Voluntary Contribution of Public Goods and Income Equality

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1. Introduction

If we consider the models of voluntary contributions of public goods, some neutrality theorem holds. Bergstrom, Blume and Varian (1986) precisely proved the neutrality theorem. In this neutrality theorem, small redistributions of incomes between contributors do not affect each individual's consumption level at the equilibrium outcome. So such redistributions do not affect each individual's utility or social welfare. So redistributions of income are meaningless from the viewpoint of social welfare.

In addition, Bergstrom, Blume and Varian (1986) proved that large redistributions of incomes change the set of the contributors and each individual's consumption level. Especially, the redistributions that increase the aggregate income of the current contributors necessarily increase the level of the public good at the equilibrium outcome.

Itaya, de Meza and Myles (1997) considered the case where only one individual contributes and the other individuals are indifferent between contributing and not contributing. Then, Itaya, de Meza and Myles (1997) analyzed the effect of redistributions that increase the income of the contributor and proved that such redistributions raise social welfare in the case of two individuals.

In this paper, we consider redistributions that are similar to those analyzed by Itaya, de Meza and Myles (1997). We show that such redistributions do not make Pareto improvement in the case of two individuals. That is, such redistributions necessarily decrease non-contributor's utility. But, in the case of more than three individuals, there is a redistribution that makes Pareto improvement. We derive the necessary and sufficient condition for Pareto improvement.

As mentioned before, the redistributions that increase the contributors' income increase the level of the public good. So, usually, many have an intuition that efficiency needs inequality. In fact, the title of Itaya, de Meza and Myles (1997) is 'in praise of inequality'. Contrary to this intuition, this paper shows that we can construct the economy where efficiency consists with equality. So inequality is not always praised.

In addition, we consider the economies where each individual's utility function is Cobb=Douglas function, assuming that there are individuals who evaluate the public good and individuals who evaluate the private good. We calculate the income distributions that have no room for further Pareto improvement and analyze who should be a contributor. In many cases, the individual with the highest evaluation of the public good should be a contributor, but, there may be the cases where the individual with the lowest evaluation of the public good should be a contributor. Moreover, we show that there is the case where transferring to the least advantaged individual improves the most advantaged individual.

2. The Model

Consider an economy where there are one private good, one public good and n individuals. Let $N = \{1,2,\cdots,n\}$. Each Individual $i \in N$ has his or her own income I^i.

Each Individual i consumes an amount of the private good x^i and contributes an amount g^i to the public good. An amount of the public good G is defined as $G = \sum_i g^i$. Let $G_{-i} = \sum_{j \neq i} g^j$. So we have $G = G_{-i} + g^i$. Each Individual i has a utility function: $u^i(x^i, G)$. We assume that the utility functions are continuous, increasing and quasi-concave. Moreover, we assume that the public good and the private good are normal good.

The units of the public good and of the private good are chosen so that prices are normalized to 1. Then, the Individual i's budget constraint is $x^i + g^i \leq I^i$. Let $I = (I^1, \cdots, I^n)$ and $E = \sum_i I^i$.

Each individual decides his or her consumption of the private good and his or her contribution to the public good. We consider the case where each individual simultaneously makes such a decision. Moreover, we assume that the structure of the economy is the common knowledge. So, we analyze the property of Nash equilibria of this game.

3. Nash equilibria and income distribution

Individual i's maximization problem is as follows:

$$\max\ u^i(x^i, G_{-i} + g^i)$$
$$\text{subject to } x^i + g^i \leq I^i,\ x^i \geq 0 \text{ and } g^i \geq 0.$$

Denote by $\left(x^i(I^i, G_{-i}), g^i(I^i, G_{-i})\right)$ the solution of this problem. We can rewrite this maximization problem as follows:

$$\max\ u^i(x^i, G)$$
$$\text{subject to } x^i + G \leq I^i + G_{-i},\ x^i \geq 0 \text{ and } G \geq G_{-i}.$$

We consider the following problem, denoting $Y^i = I^i + G_{-i}$:

$$\max\ u^i(x^i, G)$$
$$\text{subject to } x^i + G \leq I^i + G_{-i} = Y^i \text{ and } x^i \geq 0$$

and denote the solution of this problem by $x^i = \xi^i(Y^i)$ and $G = \Gamma^i(Y^i)$. Since the public good and the private good are normal, $d\xi^i/dY^i > 0$ and $d\Gamma^i/dY^i > 0$ hold. It is easy to show that $x^i(I^i, G_{-i}) = min\{I^i, \xi^i(I^i + G_{-i})\}$ and $g^i(I^i, G_{-i}) = max\{0, \Gamma^i(I^i + G_{-i}) - G_{-i}\}$. For given $I = (I^1, \cdots, I^n)$, a Nash equilibrium $\left((x^{*1}(I), g^{*1}(I)), \cdots, (x^{*n}(I), g^{*n}(I))\right)$ satisfies $x^{*i}(I) = x^i\left(I^i, \sum_{j \neq i} g^{*j}\right)$ and $g^{*i}(I) = g^i\left(I^i, \sum_{j \neq i} g^{*j}\right)$.

For given $I = (I^1, \cdots, I^n)$, we denote by $C(I)$ the set of the players who contribute at the equilibrium. With respect to the relation between income redistribution and Nash equilibria, Bergstrom et al. (1986) proved the following facts.

Proposition 1. Consider $I' = (I^{1\prime}, \cdots, I^{n\prime})$ such that $\sum_i I^{i\prime} = E$, $I^{i\prime} > I^i - g^{*i}(I)$ for any $i \in C(I)$, and $I^{i\prime} = I^i$ for any $i \in N \setminus C(I)$. Then we have $x^{*i}(I') = x^{*i}(I)$ for any $i \in N$ and $\sum_i g^{*i}(I') = \sum_i g^{*i}(I)$.

Proof: See Bergstrom et al. (1986).

From Proposition 1, the income redistribution does not affect each individual's utility. So, the redistributive policies by the government are meaningless.

Itaya et al. (1997) derived the necessity of the redistributive policies by the government in the same model. Itaya et al. (1997) analyzed the effect of income redistributions from income distributions under which one individual contributes and the other individual is indifferent between contributing and not contributing in the case where $n = 2$.

In this paper, we will analyze the same income redistributions. Suppose that only Individual i contributes and the other individuals are indifferent between contributing and not contributing under some income distribution $\hat{I}$. We can easily show that $\hat{I}$ is on the boundary of the set $Z = \{I | g^{*i}(I) > 0 \, for\, any\, i\}$. For given $(t^1, \cdots, t^2)$ with $\sum_{j \neq i} t^j = 1$, denote by $(T^i; t^1, \cdots, t^n)$ an income redistribution from $\hat{I}$. This means that $I^i = \widehat{I^i} + T^i$ and $I^j = \widehat{I^j} - t^j T^i$ for any $j \neq i$ after this redistribution.

Because only Individual i contributes in this case, $G = \Gamma^i(\widehat{I^i} + T^i)$ and $x^j = \widehat{I^j} - t^j T^i$ hold at the equilibrium and Individual j's utility is $u^j\left(\widehat{I^j} - t^j T^i, \Gamma^i(\widehat{I^i} + T^i)\right)$. Since $\hat{I}$ is on the boundary of the set Z, the marginal rate of substitution is equal to 1 ($du^j/dG = du^j/dx^j$) and $du^j/dT^i = (d\Gamma^i/dY^i - t^j)(du^j/dG)$ holds. Next is our main result.

Proposition 2. If $d\Gamma^i/dY^i > 1/(n-1)$ holds, then an income redistribution $(dT^i; t^1, \cdots, t^n)$ with $(t^1, \cdots, t^2) = (1/n-1, \cdots, 1/n-1)$ makes Pareto improvement. If an income redistribution $(dT^i; t^1, \cdots, t^n)$ makes Pareto improvement, then $d\Gamma^i/dY^i > 1/(n-1)$ holds.

Proof: If $d\Gamma^i/dY^i > 1/(n-1)$ holds, then we have $du^j/dT^i = (d\Gamma^i/dY^i - 1/n-1)(du^j/dG) > 0$ for any j. So, the income redistribution $(dT^i; 1/n-1, \cdots, 1/n-1)$ makes Pareto improvement. Suppose an income redistribution $(dT^i; t^1, \cdots, t^n)$ makes Pareto improvement. If $d\Gamma^i/dY^i \leq 1/(n-1)$ holds, then $du^j/dT^i = (d\Gamma^i/dY^i - t^j)(du^j/dG) < 0$ holds for some j or $du^j/dT^i = 0$ holds for any j. This is a contradiction. (Q.E.D.)

Iritani and Yamamoto (2005) showed that $d\Gamma^i/dY^i > 1/(\square - 1)$ is the sufficient condition for the Pareto Improvement. Our proposition proves that this is the necessary and sufficient condition.

Suppose that $n = 2$. Then the necessary and sufficient condition for Pareto improvement is $d\Gamma^i/dY^i > 1$. If the public good and the private good are normal, then we have $d\Gamma^i/dY^i < 1$. So Pareto improvement is not impossible in the case of two individuals.

From this, we can justify the viewpoint of Buchholz and Konrad (1995), Ihori(1996), Cornes and Hartley (2007), etc. They, focusing on difference of productivity between the individuals, analyzed the relation between productivity and transfers of income in the case of two individuals. As shown by Proposition 1, if there is no productivity difference like our model, it is impossible to make Pareto improvement through income redistributions. So they have to focus on the difference of productivity.

Itaya et al. (1997) proved that an income redistribution $\left(dT^i; t^1, \cdots, t^n\right)$ improve the social welfare in the case where $n = 2$. This is true. But, this improvement of social welfare does not imply Pareto improvement because Itaya et al. (1997) considered the case of two individuals.

If there are more than three individuals, circumstances change. In Proposition 1, we derive the necessary and sufficient condition for Pareto improvement. This condition can hold in the case of more than three individuals. So, without difference of productive, there may be some income redistribution that makes Pareto improvement. Under this income redistribution, a contributor receives more income and the others lose income. Based on this result, Itaya et al. (1997) argued that inequality of the income distribution is praised. In the following example, we show that Pareto improvement does not necessarily imply inequality.

Example 1. Let $n = 3$. Each Individual i's utility function is $u^i = \left(x^i\right)^{a^i} G^{b^i}$ where $a^i > 0$ and $b^i > 0$. Let $r^i = a^i/b^i$, $r^1 < r^2 < r^3$, $r^1 < 1$, $r^2 > r^1 + 1$, $r^3 > r^1 + 1$, $I^2 = (a^2/b^2)(b^1/(a^1 + b^1))I^1$ and $I^3 = (a^3/b^3)(b^1/(a^1 + b^1))I^1$. In this example, from $r^2 > r^1 + 1$ and $r^3 > r^1 + 1$, we have $I^2 > I^1$ and $I^3 > I^1$. So Individual 1 is poorest. Under the income distribution (I^1, I^2, I^3), only Individual 1 contributes and the other individuals are indifferent between contributing and not contributing (From $I^j = \left(a^j/b^j\right)(b^1/(a^1 + b^1))I^1$ ($j = 2{,}3$), individual j's marginal rate of substitution is equal to 1 at $\left(I^j, G\right)$ where G is determined by individual 1 ($G = (b^1/(a^1 + b^1))I^1$).). Moreover, $d\Gamma^i/dY^i = b^1/(a^1 + b^1) > 1/2$ holds. So, the income redistribution from Individuals 2 and 3 to Individual 1 makes Pareto improvement. This redistribution equalizes incomes between individuals. Therefore, Pareto improvement consists with equality in this example.

4. The economies with Cobb-Douglas utility functions

In this section, in order to analyze the effect of income redistribution more precisely, we assume that each Individual i's utility function is $u^i = \left(x^i\right)^{a^i} G^{b^i}$ where $a^i > 0$ and $b^i > 0$. Let $r^i = a^i/b^i$ and $R = \sum_i r^i$. We assume that $r^1 \leq \cdots \leq r^n$. For given G_{-i}, $\xi^i\left(I^i + G_{-i}\right) = \left(a^i/\left(a^i + b^i\right)\right)\left(I^i + G_{-i}\right)$ and $\Gamma^i\left(I^i + G_{-i}\right) = \left(b^i/\left(a^i + b^i\right)\right)\left(I^i + G_{-i}\right)$ hold. So, Individual i's decision is $x^i\left(I^i, G_{-i}\right) = min\left\{I^i, \left(a^i/\left(a^i + b^i\right)\right)\left(I^i + G_{-i}\right)\right\}$ and $g^i\left(I^i, G_{-i}\right) = max\left\{0, \left(b^i/\left(a^i + b^i\right)\right)I^i - \left(a^i/\left(a^i + b^i\right)\right)G_{-i}\right\}$.

The income redistribution to Individual i from the distribution on the boundary Z makes Pareto improvement, only if $d\Gamma^i/dY^i > 1/n-1$ holds. For such i, we have $r^i < n-2$. Let $L = \{i | r^i < n-2\}$. This is a set of the candidates of a contributor.

Consider the income distribution $I = (I^1, \cdots, I^n)$ where only Individual i contributes and the income redistribution $(dT^i; t^1, \cdots, t^n)$ from I. Since only Individual i is a contributor, Individual j's utility $(j \neq i)$ is $u^j = (I^j - t^j T^i)^{a^j} \{(b^i/(a^i+b^i))(I^i + T^i)\}^{b^j}$. We have $du^j/dT^i = A \cdot B \cdot (C-D)$ with $A = (I^j - t^j T^i)^{a^j - 1}$, $B = \{(b^i/(a^i+b^i))(I^i+T^i)\}^{b^j-1}$, $C = (I^j - t^j T^i)(b^j b^i/(a^i+b^i))$ and $D = a^j t^j (b^i/(a^i+b^i))(I^i + T^i)$. Set $T^i = 0$. Then, if $(I^j\, b^j b^i/(a^i+b^i)) - (I^i a^j t^j b^i/(a^i+b^i)) > 0$ holds for any j, there is an income redistribution from $I = (I^1, \cdots, I^n)$ making Pareto improvement. In other words, if for any j, $(I^j\, b^j b^i/(a^i+b^i)) - (I^i a^j t^j b^i/(a^i+b^i)) = 0$ holds (and $I^j = t^j r^j I^i$ holds), there is no income redistribution $(dT^i; t^1, \cdots, t^n)$ from $I = (I^1, \cdots, I^n)$ making Pareto improvement. (This condition is one of such conditions. For example, if $(I^j\, b^j b^i/(a^i+b^i)) - (I^i a^j t^j b^i/(a^i+b^i)) < 0$ holds for some j, there is no income redistribution making Pareto improvement.)

For each $i \in L$, we consider the income distribution $I(i) = (I^1(i), \cdots, I^n(i))$ satisfying $I^j(i) = t^j(i) r^j I^i(i)$ for any j. In order to treat each $I(i)$ $(i \in L)$ symmetrically, we assume that $t^j(i) = 1/n-1$ for any i and j. That is, we consider the income distributions $I(i) = (I^1(i), \cdots, I^n(i))$ $(i \in L)$ such that $I^j(i) = r^j I^i(i)/(n-1)$ for any i and j. (Note that we do not necessarily attain this income distribution through the income redistribution with $t^j(i) = 1/n-1$. We should consider this income distribution just as a benchmark.)

In this case, we have $I^i(i) = (n-1)E/(R - r^i + n - 1)$ and $I^j(i) = r^j E/(R - r^i + n - 1)$ for each $i \in L$. Denote by $G(i)$ and $x^j(i)$ the amount of the public good and Individual j's consumption of the private good at the equilibrium under the income distribution $I(i)$. Since only Individual i contributes under $I(i)$, we have $G(i) = (n-1)E/\{(r^i+1)(R - r^i + n - 1)\}$, $x^i(i) = r^i\,(n-1)E/\{(r^i+1)(R - r^i + n - 1)\}$ and $x^i(i) = r^j E/(R - r^i + n - 1)$.

Let $u^j(i)$ be Individual j's utility at the equilibrium under the income distribution $I(i)$. That is, $u^j(i) = (x^j(i))^{a^j} G(i)^{b^j}$ and $u^i(i) = (x^i(i))^{i} G(i)^{b^i}$.

If we want to analyze the social welfare, we have to compare each individual's utility because each individual may have a different preference. But, there is no criterion for the comparison which is unanimously accepted. In follows, we analyze each individual's consumption levels and the social welfare at the equilibrium without comparison of utilities.

Lemma 1. For any $i \in L \setminus \{i\}$, we have $x^1(i) < x^k(i)$ for any $k \neq 1$.

Proof: We have $r^1 E/(R - r^i + n - 1) < r^k E/(R - r^i + n - 1)$. (Q.E.D)

From this Lemma, if the contributor is other than Individual 1, Individual 1's consumption level of the private good is minimal in this society. The consumption level of the public good is common for every individual. So, we can say that Individual 1 is least advantaged if we focus on the consumption level.

Lemma 2. For any $i \in L$ and $j \in L$ such that $i < j$, we have $G(i) > G(j)$ and $x^k(i) < x^k(j)$ for any $k \neq i$ and $k \neq j$.

Proof: We have, letting R and n be constant, $d\{(r^i + 1)(R - r^i + n - 1)\}/dr = R + n - 2 - 2r^i$. From $r^i < n - 2$ for any $i \in L$, we have $d\{(r^i + 1)(R - r^i + n - 1)\}/dr > 0$. So if $r^i < r^j$, we have $G(i) > G(j)$. If $r^i < r^j$, we have $R - r^i + n - 1 > R - r^j + n - 1$ and $x^k(i) < x^k(j)$. (Q.E.D.)

From this Lemma, Individual k consumes more public good and less private good when the contributor's index is small. So we can say there is a trade-off in Individual k's utility as for the identity of contributors and that it is not easy question who should be a contributor.

With respect to each individual's utility, we get the following fact.

Fact 1. For any $i \in L$ and $j \in L$ with $i < j$, we have $u^i(i) > u^i(j)$.

Proof: We have $G(i) > G(j)$ from Lemma 2. Since $r^j < n - 2$, $(r^j + 1)/(n - 1) < 1$ and $((r^j + 1)/(n - 1))^{a^i} < 1$ hold. Noting that $u^i(i) = (r^i)^{a^i}(G(i))^{a^i + b^i}$ and $u^i(j) = (r^i)^{a^i}((r^j + 1)/(n - 1))^{a^i}(G(j))^{a^i + b^i}$, we get $u^i(i) > u^i(j)$. (Q.E.D)

Corollary 1. For any $i \in L \setminus \{1\}$, we have $u^1(1) > u^1(i)$.

When Individual 1 is a contributor, the amount of the public good is largest. From this, the redistribution to Individual 1 can be justified. In addition, as mentioned before, when the contributor is not Individual 1, Individual 1 is least advantaged if we focus on the consumption level. When Individual 1 is a contributor, Individual 1's utility is highest. Advocates of Rawls' Difference Principle are arguing that we should change our institutions to improve the life prospects of the least advantaged in society (Rawls (1971)). From this viewpoint, the redistribution to Individual 1 may be justified.

From Lemma 1, if the contributor is other than Individual 1, Individual 1's consumption level of the private good is minimal. But, if Individual 1 is the contributor, Individual

1's consumption level is not always minimal. See the following example.

Example 2. Let $n = 3$. Suppose that $r^1 < r^2 < 2r^1/(r^1 + 1)$. Then, $x^1(1) > x^2(1)$ holds. So if Individual 1 is the contributor, Individual 2 is least advantaged. When the contributor is not Individual 1, Individual 1 is least advantaged. The utility of the least advantaged individual in each $I(i)$, is $u^1(2)$, $u^1(3)$ or $u^2(1)$. Based on comparison among these utilities, there may be the case where we should not choose $I(1)$ and the contributor should be other than Individual 1. Moreover from Lemma 2, $x^1(2) < x^1(3)$ holds. So $u^1(2) < u^1(3)$ can hold. Then, if we should improve the least advantaged individual, Individual 3 should be the contributor in some case. That is, in this case, Rawls' Difference Principle justifies the redistribution to the most advantaged individual.

In the above analysis, we focus on the least advantaged individual. In the most cases, it is justified to transfer to the least advantaged individual. In the example 2, however, transferring to the other individual is more favorable from the same viewpoint.

Next, we focus on another individual's welfare, especially the most advantaged individual's welfare. This is exact opposite of the criterion of Rawls.

Example 3. Let $n = 3$. Suppose that $r = r^1 < r^2 = r^3 = 1$ and $a^3 = b^3 = 1$. Then $\square^3(3) = (E/(3 + r))^2$, $u^3(2) = (E/(3 + r))^2$ and $u^3(1) = (E/4)^2(2/(r + 1))$. As r goes to zero, we have $u^3(1) > u^3(2) = u^3(3)$.

From this example, there is the case where transferring to the least advantaged individual improves the most advantaged individual.

5. Concluding Remarks

In this paper, we derive the necessary and sufficient condition for Pareto improvement in the models of voluntary contribution of the public good. Contrary to existing intuition among most scholars, this condition has no direct relation to income inequality. In fact, we present the example where income equality consists with efficiency. (Dasgupta (2009) showed that income equality consists with efficiency assuming that the public good is impure. In this paper, such impurity is not assumed.)

Moreover, assuming that each individual's utility function is Cobb=Douglas function, we derive the income distributions that have no room for further Pareto improvement. Then, we analyze each individual's consumption levels and utility at the equilibrium from the viewpoint that is similar to Rawls' Difference Principle. Some of our results are as follows: In most cases, it is justified to transfer to the least advantaged individual; in some cases, transferring to the other individual is more desirable from the same viewpoint; transferring to the least advantaged individual may improves the most advantaged individual.

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