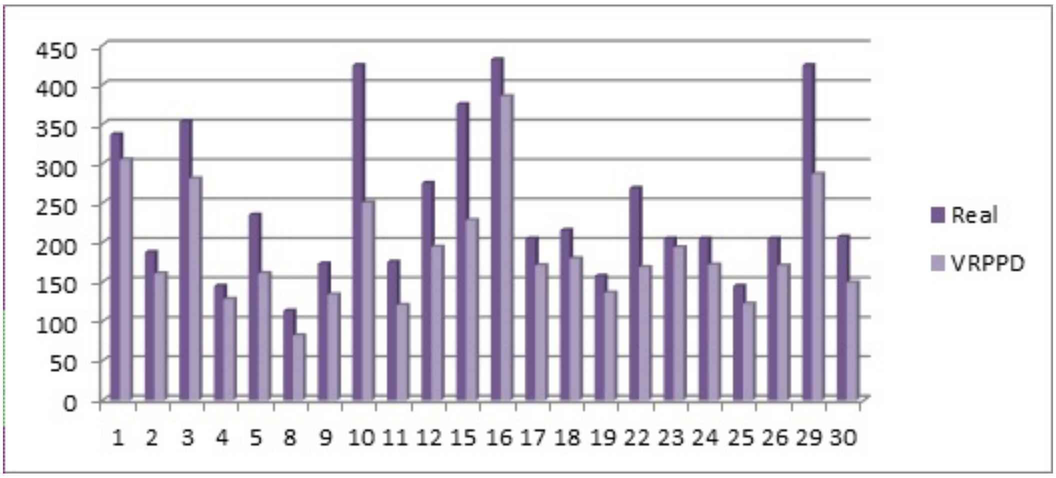


Figure 9: Comparative analysis of distribution costs
Source: Authors

Table 9 shows the average value involved, as well as the total cost of the operation for November 20XX.

| | Actual costs (€) | VRPPD costs (€) |
|---|---|---|
| Average | 248,00 | 187,00 |
| Total | 5460,00 | 4119,00 |

Table 9: Summary of distribution costs, November 20XX
Source: Authors

Comparing the results obtained there is a significant reduction of costs for the VRPPD model, showing a reduction of 1341 euros monthly compared to the costs incurred in the Real Scenario, which translates into a daily average reduction of 61 euros.

## Conclusions and recommendations

In this study it was intended to optimize the routes and fleet of the company ABC, promoting the achievement of its economic efficiency, and preserving the quality of the services it provides.

A mathematical model was developed to define optimal routes that allowed total costs to be minimized, respecting the capacity of each vehicle. This model allowed defining routes by vehicle, as well as the time and distance covered in the distribution by the several points of delivery, minimizing the logistics costs.

The study focused on the Lisbon area, and took place during the month of November 20XX, as it was at this time of the year that there was a greater demand for material.

The thousands of Lisbon customers were aggregated in the parishes to which they belong, considerably reducing the number of search points. The p-median model was then applied, dividing the clusters into smaller groups, so that the VRPPD model could provide optimal data in acceptable computational time. Once this process was completed, the VRRPD model was finally applied to all the groups defined for each distribution day, with the conclusion that there were improvements when comparing the model with the company's reality.

Regarding the first research question, it is concluded that the use of a mathematical model of route determination, improves the performance of the distribution network of ABC company.

In relation to the second research question, we can conclude that the use of a mathematical model of route determination will improve all variables studied, whether the distance traveled, the number of vehicles used, the travel time or the total cost. The study showed that all these variables will improve if the ABC company uses a mathematical model in the elaboration of the routes of its distribution vehicles.

With the use of the VRPPD model, we observe a reduction in the distance traveled, the number of vehicles used, the average time of each distribution route, and the total costs of operations compared to the reality of ABC. The application of the model also has the advantage of establishing optimal routes so that the needs of the customers are promptly fulfilled.

As a main recommendation for the future, we can highlight the application of the same model to other geographical areas of Portugal, ensuring a greater reduction of the logistics costs of the ABC company, and maintaining the satisfaction of its customers at a national level.

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