

*Call for a Change in Mathematics education: From Platonism to Social
constructivism*

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Abstract

At school, children are expected to become numerate in order to be able to function in a modern technological society and contribute to the growth of its economy. However, one of the most frequent complaints of mathematics teachers is that “forgetting is particularly common for knowledge acquired in school, and much of this material is lost within days or weeks of learning” (Rohrer & Taylor, 2006, p. 1209).

In mathematics education, as Renert (2011) noted, influenced significantly by Platonism, early mathematics was popularly viewed as consisting of abstract mathematical objects, which have no causal properties linking them to their environment. Social constructivists challenged Plato’s assumptions about mathematics for ruling out social dimensions in its teaching and learning. They argued that mathematics is the theory of form and structure that arises within language (Zakaria & Iksan, 2007) and that mathematics learning acquires an alignment with its cultural practices through communicative practices or dialogic interactions (Cobb & Bauersfeld, 1995).

Thus, in this paper, we present a theoretical synthesis of the specialized literature in the learning and teaching of mathematics, with the aim of calling for a change in mathematics education from Platonism to social constructivism. As stated by Vygotsky (1978, p. 90): “[procedure-oriented learning] does not aim for a new stage of the developmental process, but rather lags behind this process”, we argue that mathematics teaching and learning cannot afford to continue with the “teaching to the test” culture.

Keywords: mathematics education; Platonism; social constructivism

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Introduction

There is no doubt that 21st century students are in the information age, which presents different challenges to them from those students who were born in the 20th century industrial age. The successful transition to the information age from the industrial age requires to address specific challenges in the area of education and learning. In particular, schools need to help students develop 21st century learning skills such as critical thinking, creative thinking, collaborating and communicating. These skills have always been important for students, though they are particularly important in our information-based economy. For example, to hold information-age jobs, employees need to think deeply about issues, solve problems creatively, work in teams, communicate clearly, learn new technologies and deal with a flood of information. The rapid changes in our world require employees to be flexible, to take the initiative and lead when necessary and to produce something new and useful.

As Hiebert and Grouws (2007, p. 373) noted, “pressures are increasing to provide evidence-based descriptions of effective mathematics teaching”. However, they also remarked that:

Different teaching methods might be effective for different learning goals [...]. Some methods of teaching are more effective for, say, memorizing number facts, whereas other methods of teaching are more effective for deepening conceptual understanding and still other methods are more effective for acquiring smooth execution of complex procedures” (*ibid*, p. 374).

A number of researchers consider that mathematics teaching, by nature of its objectivity that was significantly influenced by Platonism, is inevitably restricted by one dominant view of mathematics relating to an objectivist stance (Kirschner, Sweller, & Clark, 2006; Muijs & Reynolds, 2001). Burton (1995, p.276) stated that adopting an objectivist stance means that “mathematical “truths” exist and the purpose of education is to convey them into the heads of the learners”. Such mathematics “absolutism” has been questioned by many researchers over the past two decades. For instance, Fan and Bokhove (2014) argued, “one should not classify knowledge or cognitive activities into different levels without taking into account contextual factors”.

Constructivist perspectives on learning and teaching have been a popular topic among mathematics educators, psychologists and researchers (Cobb et al., 1991; Francisco, 2013; Levenson, 2013; Li & Tsai, 2017; Tall, 2011; Wood & Sellers, 1997) and as a result, have contributed to shaping mathematics reform efforts in many countries around the world (e.g. Australia, Queensland Studies Authority, 2004; Brunei, Ministry of Education, 2009; The Netherlands, Van den Heuvel-Panhuizen, 2000; USA, National Council of Teachers of Mathematics, 2000; UK, Department for Education and Employment, 1999; Taiwan, Ministry of Education, 1993).

Vygotsky’s theory of social constructivism

The widespread interest in constructivism has led to many different meanings of the term. Indeed, in practice, “constructivism is not a singular theory, but a family of

related theories that are not always seen as compatible” (Efran, McNamee, Warren, & Raskin, 2014, p. 1). However, as Anthony (1996, p. 349) stated, “an important tenet of constructivism is that learning is an idiosyncratic, active and evolving process”. Pirie and Kieren (1992) also stressed that in line with constructivist perspectives, students themselves have to construct the skills and concepts of knowledge, rather than being taught by teachers.

In Piaget’s theories, children’s learning is considered from the biologist’s perspective. One of his assertions was that young children were egocentric and unable to see a situation from another’s point of view (May, 2013). Vygotsky’s idea of “social constructivism” (1978) challenged some of Piaget’s theories. Vygotsky argued that, while children did make sense of the world individually, they did not, as Piaget asserted, do it alone. He believed that children learned the world socially through the adults around them – or, as Smidt (2013) called them, “the expert others”. In this respect, communication and language are seen to be central to successful learning.

Vygotsky proposed the concept of “Zone of Proximal Development” (ZPD) and considered that a child’s cognitive development was associated with both its actual development level and potential level. The actual level can be measured by observing a child’s independent problem solving ability without any guidance or help, such as by way of a static standard testing approach. Its potential level can be observed after a child has been guided on how to perform. Potential development thus becomes actual development after the process of guidance by a more competent individual. The social communication of learners and teacher, therefore, is essential in negotiating the co-construction of a ZPD (Wertsch, 2007). As Vygotsky (1978, p. 212) argued, “this is what distinguishes instruction of the child from the training of animals”. However, unfortunately, as Moll (2014) noted, most classrooms nowadays exist, more or less in isolation, which Dewey (1980, p. 39) criticised strongly when he stated that, “all waste [in education] is due to isolation”.

As Lerman (2000) and May (2013) pointed out, Vygotsky’s social constructivism and von Glasersfeld’s radical constructivism had different perspectives on how children think and learn, with each suggesting different approaches to guiding their learning. Such a distinction between radical and social constructivism, “when seen as a dichotomy, is productive” (Lerman, 2000, p. 210). In line with Lerman (2000) and May (2013), we briefly discuss the differences between these two constructivist traditions, and then we describe a growing interest in social constructivism in mathematics education.

In general, to radical constructivists, “all understanding and all communication is a matter of interpretative construction on the part of the experiencing subject” (Olssen, 1996, pp. 276-7). This definition sees the human cognition as a closed system, in that “things don’t get in and they don’t get out” (Efran et al., 2013, p. 3), which means that people can be “triggered to learn”, but what they do with what they learn lies within their internal structure. Also, “radical constructivism considers absolute meanings for words unattainable” (Loria, 1995, p. 156), suggesting that learners speak their own, as well as private, languages; and also their personal histories influence them to create unique meanings, even though the words they use may be familiar ones.

By contrast, social constructivists highlighted the important role the “expert other”

plays in the learning process. “Expert others” were not simply to await children’s readiness, but to intervene in order to support children towards further stages of understanding (Smidt, 2013). Such social interactions represent the “primacy of relational, conversational, social practices [which are] the source of individual psychic life” (Stam, 1998, p. 199). As Ernest (1998) stated, children will not develop the social meaning of important symbol systems and the ways to use them if they are not provided with a social situation of development. In essence, therefore, interaction and collaboration is seen as a crucial tool to help a learner’s potential cognitive development to become actual development (Wood & Sellers, 1997).

In mathematics education, as Renert (2011) noted, influenced significantly by Platonism, early mathematics was popularly viewed as consisting of abstract mathematical objects, which have no causal properties linking them to their environment. Hence, social constructivists challenged Plato’s assumptions about mathematics for ruling out social dimensions in its teaching and learning. They argued that mathematics is the theory of form and structure that arises within language (Ernest, 1991; Zakaria & Iksan, 2007) and that mathematics learning acquires an alignment with its cultural practices through communicative practices or dialogic interactions (Cobb & Bauersfeld, 1995; Wertsch & Toma, 1995).

Conclusion

Problem solving has also been a focus in mathematics education for many years (Fan & Zhu, 2007; Lesh, & Zawojewski, 2007; Singer & Voica, 2013). Not only is teaching students to solve problems important to them learning mathematics, learning about mathematics through problem solving is equally important (Ontario Ministry of Education, 2005). However, at school, problem solving is often based on a narrower spectrum of story or word problems, which function more like an exercise for students to perform, rather than a challenge for them to solve real-life situation problems (Singer & Voica, 2013).

The aspect that emphasises a focus on classroom talk to encourage students to construct concepts, rather than being taught by their teachers, is not new (Alexander, 2008; Gillies, 2014; Hennessy, 2014). Many researchers in mathematics education by using a variety of approaches, have made many contributions towards providing models to link students’ procedural and conceptual understanding (Fan & Bokhove, 2014; Li & Stylianides, in press; Saxe, Diakow, & Gearhart, 2013).

Mathematics education has often been described as “being at war” in the US; for instance, the “math wars” waged over the Curriculum and Evaluation Standards for School Mathematics in the US in 1989 are prominent examples of bitter disagreements over teaching approaches. However, more recently, the idea of knowledge being constructed by a learner, rather than transmitted by a teacher, has become a widely accepted position in mathematics education (Brough & Calder, 2012; Schoenfeld & Kilpatrick, 2013; Schukajlow et al., 2012; Zazkis, 2011). For example, Schukajlow et al. (2012) compared teacher- and student-centred programmes for teaching the solving of modelling problems to students aged between 14 and 15 in Germany and found that student-centred teaching methods improved both students’ achievements and enjoyment. Robison (2012) called for further research on designing accessible resources to support student-centred approaches in mathematics teaching

since she was concerned that this kind of approach could be challenging, especially in mathematics, due to the difficulty of producing materials in a format that could be adapted for students with a range of additional needs.

In many countries, “teaching to the test” was a common phenomenon, which made mathematics teaching “instructive-oriented” (Li & Tsai, in press; Schoenfeld & Kilpatrick, 2013; Tsai & Li, in press). Indeed, teachers can impart knowledge of many skills and strategies to students. However, unless their students have been actively, and personally, involved in planning, monitoring and reviewing their own learning, the skills and strategies imparted to the students may not be effective. Consequently, in line with the findings in the literature (Ball, 2009; Goos, 2004; Moll, 2014; Webb, 2009), we argue that, when the role of teachers is not a deliverer of pre-packed knowledge, but as a member in a community of learners, a genuine, trusting learning relationship between teachers and students develops. As Webb (2009, p. 21) argued, “changes occurred as a result of the teacher learning how to listen to students and [by] relinquishing control over the students’ methods”. Hence, teachers need to develop awareness that, if their students’ initial understanding is not meaningful, it will not be possible for them to grasp new concepts and information, or they may learn just enough to pass a test, but fail to apply their knowledge outside the classroom (Bransford, Brown, & Cocking, 1999).

This paper by no means claims to be a fully comprehensive study of how mathematics education ought to change in this information age. However, its findings may offer relevant information regarding the teaching and learning of mathematics in a classroom context. Ongoing research into mathematics classroom practice will no doubt contribute further to an understanding of this highly complex and demanding area of education. This would also require deep reflection on the part of researchers and teachers as to how teaching strategies may be adopted by teachers to develop their students’ mathematical knowledge in particular, and meaningful understanding in general.

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