Quantifying Uncertainty to Plan in Dynamic Environments

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Abstract
Uncertainty is defined by the lack of information to know the future state of a system. This definition highlights the importance of information in an era of constant change and turbulence. It is not surprising that greater the uncertainty in the environment, the greater the value and importance that the management of information takes.

Even so, information, being such a valuable resource, is treated empirically and qualitatively although there are formulas to quantify it. This article seeks to provide tools to quantify uncertainty so that it can be included in the planning process and scenario projections.

Keywords: Uncertainty, forecasting, dynamic environment, entropy, information, planning

Introduction

The first attempts to quantify the amount of information in a message come from Shannon. This quantification comes from the principle that the more probable is an event, less information will have the message that anticipates it. If a message is issued forecasting that the Earth will continue to rotate around the sun tomorrow, it is a message with very low content of information because it does not contribute anything new to our knowledge. If a message is issued forecasting that a meteor will crash into the Earth, the message will have a high content of information because it is forecasting a very low probability occurrence. This way, information can be quantified through the probability of occurrence of the event that forecasts.

The measurement unit of information is the bit, a term that comes from binary digit which measures the amount of positive choices (1) or negative (0) in a binary sequence.

If I have eight future scenarios of equal probability of occurrence (1/8 each), I will need three stages to determine which scenario to choose at end. The first stage consists in ruling out if the scenario to happen is among the first 4 or the second 4 scenarios, the second stage consists in choosing between the first two or the last two and the last stage consists in choosing between the last two scenarios that are left. Each stage involves a binomial decision of yes or no, which provides one bit of information, the three stages provide 3 bits which is the enough amount of information needed to rule out the correct setting. The mathematical operation that tosses this result for 8 possibilities is the logarithm base 2 of 8, so Log2 8 = 3
However, the information does not remain static over time, because as this progresses, the information keeps getting outdated. Likewise, as we seek to predict more distant scenarios in time, uncertainty grows. This loss of information over time is known as entropy.

Following this reasoning, Grassberger proposes to quantify the loss of information through Kolmogorov entropy shown in equation (1)

$$K_n = - \sum P_{1\ldots n} \log P_{0\ldots n} \qquad (1)$$

P_0 being the probability that the system will be found in the predicted scenario.
According to this formula, the higher uncertainty degree is when two scenarios have the same probability of occurrence and the lower uncertainty degree, when one has 100% probability of occurrence and the other 0% probability of occurrence (absolute certainty). In the latter case, entropy is zero, no loss of information is documented. The entropy will reach its maximum value when both scenarios have equal probability of occurrence (50%) because the occurrence of one or the other is given almost by chance, there is no indication that makes me favor a higher probability of occurrence of a scenario over the other. This distribution of probabilities is shown in Fig. 1, for different combinations of occurrence probabilities between two scenarios.

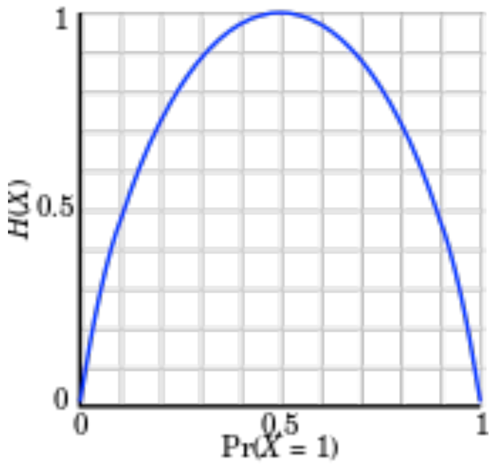

Figure 1. Entropy of a Binary Source according to its Probability of Occurrence.
Source: Zanuy, MF (2001). Communication systems. Marcombo.

This example provides the first conclusion to reduce the uncertainty at the time of projecting scenarios: the way to reduce the entropy increase in the projection of a scenario will be the development of an analysis that allows to favor a higher probability of occurrence of a scenario over another, considering that a closer to 100% occurrence of one of the scenarios grant a lower entropy in the calculation.

How long to plan? Calculation of Maximum Planning Horizon
To set the maximum planning horizon, it should be used the Lyapunov exponent. Kolmogorov entropy can be approximated by the sum of the Lyapunov exponents. In the case shown, where the system is unidimensional, Kolmogorov entropy coincides with the Lyapunov exponent. This relationship is important because according to Prigogine, the inverse of Lyapunov exponent shows the maximum planning horizon also called time or Lyapunov.

In the example of the case, where I measured a loss of information of 3 bits in a year. Its inverse is the Lyapunov time and displays the maximum projection time until all system information is lost.

Time = 1/3 = 0.333 years approximately 4 months

According to this result, any credible projection may be done within the next four months to the current rate of loss of information. From this limit, it will not be able to make reliable predictions about the variable being studied, in this case, the projection of sales. In conclusion, before studying how to make a reliable long-term projection, it should be calculated if it is possible to make any long-term projection.

Calculating the probability of occurrence of a scenario
In the case of complex systems like those being studied, it is a mistake to project scenarios based on linear regression calculations. For a system with these characteristics, it is much more useful to look forward to increasing the level of reliability of a projection than to look back. According to Stewart in the twenties, Yule was able to quantify the extent to which the data from the current year about the sunspots, provided more information on the number of spots that would appear in the following year than the data from the last 10 years. This shows that forecasting

scenarios in a turbulent environment, the previous event influences more than the ten previous events. The technique for calculating the probability of occurrence of an event based on the preceding event is known as Markov chain.

A Markov chain is a sequence of events in which the probability of each outcome for an event depends only on the immediately preceding event. The main condition for applying the Markov chain is that the transition probability remains constant in time.
To outline the Markov chain, an example is proposed where there are only two possibilities: Sales can be raised or lowered.

If a projection for the next two trimesters wants to be done, four scenarios will be obtained: 1) sales go up and then down, 2) sales go up and then up again, 3) sales drop and then rise and 4) sales drop and then drop again. If a scenario projection in three trimesters is wanted, eight scenarios will be obtained according to Fig 2.

Logically, the higher the projection horizon, the higher the number of final scenarios, therefore, greater uncertainty about the final state of the system.

According to the equations previously shown, these conclusions can be translated in terms of information and entropy. Taking the current trimester as T, it will seek to calculate the entropy for trimesters T+1, T+2 and T+3. For this, it must be defined the transition probabilities, measuring the probability that sales rise after increasing in the previous trimester, the probability that sales drop after increasing in the previous trimester, the probability that sales rise after dropping the previous trimester and the probability that sales drop after dropping the previous trimester.

Consider that according to historical records, in 70% of cases, sales have risen again after increasing in the previous trimester and in 30% of cases, sales have dropped after increasing in the previous trimester. Likewise, in 60% of cases, sales have risen after dropping the previous trimester and in 40% of cases, sales have continued to drop after dropping the previous trimester. Transition probabilities would be as shown in Fig 3.

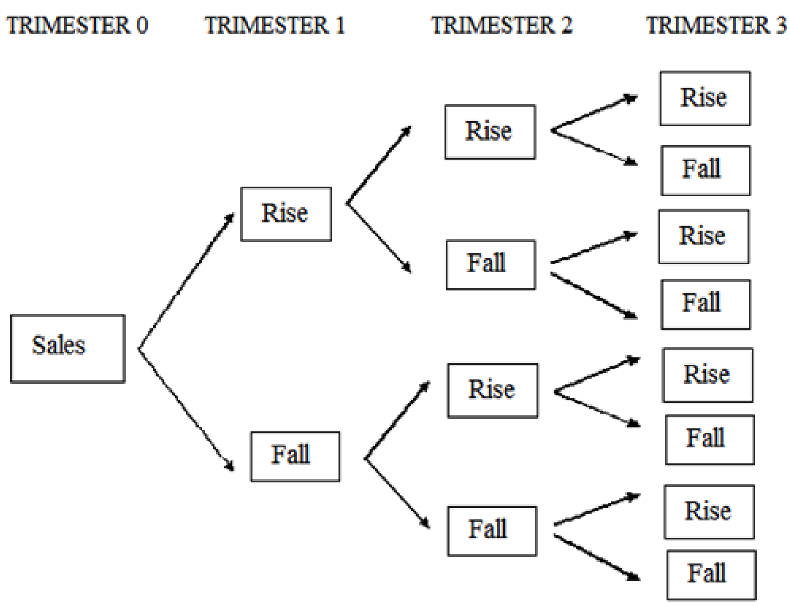

Figure 2. Sales Projection in Three Trimesters

According to Fig. 3 it is possible to answer the question: what is the probability that sales fall in the trimester T + 3, after rising in the trimester T + 1 and rising again in the trimester T + 2? The answer is obtained by multiplying the transition probabilities to reach to that event, in this case, P = 0.7 x 0.7 x 0.3 = 14.7%.

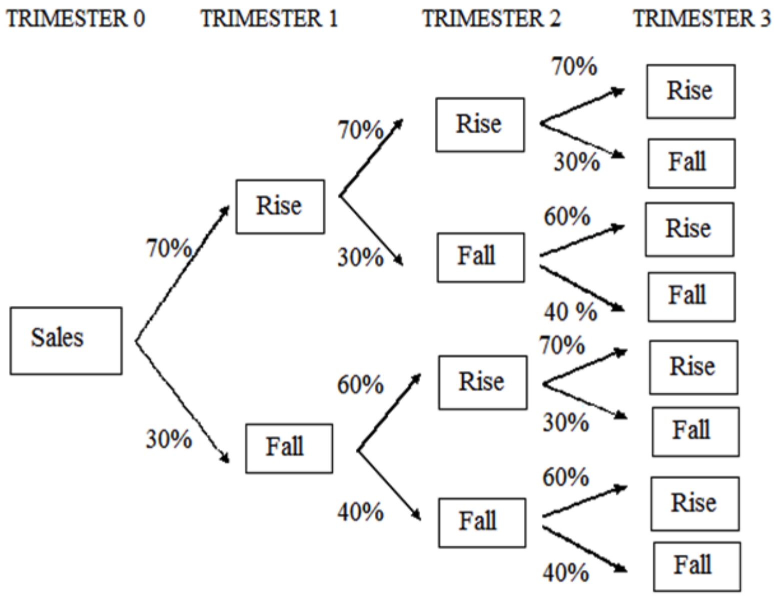

Figure 3. Probability of Future Scenarios

Once calculated the probabilities of each event, it is proceed to calculate the loss of information or entropy between a trimester and another. Kolmogorov entropy for each of the trimesters according to the probabilities shown would be:
Kolmogorov entropy for trimester T + 1 = $- [0.7 * \log_2 (0.7) + 0.3 * \log_2 (0.3)] =$ 0.881 bit
Kolmogorov entropy for trimester T + 2 = $- [0.49 * \log_2 (0.49) + 0.2\ 1 * \log_2 (0.2\ 1) + 0.18 * \log_2 (0.18) + 0.12 * \log_2 (0.1\ 2)] = 1.79$ bits

Kolmogorov entropy for trimester T + 3 = $- [0.343 * \log_2 (0.343) + 0.147 * \log_2 (0.147) + 0.126 * \log_2 (0.126) + 0.084 * \log_2 (0.084) + 0.126 * \log_2 (0.126) + 0.054 * \log_2 (0.054) + 0.072 * \log_2 (0.072) + 0{,}048 * \log_2 (0.048)] = 2.7$ bits

This calculation of entropy shows information lost in the system, so the entropy difference between trimesters shows the loss of information from trimester to trimester, as it follows:

K3 - K2 = 2.7 - 1.79 = 0.91 bits approximately 1 bit
K2 - K1 = 1.79 - 0.881 = 0.91 bits approximately 1 bit
It follows that 1 bit of information of the trimester T+1 is approximately lost to trimester T+2 and 1 bit of the trimester T + 2 to trimester T + 3.

This calculation shows the increase in entropy period to period. The greater planning horizon, the less accurate will the predictions be, which is intuitively sensed by managers who are reluctant to make long-term plans. However, the response to this growing uncertainty is not in stop making long-term plans, neither by extending the historical database on which the projection is made, but decreasing the entropy increase that occurs in time. Even so, the natural reaction to make more precise future predictions has always been to expand the historical database on which the prediction is actually done, this strategy has not lessened the fear of managers to deal with uncertainty when making long-term plans.

The goal: Reducing Entropy

By quantifying the loss of information through entropy, it is also possible to quantify the degree of uncertainty when planning. Reducing uncertainty is equivalent to reducing the entropy in the prognostication, so this should be the new target projection of scenarios.

The Kolmogorov entropy formulas demonstrate that the larger number of projected scenarios, the greater the increase in entropy of the system. If instead of projecting two scenarios, five future scenarios (all with the same probability of occurrence) are projected, the increase of entropy is greater, since to a higher number of scenarios, uncertainty about what will happen is increased.

Kolmogorov entropy for two scenarios with equal probability of occurrence:
$K = - [0.5 * \log_2 (0.5) + 0.5 * \log_2 (0.5)] = 1$ bit

Kolmogorov entropy for five scenarios with equal probability of occurrence:
$K = - [0.2 * \log_2 (0.2) + 0.2 * \log_2 (0.2) + 0.2 * \log_2 (0.2) + 0.2 * \log_2 (0.2) + 0.2 * \log_2 (0.2)] = 2.32$ bit

The conclusion that emerges from this analysis is that the smaller number of future scenarios, lesser uncertainty in the projection. This conclusion may go against the popular logic, as it always has been argued that while greater range of possibilities to consider, the more prepared you are for the future. The solution to this paradox is an investment in the conclusion: it is not that more information is possessed by handling a smaller amount of future scenarios, but fewer future scenarios are managed by doing a more detailed prospective and information with better quality is possessed. In short, the less we know, the more the spectrum of future possibilities will be displayed.

The new objective of the strategic foresight should abide in limiting the number of scenarios based on more detailed analysis in each of its stages. This reduction in number of projected scenarios can affect more in reducing uncertainty than a correct calculation of occurrence of each stage, so that the objective of limiting the number of scenarios is the most important to reduce the entropy of the system.

Conclusions

Information is the backbone of value generation of our time. Therefore it should be quantified within the planning process rather than being treated empirically. Likewise scenarios projections should not ignore the uncertainty and turbulence of the environment but involve them in the planning process.

To design a more accurate long-term planning, two steps must be performed. The first is to set the maximum planning horizon and the second one is to reduce the entropy increase over time within that horizon.

The new objective of the strategic foresight should consist in limiting the number of scenarios based on more detailed analysis in each of its stages. This reduced number of projected scenarios will influence in the reduction of uncertainty, so that the objective of limiting the number of scenarios will be the most important to reduce the entropy of the system.

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